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MODELLING AND COMPUTER SIMULATION OF A TRAILER TRUCK MOVEMENT ALONG INCLINED PLANE

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Abstract: The paper presents mechanomathematical model of a trailer truck in motion after a lateral stability loss in a crash, turn or stopping taking into consideration the redistribution of normal reactions in the wheels, wheels rotation and the variable character of the coefficient friction between the tyres and the asphalt. *Expertcar* computer simulation programme has been created in *Matlab* to reconstruct the motion of a body consisting of two vehicles or one vehicle in a given road accident.

AMS Subject Classification: 68U20, 65C20, 00A72
Key Words: accident reconstruction, road accident, vehicle, trailer truck, motion, coefficient of friction, friction forces, collision, computer simulation, stability

1. Introduction

The study of road accidents with trailer trucks in specific conditions is a highly complicated process due to its complex dynamics compared to an ordinary vehicle. Works [1], [2] show a mechanomathematical model of a single vehicle moving along inclined plane with blocked wheels. Computer simulation of the motion has been created based on the model. It identifies the vehicle motion

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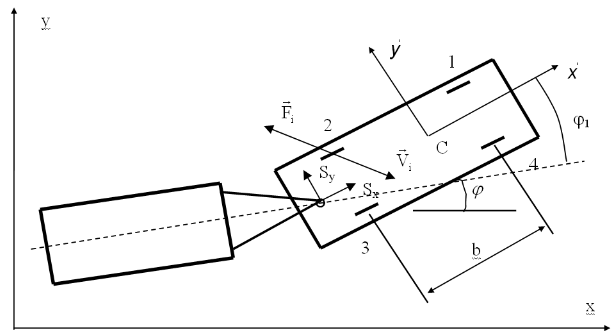

Figure 1: Scheme of a mechanical system consisting of a truck and trailer

on the left-over tyre traces (*Expertcar* computer programme). The same model solves the problem of a crash between two vehicles.

The aim of the paper is to create a mechanomathematical model of a trailer truck analysing the wheels rotation and the motion of the staff as well as identifing the driver's actions.

2. Mechanomathematical Model

Figure 1 shows a scheme of the mechanical system consisting of two-axis trailer truck.

Setting up the differential equations of the movement of the mechanical system the following assumptions have been made:

1. It has been accepted that the motion of the wheels of the trailer truck is carried along a random inclined plane with angles of the inclination of axes x and y according to the horizon, respectively α and β.

2. The motion of a given vehicle is accepted to be possible when three of the wheels are separated from the road but the motion of the two bridges remains close to the planar.

3. The motion neglects the least possible oscillations of the mechanical system round its dynamic equilibrium. The grounds for that is the negligible small loss of kinaesthetic energy in mechanical work of the interior resistant and elastic forces at the small oscillations of the underspring masses in comparison to the total work of friction forces between the tyres and the road macadam. The assumption is real and is based on the negligent influences of the side

displacement of the mass centre of the underspring masses in their plane of motion upon the value of normal reactions in the wheels.

4. The mechanical relation between the tyres and the road macadam is based on the model of friction circle.

Summarised coordinates of the mechanical system are: x and y are the coordinates of the mass centre of the tug; ϕ is the twirl angle; the angle between the axes of the tug and the trailer is ϕ_1 and the twirl angles of the separate wheels are γ_i $(i = 1 \div 8)$.

The differential equations of the motion of the transport composite are the following:

$$m\ddot{x} = \sum_{i=1}^{4} \left[\mathrm{F_{ix}}\right] + \mathrm{mg.sin}\alpha + \mathrm{S_x}\,, \tag{1}$$

$$m\ddot{y} = \sum_{i=1}^{4} \left[\mathrm{F_{iy}}\right] + \mathrm{mg.sin}\beta + \mathrm{S_y}\,, \tag{2}$$

$$I\ddot{\varphi} = \sum_{i=1}^{4} \left[\begin{array}{c} \mathrm{F_{iy}}(x_i' \cos\varphi - y_i' \sin\varphi)\text{ -} \\ \text{- }\mathrm{F_{ix}}(x_i' \sin\varphi + y_i' \cos\varphi) \end{array}\right] \text{ - } \mathrm{S_y.l_s}\,, \tag{3}$$

$$m_1\ddot{x}_1 = \sum_{i=5}^{8} \left[\mathrm{F_{ix}}\right] + \mathrm{m_1 g.sin}\alpha \text{ - } \mathrm{S_x cos}\varphi_1\,, \tag{4}$$

$$m_1\ddot{y}_1 = \sum_{i=5}^{8} \left[\mathrm{F_{iy}}\right] + \mathrm{m_1 g.sin}\beta - \mathrm{S_y.cos}\varphi_1\,, \tag{5}$$

$$\begin{array}{l} I_{\mathrm{r}}(\ddot{\varphi} + \ddot{\varphi}_1) = \sum_{i=5}^{8} \left[\begin{array}{c} \mathrm{F_{iy}}.[x_i' \cos(\varphi+\varphi_1) - y_i' \sin(\varphi+\varphi_1)]\text{ -} \\ \text{- }\mathrm{F_{ix}}.[x_i' \sin(\varphi+\varphi_1) + y_i' \cos(\varphi+\varphi_1)] \end{array}\right] \\ +\mathrm{S_x sin}\varphi_1.\mathrm{l_c} \text{ - } \mathrm{S_y cos}\varphi_1.\mathrm{l_c}\,, \end{array} \tag{6}$$

$$\begin{array}{l} \mathrm{I_i}\ddot{\gamma}_{\mathrm{i}} = \mathrm{F_{i\tau}.R_i} + \mathrm{sign}(\dot{\gamma}_{\mathrm{i}}).\left[\text{ - }\mathrm{f_i.N_i}\text{ - }\mathrm{M_{is}}\right], \\ \mathrm{M_is} = \mathrm{k_i.M_{is\ max}}\,, \end{array} \tag{7}$$

where m and m_1 are the masses of the tug and the trailer respectively; I and I_r - inertial moments of the tug and the trailer according to the central axis perpendicular to the plane of motion; α, β - angles of the inclined road on axes x and y, respectively, positive at going down and negative at going up; N_i (i=1...8) - normal reaction in the wheels; x_i' ,y_i'– coordinates of the wheel centres in a mobile coordinate system unchangeably connected to the given vehicle; S_x, S_y - projections of the articular force applied to the tug; l_s, l_c - distances of mass centres of the tug and the trailer respectively to the point of suspension of the trailer; $\vec{F}_{\mathrm{i}\tau}$- friction forces in the wheels; I_i - applied inertial moments of the

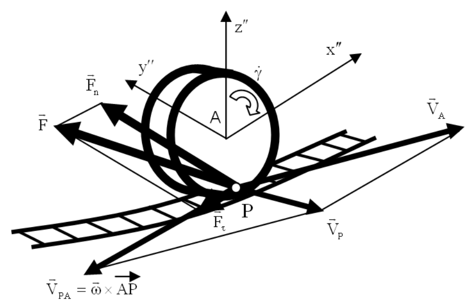

Figure 2: Forces acting on the wheels

wheels to their rotation axes; $\vec{F}_{1\tau}$ - tangent components of friction forces; R_i - dynamic radiuses of the wheels; f_i - friction coefficients in rolling; M_{is} - braking moments on the wheels; k_i - coefficients characterizing real braking moments according to the maximal ones M_{ismax}.

Figure 2 shows the acting forces at any of the wheels in compliance with the plan of speeds of the central point from the contact tyre spot.

The system of differential equations is enriched with two more systems of equations for each vehicle, as follows:

$$b.N_1 + b.N_4 = -m.(\ddot{x}\cos\varphi + \ddot{y}\sin\varphi).h_c + I_{x'z'}.\dot{\varphi}^2 - \mathrm{G_{z'}}.x'_2 \text{ - } \mathrm{G_{x'}.h_c} + \mathrm{mom_1(S_x,S_y)}\,, \quad (8)$$

$$-2.y'_1.N_1 - 2.y'_2.N_2 = m.(-\ddot{x}\sin\varphi + \ddot{y}\cos\varphi).h_c + I_{x'z'}\ddot{\varphi} - \mathrm{G_{z'}}.y' \text{ - } \mathrm{G_{y'}.h_c} + \mathrm{mom_2(S_x,S_y)}\,, \quad (9)$$

$$N_1 + N_2 + N_3 + N_4 = \mathrm{G_{z'}}\,, \quad (10)$$

$$\frac{N_1}{c_1} - \frac{N_2}{c_2} + \frac{N_3}{c_3} - \frac{N_4}{c_4} = 0\,. \quad (11)$$

Here:

$$\mathrm{G_{x'} = G_x.cos}\varphi + \mathrm{G_y.sin}\varphi\,, \quad \mathrm{G_{y'}} = -\mathrm{G_x.sin}\varphi + \mathrm{G_y.cos}\varphi\,, \quad \mathrm{G_{z'} = G_z}\,,$$

$$\mathrm{G_x = mg.sin}\alpha\,, \quad \mathrm{G_y = mg.sin}\beta\,, \quad \mathrm{G_z} = -\mathrm{mg}.\sqrt{1 \text{ - } (\sin^2\alpha + \sin^2\beta)}\,,$$

where h_c is the height of the mass centre of the given vehicle; $I_{x'z'}$ - the centrifugal inertial moment; b-longitudinal base; c_i ($i = 1 \div 4$) applied vertical elastic constants of tyre suspension.

Equations (8) and (9) $\mathrm{mom}_{1,2}(\mathrm{S}_x,\mathrm{S}_y)$ are functions of the projections of the articular force characterizing its moment according to the characteristic axes of the relevant vehicle. The tug accomplishes the following functions:

$$\mathrm{mom}_1(\mathrm{S_x,S_y}) = \mathrm{S_x.h_0}\,, \quad \mathrm{mom}_2(\mathrm{S_x,S_y}) = -\mathrm{S_y.h_0}\,, \tag{12}$$

while the trailer:

$$\begin{aligned} \mathrm{mom}_1(\mathrm{S_x,S_y}) &= (-\mathrm{S_x}.\cos\varphi_1 - \mathrm{S_y}.\sin\varphi_1).\mathrm{h_0}\,, \\ \mathrm{mom}_2(\mathrm{S_x,S_y}) &= (-\mathrm{S_x}.\sin\varphi_1 + \mathrm{S_y}.\cos\varphi_1).\mathrm{h_0}\,, \end{aligned} \tag{13}$$

where h_o is the height of the articulation.

The projections of the friction forces in equations (1)-(6) for each wheel taking into consideration the friction wheel model are defined as follows:

$$\mathrm{F_x} = \text{-}\mu.\mathrm{N}.\frac{\mathrm{V_{Px}}}{\mathrm{V_P}}\ ;\ \mathrm{F_y} = \text{-}\mu.\mathrm{N}.\frac{\mathrm{V_{Py}}}{\mathrm{V_P}}\,, \tag{14}$$

where the projections of contact-spot speed are as follows:

$$V_{\mathrm{P}_x} = \text{ - }\mathrm{R}.\cos\varphi_\mathrm{S}.\dot\gamma + \mathrm{V_{Ax}}\,, \quad V_{\mathrm{P}_y} = \text{ - }\mathrm{R}.\sin\varphi_\mathrm{S}.\dot\gamma + \mathrm{V_{Ay}}\,. \tag{15}$$

The projections of the speed of the wheel centres in (15) are defined according to the speed distribution rule based on the formulae:

$$\begin{aligned} V_\mathrm{Ax} &= \dot x - \dot\varphi.(x'\sin\varphi + y'\cos\varphi)\,, \\ V_\mathrm{Ay} &= \dot y + \dot\varphi.(x'\cos\varphi - y'\sin\varphi)\,. \end{aligned} \tag{16}$$

The tangential component of the friction forces in equation (17) is derived from the expression:

$$\mathrm{F}_\tau = \mu.\mathrm{N}\frac{\mathrm{V_{Px}}}{\mathrm{V_P}}\cos(\varphi+\theta) + \mu.\mathrm{N}\frac{\mathrm{V_{Py}}}{\mathrm{V_P}}\sin(\varphi+\theta), \tag{17}$$

where $\mu(\mathrm{V}_P)$ is a friction coefficient depending on the speed of the contact spot sliding V_P (Figure 3), introduced graphically or analytically; θ(t) is the of swirl of the corresponding controlled wheel around its axis z", given graphically and dependent on time (for the rear wheels $\theta = 0$).

Thus, we get a system of 22 differential equations with indefinite motion law x=x(t), y=y(t), $\varphi = \varphi$(t), $\varphi = \varphi_1$(t), γ_i(t) ($i = 1 \div 8$), normal reactions N_i=N_i(t) and projections of the articulate force S_x(t), S_y(t).

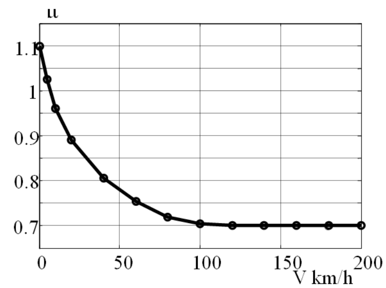

Figure 3: Friction coefficient depending on the sliding speed of the contact point tyre on a dry asphalt

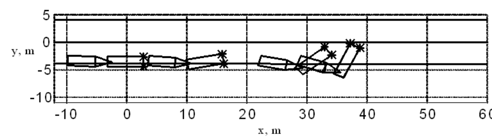

Figure 4: Motion of the road composites at a loss of stability

3. Problem Solution

The system of differential equations is solved by means of *Matlab* programme product where the algorithm is realized by a series of subprogrammes, representing specilized m-files. Based on the mechanomathematical model expertcar computer programme has been updated [1, 2] introducing additional function blocks.

Case 1. Figure 4 shows the motion of a trailer truck, where the right wheels are on the banquet and the driver attempt to stop the vehicle. The masses of the tug and the trailer are m = m_1= 6000 kg. The asymptotic coefficient of friction on the asphalt is accepted to be constant μ=0,6, while on the banquet it is μ=0,4.

Figure 5 represents the summed up speeds of the mechanical system; Figures 6 and 7 show the normal reactions of the tug and the trailer respectively. The projections of the articulate force are given in Figure 8.

Case 2. In ordinary vehicle having a mass of 1550 kg and inertial moment I of 1990 $kg \times m^2$ along a horizontal road after a left turn leaves straight away even the slow lane and crashes its right side into a road tree. While in motion the coefficient of friction between the tyres and the road macadam is accepted to be a variable and the asymptotic coefficient of longitudinal friction on the

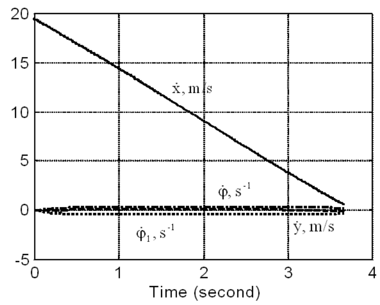

Figure 5: Summed up speeds of the mechanical system

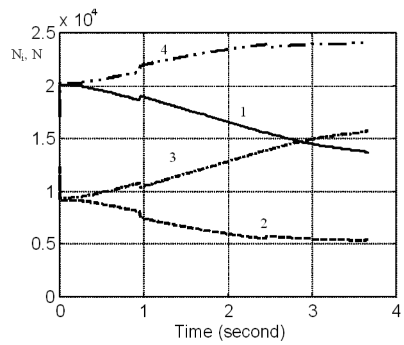

Figure 6: Normal reactions of the tug

asphalt is: μ=0,7, while on the banquet it is : μ=0,4. The front right wheel has blocked due to suspension deformation and the rear right wheel has 50% load of max braking moment because of suspension deformation and the mud guard.

Figure 9 shows a computer simulation of the vehicle in motion after a tree crash realized by *Expertcar* computer program.

The summarized coordinates of the vehicle after the crash, i.e. the coordinates of its mass centre and the twirl angle in relevance to the road axis are as follows:

$$x_1 = 12,88m\,,\quad y_1 = 2,62m\,,\quad \varphi_1 = 59^0.$$

The initial conditions of the vehicle final position are the following:

$$\mathrm{x}_0 = -0,3\ \mathrm{m}\,, y_0 = -1,8\ \mathrm{m}\,,\quad \varphi_0 = 45^0\,,$$

$$V_{0\mathrm{x}} = 11,39\ \mathrm{m/s} = 41,0\ \mathrm{km/h}\,,\quad V_{0\mathrm{y}} = 4,92\ \mathrm{m/s} = 17,7\,\mathrm{km/h}\,,$$
$$\omega_{0\mathrm{z}} = 4{,}58\ \mathrm{s}^{-1}\,.$$

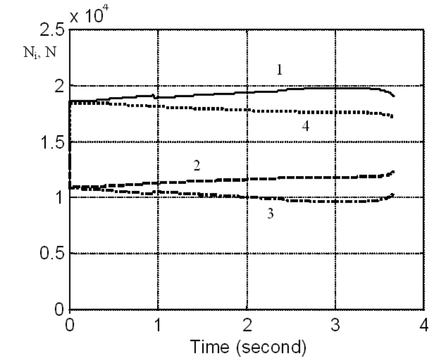

Figure 7: Normal reactions of the trailer

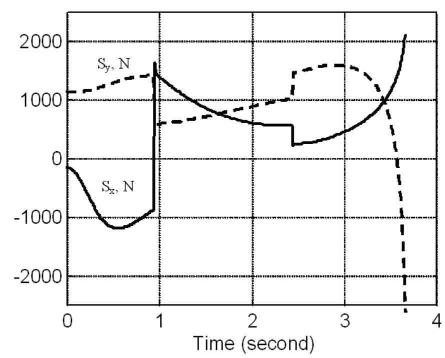

Figure 8: Projections of the articulate force

Figures 10-12 show the main results derived from the computer simulation.

Computer simulation results and mechanics rules provide us with the speed of the vehicle before the crash into the tree. It is as follows:

$$V_0 = 17,70 \text{ m/s} = 63,7 \text{ km/h}.$$

4. Conclusion

The aforementioned study has a wide application in identifying road accidents when there is stability loss with different vehicles, when two vehicles crash or when there is a crash into an obstacle. Computer simulation allows determining

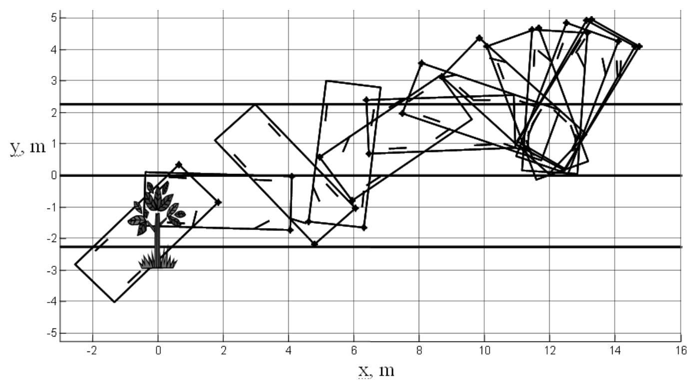

Figure 9: Computer simulation of a vehicle movement after a tree crash (positions in every 0,2 s interval)

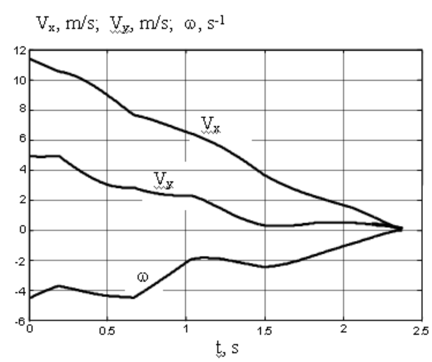

Figure 10: Mass centre speed projections after the crash and its angular speed

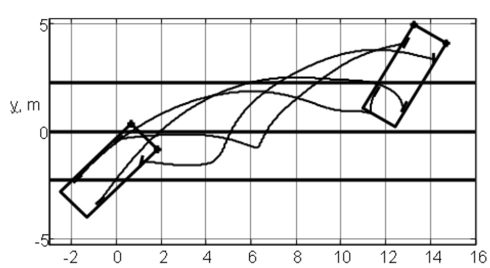

Figure 11: Wheel-centre trajectories

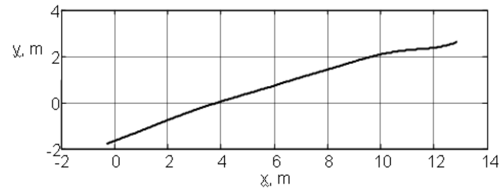

Figure 12: Vehicle mass centre trajectory after

of the initial motion conditions as well as the speed of the vehicles before the accident. The carried out mechanomathematical model has proved its validity and reliability in identifying a great deal of road accidents.

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