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ON THE VERTEX-DISTINGUISHING EDGE COLORING OF $S_n \vee K_{n,n}$

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Abstract: Let $G(V,E)$ be a connected graph. A k-proper edge coloring f of $G(V,E)$ is said to be a k-vertex-distinguishing edge coloring iff $C(u) \neq C(v)$ for $\forall u,v \in V(G)$, $u \neq v$, where $C(u) = \{f(uv)|uv \in E(G)\}$; and $\chi'_{vd}(G) = \min\{k|\text{there exists a } k\text{-VDEC of } G\}$ is called the vertex-distinguishing edge chromatic number. In this paper, we obtain the vertex-distinguishing edge chromatic number of the join graphs $S_n \vee K_{n,n}$.

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1. Introduction

It is a very hard to solve the vertex-distinguishing edge coloring (or strong coloring) of graphs introduced from the theory of network. It is also hard to solve the adjacent strong edge coloring. In this paper, we study the vertex distinguishing edge coloring of the graphs $S_n \vee K_{n,n}$.

Definition 1.1. (see [2]) Let $G(V,E)$ be a connected graph. A k-proper edge coloring f of $G(V,E)$ is said to be a k-vertex-distinguishing edge coloring (abbreviated as k-VDEC) iff $C(u) \neq C(v)$ for $\forall u,v \in V(G)$, $u \neq v$, where

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$C(u) = \{f(uv)|uv \in E(G)\}$; and $\chi'_{vd}(G) = \min\{k|\text{there exists a } k\text{-VDEC of } G\}$ is called the vertex-distinguishing edge chromatic number.

Definition 1.2. Let $G(V,E)$ be a simple graph and n_i the number of vertices with degree i. The combinatorial degree of graph G is defined as

$$\mu(G) = \max\left\{\min\left\{\lambda\middle|\binom{\lambda}{i} \geq n_i, \delta \leq i \leq \Delta\right\}\right\},$$

where δ and Δ denote the minimum and maximum degree of graph $G(V,E)$, respectively.

Conjecture 1.1. (see [2]) *For a connected graph $G(V,E)$ with $|V| \geq 3$, we have*

$$\mu(G) \leq \chi'_{vd}(G) \leq \mu(G) + 1.$$

It is clear that the left hand inequality of the Conjecture 1.1 is trivial.

Let G and H be two simple graphs. The join graph (see [1]) of the graphs G and H, which is denote by $G \vee H$, is such a graph that $V(G \vee H) = V(G) \bigcup V(H)$ and

$$E(G \vee H) = E(G) \bigcup E(H) \bigcup \{uv|u \in V(G) \text{ and } v \in V(H)\}.$$

In this paper, the general result with respect to the vertex-distinguishing edge chromatic number of $P_m \vee P_n$ is studied. The undefined terminologies and notations in this paper are refereed to references [1].

2. Main Results

Lemma 2.1. *For a connected graph $G(V,E)$ with $|V| \geq 3$, we have*

$$\mu(G) \leq \chi'_{vd}(G).$$

Lemma 2.2. (see [3], [4]) *For the complete graph K_n, we have*

$$\chi'_{vd}(K_n) = \begin{cases} n+1, & \text{if } n \equiv 0(\bmod\ 2); \\ n, & \text{otherwise.} \end{cases}$$

Lemma 2.3. (see [3], [4]) *For the balanced complete bipartite graph $K_{n,n}$, we have*

$$\chi'(K_{n,n}) = n.$$

Theorem 2.1. *For $S_n \vee K_{n,n}$, then*

$$\chi'_{vd}(S_n \vee K_{n,n}) = \begin{cases} 5, & \text{if } n = 1; \\ 3n, & \text{if } n \geq 2. \end{cases}$$

Proof. We easily know $\mu(S_N \vee K_{n,n}) = 3n$. Suppose that

$$V(S_n) = \{w_i | i = 0, 1, 2 \cdots, n\}, \quad E(S_n) = \{w_0 W_i | i = 1, 2 \cdots, n\},$$

$$V(K_{n,n}) = \{v_i | i = 1, 2 \cdots, n\} \cup \{u_i | i = 1, 2 \cdots, n\},$$

and $C = \{\alpha_0, \alpha_1, \cdots, \alpha_{n-1}, \beta_0 \beta_1, \cdots, \beta_{n-1}, \gamma_0 \gamma_1, \cdots, \gamma_{n-1}\}$, $\overline{C}(y) = C \backslash C(y), y \in V(S_n \vee K_{n,n})$.

Case 1. If $n = 1$, then

$$S_1 \vee K_{1,1} = K_4 .$$

It is ture by Lemma 2.2.

Case 2. If $n \geq 2$, due to Lemma 2.1, we just prove that there exists a $3n$-VDEC of the graph $S_n \vee K_{n,n}$.

Let f be as follows:

Reffering to Lemma 2.3, we color properly $K_{n,n}$ with $\gamma_0, \gamma_1, \cdots, \gamma_{n-1}$:

$f(w_i v_j) = \alpha_{i+j-1(mod\ n)}, \quad i = 0, 1, \cdots, n-1; \quad j = 1, 2, \cdots, n.$
$f(w_n v_i) = \beta_{i-1}, \quad i = 1, 2, \cdots, n.$
$f(w_i u_j) = \beta_{i+j-1(mod\ n)}, \quad i = 0, 1, \cdots, n-1; \quad j = 1, 2, \cdots, n.$
$f(w_n u_i) = \alpha_{i-1}, \quad i = 1, 2, \cdots, n.$
$f(w_0 w_i) = \gamma_{i-1}, \quad i = 1, 2, \cdots, n.$

For f, we have:

$\overline{C}(w_0) = \emptyset, \overline{C}(w_i) = \{\gamma_0, \gamma_1, \cdots, \gamma_{i-2}, \gamma_i, \gamma_{i+1} \cdots, \gamma_{n-1}\}, i = 1, 2 \cdots, n.$
$\overline{C}(v_1) = \{\beta_1, \beta_2 \cdots, \beta_{n-1}\}$
$\overline{C}(v_i) = \{\beta_0, \beta_1 \cdots, \beta_{i-2}, \beta_i, \beta_{i+1} \cdots \beta_{n-1}\}, i = 2, 3 \cdots, n.$
$\overline{C}(u_1) = \{\alpha_1, \alpha_2, \cdots, \alpha_{n-1}\}.$
$\overline{C}(u_i) = \{\alpha_0, \alpha_1 \cdots, \alpha_{i-2}, \alpha_i, \alpha_{i+1} \cdots \alpha_{n-1}\}, i = 2, 3 \cdots, n.$

It is obviously that f is an $3n$-VDEC of $S_n \vee K_{n,n}$.

So, the theorem is proven. □

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