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COINTEGRATION ANALYSIS AS A TOOL TO MEASURE THE PURCHASING POWER OF UKRAINIAN MEAN TOTAL WAGES

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Abstract: The main purpose of this paper is to propose [26] methodology of linear cointegration analysis as a tool to measure the *purchasing power* of the Ukrainian *mean monthly total wages* (MTW) within the period of November 1993 to December 2004, expressed in terms of *prices* of some *basic food products* or of the *consumer price index* (CPI). Nineteen monthly price time series of *basic food products*, the CPI and the MTW, published by the *State Committee of Statistics of Ukraine (Derzhkomstat)* have been selected. Preliminary studies of stationarity are undertaken to establish their degree of integratedness with a battery of adequate unit root tests. A cointegration analysis of couples of one of these price time series of some *basic food products*, respectively the CPI, and the MTW time series are realised, yielding linear functions between both selected series, establishing tentative long-run equilibrium relations for subperiods *shorter* than the interval 1993 to 2004. These cointegration analyses show that the *purchasing power* of the MTW, expressed in terms of the *prices* of some *basic food products*, respectively the CPI, gradually increased from 1993

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to 2004.

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1. Introduction

This investigation is an integral part of a more general study of Ukrainian economic indices undertaken in the framework of the international project "Analysis of Economic and Environmental Time Series", initiated by researchers of the University of Fribourg, Switzerland, the V.M. Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, the Sevastopol National Technical University, Ukraine, and the Bourgas Free University, Bulgaria, [10], [12], [41], [37], [11]. Price time series of popular Ukrainian basic food products have been selected to investigate the purchasing power of Ukrainian *mean monthly total wages*[1]. This contribution presents an extension of the approaches to analyse the *purchasing power* of Ukrainian *mean monthly total wages* since 1991, see [13], [14], [11].

2. The Notion of Purchasing Power

In economics, *purchasing power* refers to the amount of goods and services a given amount of money or, more generally, liquid assets can buy. If money income stays the same, but the price of most goods go up, the effective purchasing power of that income falls.

2.1. Transitional Economy and Hyperinflation, Evolution of Purchasing Power

In the decade 1991-2004 the Ukrainian economy in transition was severely touched by periods of hyperinflation and inflation. Since the year 2000, there are severe fluctuations in the prices. The evolution of the prices of primary goods was dramatic. The monthly Ukrainian *consumer price index* (CPI) of 1992, measuring the general price level of consumer goods and services, has to

[1]Moreover, the present prices of basic food products are used for the calculation of one of the most important economic indicator, the Consumer Price Index (CPI). An important part of the basket of goods of the Ukrainian CPI consists of bread and bakery: 7.615 %, see [38].

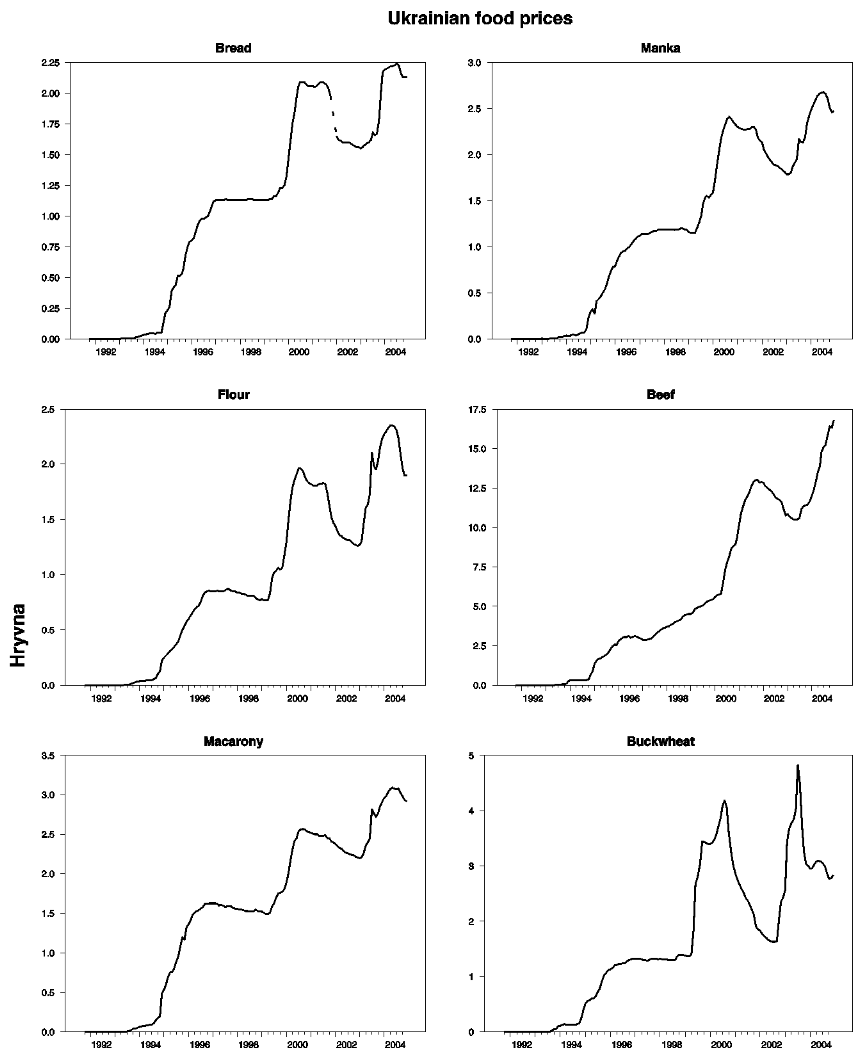

Figure 1: Period: November 1991 to December 2004

be multiplied by a factor of about 5'000 to attain the Ukrainian CPI of the same month nine years later. The households and individuals had permanently to worry about the development of their own monthly wages, in relation to the development of the prices of the goods, necessary for the life of every day.

Irving Fisher's definition [19] of *purchasing power*[2] is taken as a reference

[2] 'The *purchasing power* of money is indicated by the quantity of other goods which a given

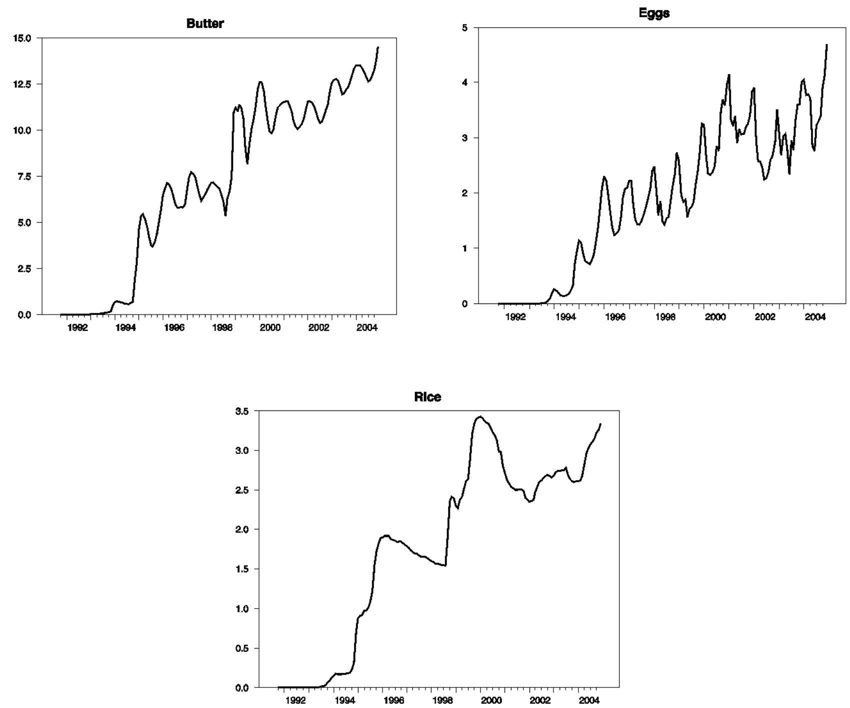

Figure 1: Continuation: Period: November 1991 to December 2004

to answer two questions:

1) What quantity of given basic food products a representative household or individual can purchase with the *earning power* of its labour, represented by the mean monthly total wages, at any cross-sectional time point within the decade 1993-2004?

2) Was the *price level*, measured by the CPI, in equilibrium with the MTW within some appropriately selected subperiods of the investigated decade?

The answers to these questions will be developed on the basis of a data-driven econometric analysis, using the concept of cointegration. The data base is presented in the next subsection.

quantity of money will buy.' ... 'The *purchasing power* of money is the reciprocal of the *level of prices.'*, see Irving Fisher ([19], p. 13, 14).

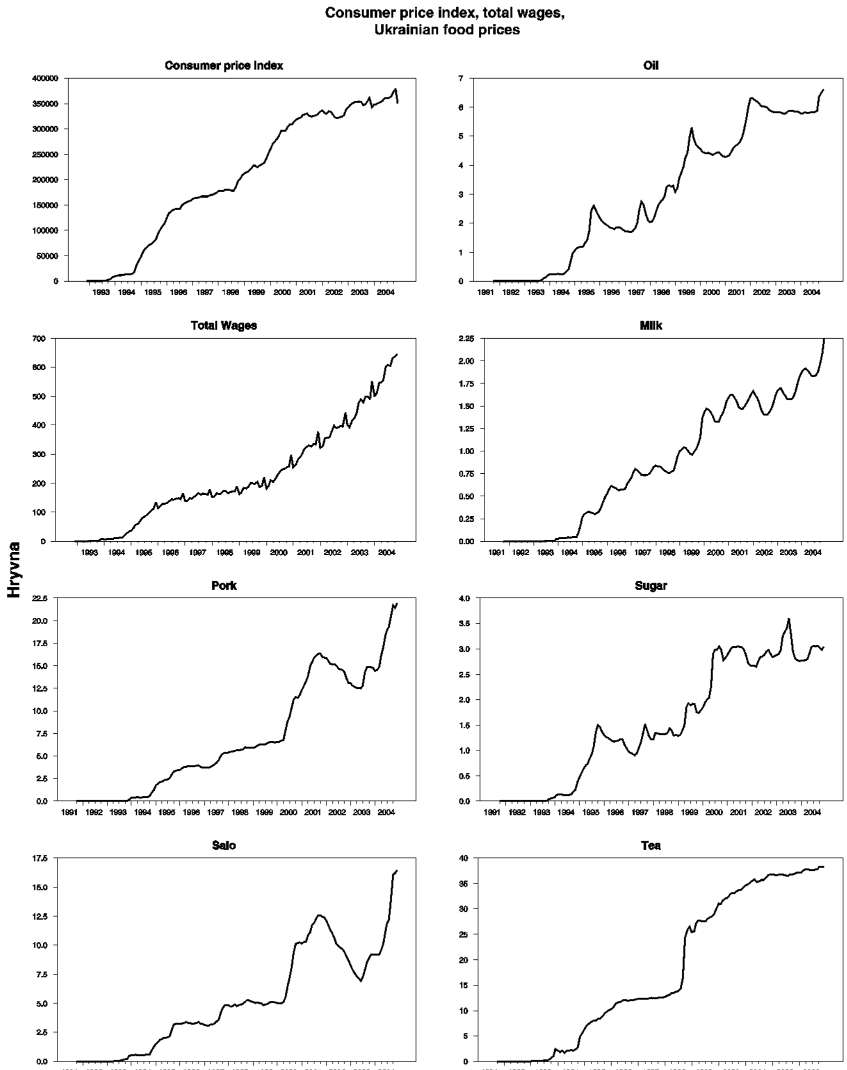

Figure 2: Period: November 1991 to December 2004

2.2. Ukrainian Consumer Price Index, Mean Monthly Total Wages and Selected Prices of Basic Food

The period of the monthly items of each series extends from $\tau = 1$ (1991:10) to $T = 159$ (2004:12). With $\tau \leq t'$, $T' \leq T$, consider a set of 19 monthly price time

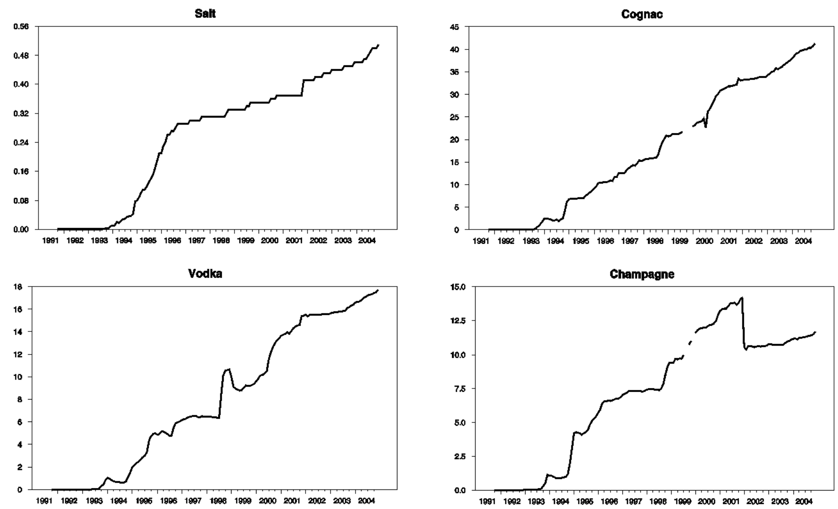

Figure 2: Continuation: Period: November 1991 to December 2004

series of Ukrainian *basic food products*, published by *Derzhkomstat*, generally covering an interval $[t', T'] \subseteq [t, T]$, denoted by $y_{ti},\ i = 1, \ldots, 19,\ \tau = t', \ldots, T'$, starting with the prices of white bread y_{t1}, ending with the prices of champagne y_{t19}, and the *Ukrainian consumer price index* (CPI) y_{t20} and finally with the *mean monthly total wages* (MTW) time series y_{t21}, see Figure 1 and Figure 2. The prices are given in Hryvnas[3]. If an algebraic expression concerns some of the 21 series, the notation is y_t.

First, the authors consider as given *quantity of other goods* the quantity of some of the 19 *basic food products* or the basket of goods, on the basis of which the Ukrainian CPI is calculated, and second, as given *quantity of money*, the *mean monthly total wages* of individuals. Therefore, an investigation of the *purchasing power* is a cross-sectional analysis with the aim to evaluate the total amount of *one* basic food or the number of baskets of goods that can be acquired by an individual or a household with the amount of money, available in this month, represented by the MTW within a subperiod shorter than the period 1991 to 2004.

A preliminary visual study of the nineteen price time series gives rise to the conclusion of a strong affinity between the time series, see Figure 1 and Figure 2, exhibiting strikingly similar structures. The large slope in the first

[3]Data before January 1997 when *Hryvna* has replaced the earlier *Karbovanets* are recalculated by the State Committee of Statistics of Ukraine (*Derzhcomstat*).

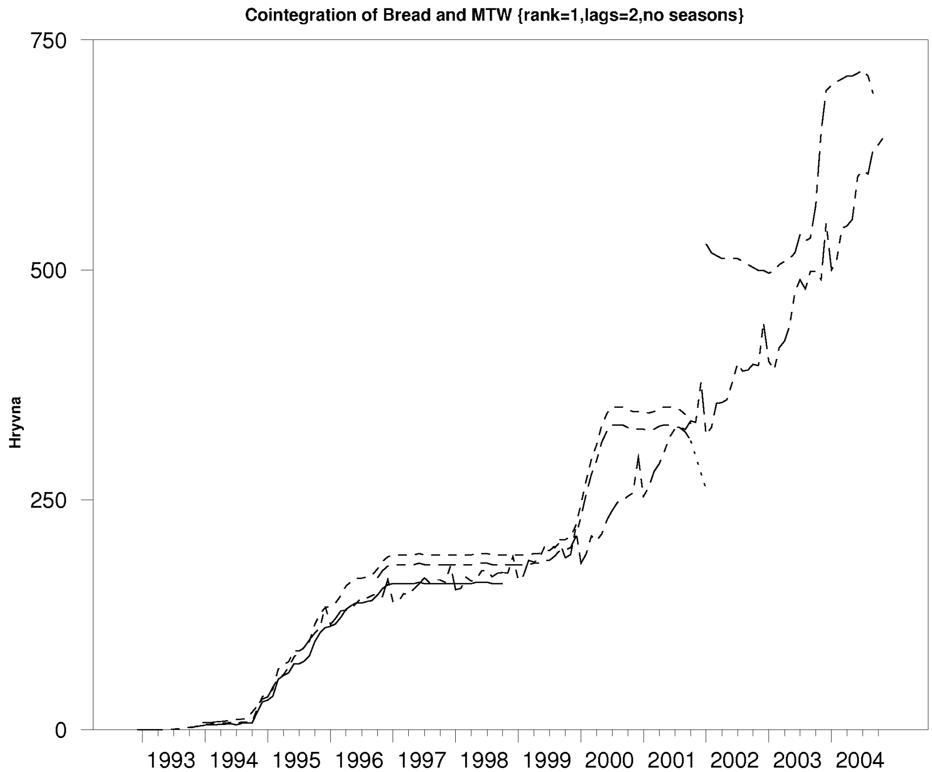

Figure 3: Consumer bread prices cointegrated with mean total wages

part of the curve, reflecting inflation processes, change into a rather flat second part after introducing Hryvna in January 1997, and then, as a sign of general economic instability, we observe a sudden jump in 2000 changing into variations until 2004. Apparently, so called *structural breaks* cannot be excluded within the period of analysis.

There are good reasons to think that some of these series are cointegrated. This subject is introduced in the next subsection.

3. The Investigation of Long-Run Equilibrium Relations

The goal of cointegration analysis of economic time series is to discover *long-run equilibrium relationships* with interpretable coefficients between two or more

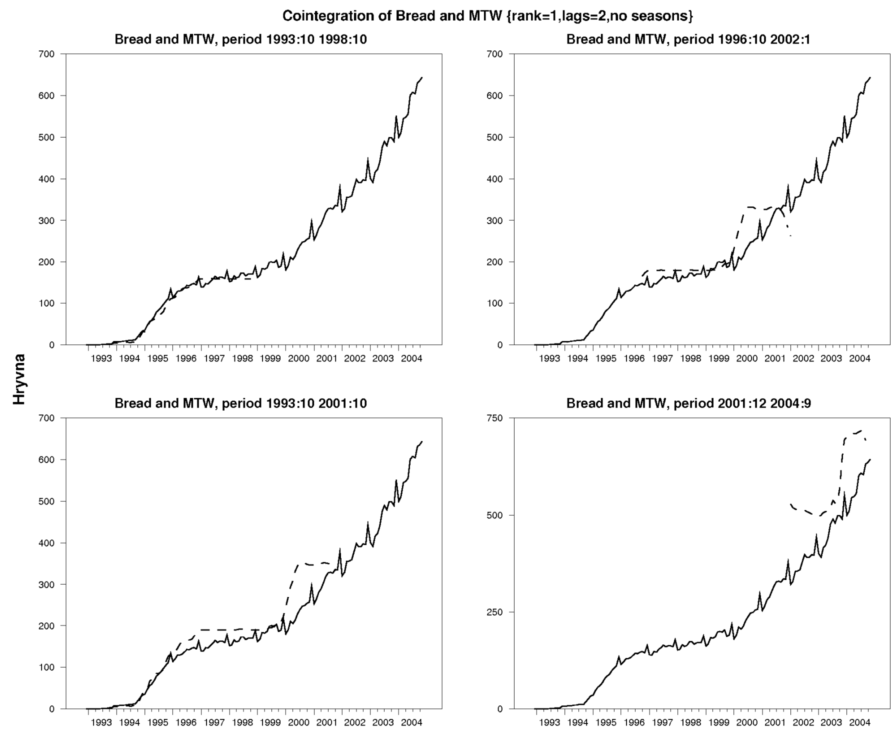

Figure 4: Consumer bread prices cointegrated with mean total wages

economic variables. Such analyses are of overall importance to verify or to falsify *equilibrium conjectures* in the cadre of *economic theories*.

It is well known that the property of non-stationarity of economic series is a crucial counter argument or, equivalently, the question of presence of unit roots in that time series. Twenty years ago, the concept of deterministic trends of economic time series belonged to the unquestioned *stylised facts* of economics. In econometrics, methods of modelling and estimating deterministic trends belonged to the standard procedures of textbooks and curricula. But Nelson and Plosser [34] found that most macroeconomic variables have an univariate time series structure with a unit root. Cochrane [5], introducing a variance ratio test, showed that Nelson's and Plosser's time series generally contain a random walk part together with an additive stationary part. Perron [36], modelling the Great Crash of 1929 and the slow down in growth after 1973 as exogenous,

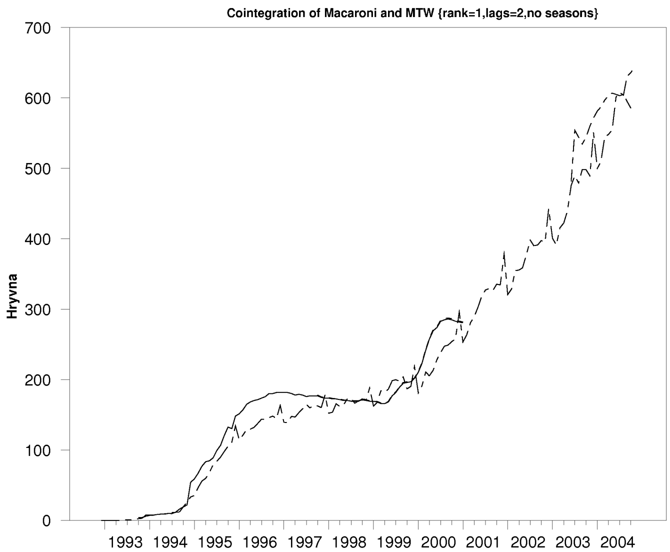

Figure 5: Macaroni prices cointegrated with mean total wages

concluded that most macroeconomic time series are not characterised by the presence of a unit root.

Also, some trials to build ARIMA time series models, but not reported here[4] have indicated the presence of unit roots.

4. Statistical Adequacy or the Empirical Validity of Probabilistic Assumptions

As it has been summarised, the debate on whether macroeconomic series are trend or difference stationary (unit root) remains unsolved. For this reason, a detailed analysis is proposed, taking into account the actual know-how in this domain, specially the methodology of miss specification tests reintroduced by Andreou and Spanos [1].

Andreou and Spanos [1] proposed to apply statistical adequacy arguments, meaning investigation of the empirical validity of the probabilistic assumptions of the statistical model, tentatively describing the data to analyse, before validation of models. On the other hand, Harvey [23] proposes structural time

[4] All the calculations are realised with the software packages RATS and CATS, see [8], [16], [22], GiveWin and PcGive, see [9].

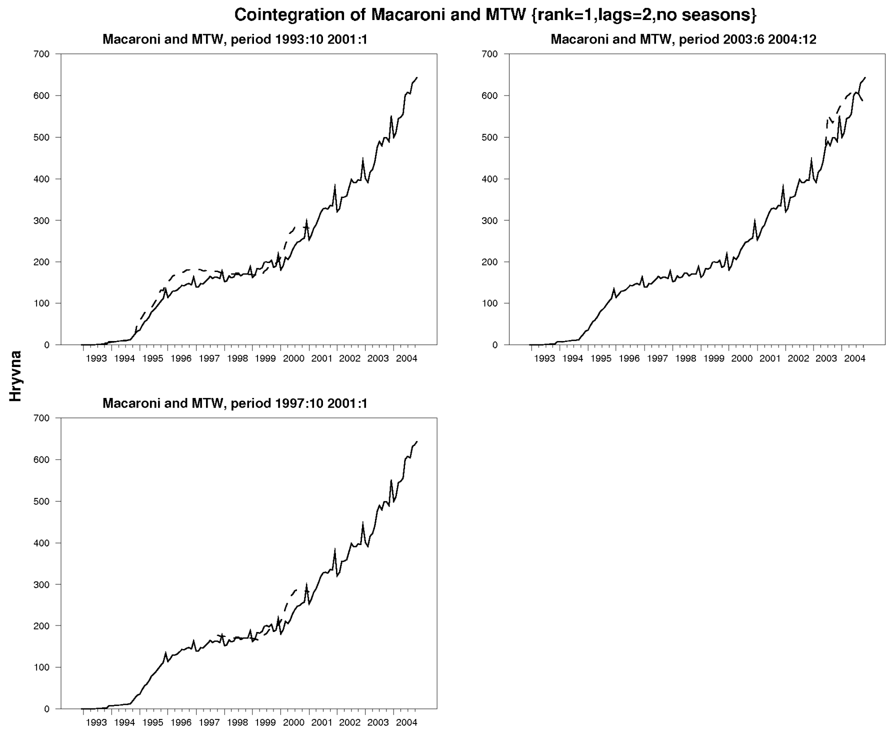

Figure 6: Macaroni prices cointegrated with mean total wages

series models and Kalman filters to get rid of the problems of economic shocks, like the Great Crash of 1929 and the Oil Price Shock of 1973.

Andreou and Spanos set up a traditional AR(p) model

$$y_t = \alpha + \mu t + \Psi D_t + \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t \tag{1}$$

that is viewed as an internally consistent set of probabilistic assumptions, relating the observations to the stochastic process to be questioned. Spanos requests in his *statistical adequacy analyses* to look for five properties either of the data or resulting in the residuals: (1) normality, skewness and kurtosis of the process y_t, (2) linearity, (3) homoskedasticity, (4) $t-$invariance of the parameters, (5) martingale difference process of the residuals.

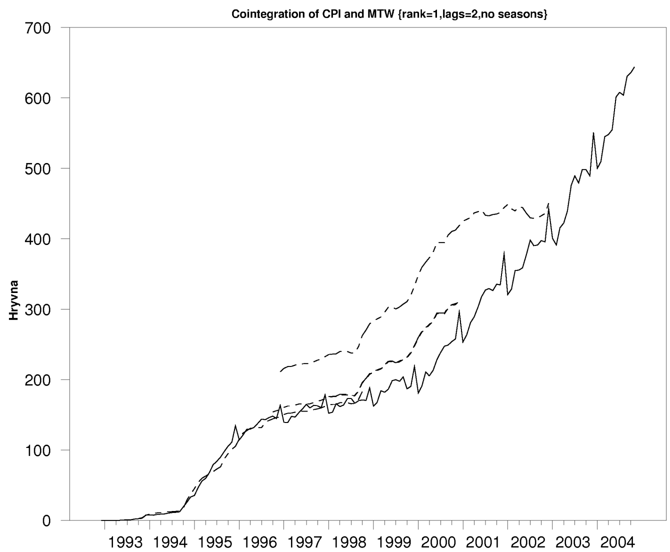

Figure 7: Consumer Price Index cointegrated with mean total wages

These popular preliminary but necessary cumbersome unit root tests are relegated to the Appendix of this paper. But the results of these unit root analyses are directly used in the next section.

5. Determining the Purchasing Power by Pairwise Cointegration Analysis

The 21 y_t appear to be integrated $I(1)$. The first condition to realise a cointegration analysis is fulfilled. The methodology of Johansen is applied for the cointegration analysis of the pairs (MTW; y_{ti}), $i = 1, .., 20$. That means, one tries to validate equilibrium equations for each of the mentioned couples

$$y_{t21} - \beta_i y_{ti} = 0, \quad i = 1, ..., 20, \tag{2}$$

the coefficients β_i being constant. Indeed, in terms of a standard cointegration analysis, the equations (2) are normed cointegration relations. To perform such an analysis, first a vector autoregression (VAR) model has to be set up. This procedure is presented in the next subsection.

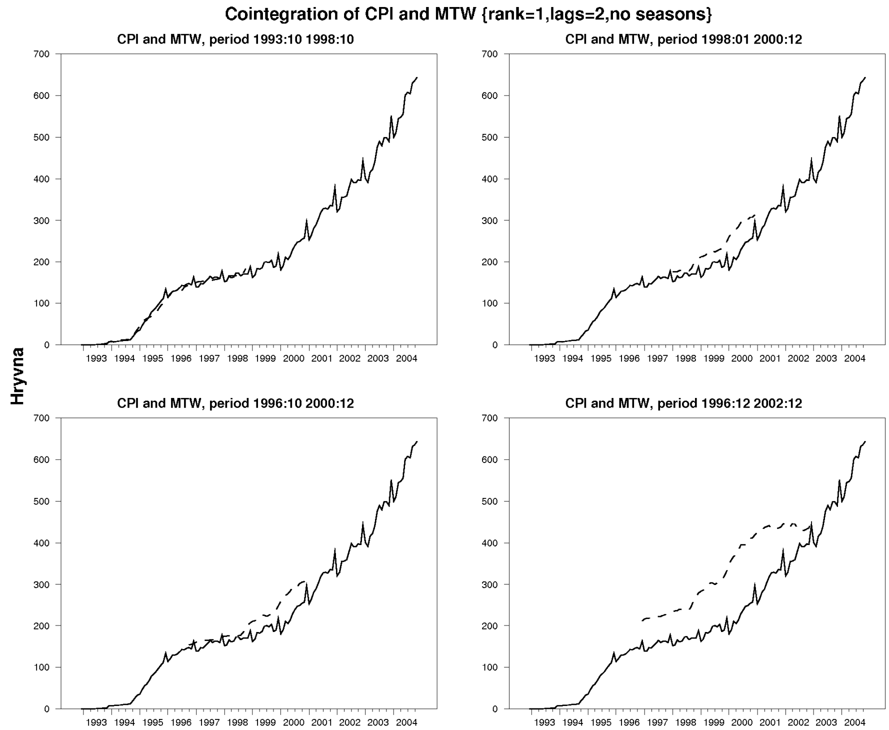

Figure 8: Consumer Price Index cointegrated with mean total wages

5.1. Vector Autoregression (VAR) Models

At the starting point of this cointegration analysis stands the construction of $p-$order vector autoregressive $VAR(p)$ models for two time series, $y_{t21} = MTW$ and one of the series $y_{ti}, i = 1, ..., 20$. Presently, (2×1) vectors $\mathbf{y}_t = [y_{t21}, y_{ti}]'$, $i = 1, \ldots, 20$ are defined. General VAR(p) models are set up,

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\mu} t + \boldsymbol{\Psi}\mathbf{D}_t + \sum_{j=1}^{p} \mathbf{A}_j \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t \; ; \; t = 1, ..., T, \quad \boldsymbol{\varepsilon}_t \sim niid(\mathbf{0}, \boldsymbol{\Sigma}). \quad (3)$$

Restrictions to Constants and Trends. An important feature of the vector autoregression model (3) is the interpretation of the intercept $\boldsymbol{\alpha}$, the trend component $\boldsymbol{\mu}$ and the seasonal dummies $\boldsymbol{\Psi}\mathbf{D}_t$ in terms of dynamic effects.

Furthermore, the present analysis shows that a quadratic trend in the *levels* of the time series y_t, described by a *linear trend* in the *differences* of y_t, is not a sensible long-run outcome outside the samples.

Two statistical tests are initiated, trying to get rid of the intercept $\boldsymbol{\alpha}$ and trend $\boldsymbol{\mu}$. A Likelihood-Ratio (LR) test, see [33] and [40] is realised and Akaike's Information Criterion (AIC) is applied. The results of the tests are presented in Table 1. Concerning the AIC criterion, when the values of col. (3) are smaller than those of col. (2), the more parsimonious model without the intercepts $\boldsymbol{\alpha}$ and trends $\boldsymbol{\mu}$ is maintained. This is *not* the case for 12 series. Concerning the LR test, the $p-$value must be higher than $\alpha = 0.05$, then the reduction is *successful*. This is the case for 15 series.

For *oil*, *sugar*, *tea*, *vodka* and *cognac*, one cannot get rid of the intercepts $\boldsymbol{\alpha}$ and the trends $\boldsymbol{\mu}$, because both test issues are negative. These 5 food price series are eliminated from the ongoing cointegration analysis. For the other 15 series the VAR(p) models

$$\mathbf{y}_t = \boldsymbol{\Psi}\mathbf{D}_t + \sum_{j=1}^{p} \mathbf{A}_j \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t \quad ; \quad t = 1, ..., T \quad \boldsymbol{\varepsilon}_t \sim niid(\mathbf{0}, \boldsymbol{\Sigma}), \tag{4}$$

are accepted. This result corresponds correctly with the claims of purchasing power analysis, asking the restriction to set *no* constant and *no* trend into the tentative cointegration vector (2), see also Dennis [6], p. 4-7.

5.2. The Vector Error-Correction Model (VECM)

It is actually well-known and a question of simple algebra to show that a VAR representation (4) can be written equivalently as

$$\Delta\mathbf{y}_t = \boldsymbol{\Psi}\mathbf{D}_t + \mathbf{A}\mathbf{y}_{t-1} + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta\mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{5}$$

a *vector error-correction model* (VECM) or *error-correction representation* (see [24], pp. 636, 638). From the unit root analyses presented in the Appendix, one concludes that the series $y_{it}, i = 1, \ldots, 21$ all appear to be $I(1)$ processes. Thus, the *Granger Representation Theorem* is applicable, stating that for any set of $I(1)$ variables, an *error-correction representation* implies *cointegration* (see [16], p. 154) or that *error-correction* and *cointegration* are equivalent representations (see [15], p. 371). Consequently, cointegration is equivalent to the property of stationarity of the variables on both sides of the VECM (5), including the

MTW and (1)	AIC with $\boldsymbol{\alpha}, \boldsymbol{\mu}$ (2)	AIC without $\boldsymbol{\alpha}, \boldsymbol{\mu}$ (3)	LR $\chi^2(4)$ (4)	$p - value$ (5)
Bread	-358.57	-357.04*	4.809980	0.307356
Flour	-215.10	-210.61*	7.418164	0.115373
Macaroni	-215.39	-216.24	2.752509	0.600058
Semolina	-291.31	-288.45	5.995038	0.199519
Beef	90.59	90.10	3.073304	0.545634
Buckwheat	114.66	114.19	5.705401	0.222256
Butter	366.26	366.90*	4.050186	0.399257
Eggs	151.71	151.74*	3.512400	0.475995
Rice	-129.55	-130.98	2.249374	0.690000
Pork	246.07	244.69	2.295226	0.681638
Salo	222.77	220.16	1.217719	0.875173
Oil	26.89	36.43*	11.826263	0.018691*
Milk	-476.91	-478.05	2.502316	0.644221
Sugar	-63.86	-50.80*	14.902882	0.004907**
Tea	485.57	496.19*	12.610973	0.013341*
Salt	-805.15	-804.67*	3.905465	0.418951
Vodka	237.24	246.49*	11.572449	0.020831*
Cognac	408.95	417.17*	10.576154	0.031764*
Champagne	317.30	316.90	3.116447	0.538531
CPI	2941.31	2942.08*	4.166329	0.383963

Table 1: Likelihood-Ratio test for reduction of $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ in the VAR(2)-model of MTW and one of the price series of col. (1)

linear combination $\mathbf{Ay}_{t-1}$. Thus, in a VECM there is stationarity either by differencing or by linear combinations.

Thus, this result is really comfortable. Another way of thinking starts again from the VAR model (3) that is reduced, due to the LR test results presented in Table 1, to the VAR model (4) with no intercept an trend and equivalent to the VECM model (5). As there is no intercept and no trend in (5), the cointegrating relations given by $\boldsymbol{Ay}_{t-1}$ are equivalent to each of the purchasing power relations (2).

The seasonal dummies $\boldsymbol{\Psi} D_t$ are only set, if seasonality is visible in the graphs, see Figure 1 and Figure 2 and the appropriate number of lags is either set to $p = 2$, when there is no need to grasp more seasonality, or to $p = 6$, when some more seasonality is intended to be grasped.

MTW and (1)	E.V. (2)	λ_{trace} (3)	$\lambda_{trace}95$ (4)	k (5)	LB $\chi^2(k)(p\text{-val})$ (6)
Bread	0.167	12.450	12.282	50	85.452
1993:10-1998:10	0.032	1.639	4.071		(0.001)
Flour	0.132	13.762	12.282	58	59.482
1993:10-2000:12	0.028	2.333	4.071		(0.421)
Macaroni	0.173	18.356	12.282	78	156.748
1993:10-2001:1	0.023	2.020	4.071		(0.000)
Semolina	0.223	18.873	12.282	66	76.660
1993:10-2000:1	0.002	0.182	4.071		(0.174)
Beef	0.114	9.476	12.282	54	79.449
1993:10-1998:12	0.034	2.102	4.071		(0.014)
Eggs	0.227	22.691	12.282	66	111.553
1993:10-2000:1	0.048	3.353	4.071		(0.000)
Pork	0.109	12.269	12.282	66	87.105
1992:12-2000:12	0.020	1.793	4.071		(0.042)
Milk	0.166	11.680	12.282	46	65.731
1993:10-1998:6	0.030	1.695	4.071		(0.030)
CPI	0.212	12.528	12.282	50	48.797
1993:10-1998:10	0.033	1.808	4.071		(0.522)

Table 2: Cointegration of *mean total wages* with *food price* time series, first period

Determination of the Cointegrating Rank. The rank $r = \rho(\mathbf{A})$ of the (2×2) matrix $\mathbf{A}$, denotes the *cointegrating rank* and is the dimension of the *cointegrating space* (see [27], p. 37). As there are $n = 2$ variables in (5), the rank $r = \rho(\mathbf{A})$ is at most $n - 1 = 1$. Therefore, the rank $r = \rho(\mathbf{A})$ is either 0 or 1. There is the *maximum eigenvalue test* λ_{trace} of [27] and (see [15], p. 391) to determine the cointegrating rank for the pairs $(y_{t1}; y_{t21})$, $i = 1, \ldots, 20$. Again, as the purchasing power is analysed (2), the cointegrating space is determined with no deterministic component, see J.G. Dennis [6], as it has been discussed earlier.

The null hypothesis of no cointegration $H_0 : r = 0$ against the alternative hypothesis of *one* cointegrating vector $H_1 : r = 1$ is analysed for several subperiods within the interval 1991-2004. Tables 2-5 present the results of this testing for one up to a maximum of four subperiods. When the λ_{trace} statistic, col. (3) is greater than the $\lambda_{trace95}$ critical value, col. (4), the corresponding

MTW and (1)	E.V. (2)	$\lambda_{trace}*$ (3)	$\lambda_{trace}95$ (4)	k (5)	LB $\chi^2(k)(p\text{-val})$ (6)
Bread	0.262	12.547	12.282	22	33.520
2001:12-2004:09	0.096	3.129	4.071		(0.055)
Flour	0.355	7.893	12.282	-10	6.788
2002:12-2004:08	0.084	1.313	4.071		(NA)
Macaroni	0.529	15.166	12.282	10	13.645
2003:06-2004:11	0.177	3.111	4.071		(0.190)
Semolina	0.409	19.445	12.282	6	12.607
1998:01-2000:12	0.114	3.644	4.071		(0.050)
Beef	0.255	13.038	12.282	18	27.903
2002:09-2004:11	0.205	5.685	4.071		(0.064)
Eggs	0.300	37.109	12.282	90	123.125
1993:10-2001:12	0.026	2.562	4.071		(0.012)
Pork	0.087	11.777	12.282	102	97.416
1992:12-2003:12	0.002	0.248	4.071		(0.610)
Milk	0.457	14.013	12.282	10	71.511
2003:04-2004:11	0.154	3.014	4.071		(0.000)
CPI	0.240	19.556	12.282	62	165.09
1996:12-2002:12	0.002	0.112	4.071		(0.000)

Table 3: Cointegration of *mean total wages* with *food price* time series, second period

eigenvalue, col. (2) is significantly different from zero, otherwise the eigenvalue, col. (2) can be considered as zero. The rank is also the number of significant eigenvalues. This means here: rank $\rho = 0$ or $\rho = 1$. The alternative hypothesis $H_1 : r = 1$ cannot be rejected in the cases of the couples of *mean monthly total wages (MTW)* with *bread, flour, macaroni, semolina, eggs, milk* for the first subperiod and in the cases of the couples of MTW with *semolina, eggs, milk, CPI* for the second subperiod. On the other hand, the alternative $H_1 : r = 1$ must be rejected in the cases of the couples of MTW with *beef, pork, CPI* in the first subperiod and in the cases of the couples of MTW with *bread, flour, macaroni, beef, pork* in the second subperiod.

In Tables 2-5 following information is presented: the subperiods of the cointegration analysis, col. (1), the eigenvalues of matrix $\mathbf{A}$, col. (2), the λ_{trace*}-statistic, col. (3), the critical values of the Ostermann-Lenum (1992) test at a significance level of $\alpha = 5\%$, col. (4). The graphs of the *autocorrelation*

MTW and (1)	E.V. (2)	$\lambda_{trace}*$ (3)	$\lambda_{trace}95$ (4)	k (5)	LB $\chi^2(k)(p\text{-val})$ (6)
Bread	0.128	14.635	12.282	86	142.719
1993:10-2001:10	0.017	1.639	4.071		(0.000)
Flour	0.082	10.551	12.282	66	67.986
1993:10-2001:10	0.030	2.742	4.071		(0.409)
Macaroni	0.249	12.412	12.282	30	33.657
1997:10-2001:01	0.040	1.551	4.071		(0.295)
Semolina	0.233	14.353	12.282	42	34.141
1995:10-2000:01	0.022	1.095	4.071		(0.801)
Beef	0.271	10.569	12.282	26	75.991
1997:12-2000:09	0.013	0.435	4.071		(0.000)
Eggs	0.433	34.226	12.282	42	83.049
1993:10-1997:12	0.122	6.392	4.071		(0.000)
Pork	0.070	6.244	12.282	42	63.877
1992:12-1998:12	0.020	1.358	4.071		(0.016)
Milk	0.135	11.714	12.282	58	69.109
1993:10-1999:06	0.029	2.005	4.071		(0.151)
CPI	0.346	24.759	12.282	42	123.390
1996:10-2000:12	0.078	3.982	4.071		(0.000)

Table 4: Cointegration of *mean total wages* with *food price* time series, third period

MTW and (1)	E.V. (2)	$\lambda_{trace}*$ (3)	$\lambda_{trace}95$ (4)	k (5)	LB $\chi^2(k)(p\text{-val})$ (6)
Bread	0.212	14.874	12.282	50	10.445
1996:10-2002:10	0.014	0.847	4.071		(0.000)
Semolina	0.266	7.910	12.282	18	17.344
2002:08-2004:10	0.007	0.173	4.071		(0.500)
CPI	0.362	19.090	12.282	26	49.774
1998:01-2000:12	0.105	3.790	4.071		(0.003)

Table 5: Cointegration of *mean total wages* with *food price* time series, forth period

i (1)	period $[t', T']$ (2)	β_i (3)	ρ (4)	period (5)	β_i (6)	ρ (7)
Br	1993:10-1998:10	141	1	1993:10-2001:10	168	1
1	1996:10-2002:01	159	1	2001:12-2004:09	320	1
Fl	1993:10-2000:12	176	1	1993:10-2001:10	176	0
2	2002:12-2004:08	238	0			
Mac	1993:10-2001:01	111	1	1997:10-2001:01	112	1
3	2003:08-2004:11	197	1			
Sem	1993:10-2000:01	134	1	1995:10-2001:01	128	1
4	1998:01-2000:12	154	1	2002:08-2004:10	245	0
Be	1993:10-1998:12	47	0	1997:12-2000:09	43	0
5	2002:09-2004:11	40	1			
Eg	1993:10-1997:12	85		1993:10-2000:01	92	1
8	1993:10-2001:12	129	1			
Po	1992:12-1998:12	34	0	1992:12-1998:12	36	0
10	1992:12-2003:12	32	0			
Mil	1993:10-1998:06	207	0	1993:10-1999:06	193	0
13	2003:04-2004:11	247	1			
CPI	1993:10-1998:10	930	1	1996:10-2000:12	992	1
21	1998:01-2000:12	996	1	1996:12-2002:12	1334	1

Table 6: Cointegration: Period and cointegration factors β_i

functions of the residuals have been checked. A multivariate Ljung-Box test (LB) based on the estimated auto- and cross-correlations of the first [T/4] lags, see [30] and [25] are performed with its degree of freedom, col. (5) and the *LB* statistic with corresponding $p-value$, col. (6) are presented. There is absence of serial correlation in the residuals, when those $p-values$ are far from zero. This is only the case in the first period for *flour, semolina and CPI*, in the second period for *macaroni and pork*.

Clearly, one is interested in the existence of more than two subperiods within the interval 1991-2004 of *cointegration* between the MTW and one of the 19 price time series of *basic food products* or the *CPI*. Tables 4, 5 complete the information of the analysed subperiods. For some couples $(y_{t21}; y_{ti})$, $i = 1, \ldots,$ 20 up to four subperiods have been investigated. The results are summerised in Table 6, reporting the period, col. (2) and (5), the factor β_i of the purchasing power relation (2), col. (3) and (6), and the rank ρ, col. (4) and (7). If cointegration is verified, then a *long-run relationship* exists between the MTW time series y_{21} and the *basic food product* price time series $y_{ti}, i = 1, ..., 19,$

respectively the CPI time series y_{t20}. Therefore, the existence of estimated *long-run relationships*

$$\hat{y}_{t21i} := \beta_i y_{ti}, \qquad i \in \{1, \ldots, 20\} \tag{6}$$

between the MTW of individuals and some of the 19 price time series or the CPI, as presented in Table 6, are postulated. The couples $(y_{t21}; \hat{y}_{t21i})$, $i \in \{1, 3, 21\}$, that means $\{MTW, bread\}$, $\{MTW, macaroni\}$, $\{MTW, CPI\}$ are shown as examples of cointegrated series in Figure 3 - Figure 8. The cointegration relations (6) are presented in the form of an *affine transformation* of the series y_{ti} into an estimated series $\hat{y}_{t21i}$ with the so-called normalised cointegrating factor β_i, issued from the power purchase analysis (2), see Table 6, col. (3), (6). This means that for each cross sectional time t within the indicated subperiods $[t', T']$, the series $\hat{y}_{t21i}$ is an approximation of the series y_{t21} of MTW.

6. Conclusion

The present investigation shows through the unit root tests of Schmidt-Phillips, Sargan-Bhargava, the joint F. Wald test to attain parsimonious models, and the augmented Dickey-Fuller test that the 21 investigated Ukrainian economic time series appear to be non-stationary and integrated I(1) within the period 1991:10-2004:12. The subsequent cointegration analyses of the couples of *mean monthly total wages (MTW)* with *bread*, respectively with *flour*, *macaroni*, *semolina*, *beef*, *eggs*, *pork*, *milk* and the *consumer price index (CPI)*, realised for specifically chosen subperiods $[t', T'] \subseteq [1991:10, 2004:12]$ are reported. Those for which through the λ-trace test the null hypothesis for rank $\rho = 1$ of cointegration could not be refused, are of special interest. The normality assumption for the distribution of the residuals of the models is only verified in a small number of cases. This is not surprising, but represents a weakness of these models. The long-run relationships between the couples of *MTW* with *bread*, respectively with *macaroni* or *CPI*, estimated within subperiods shorter than 1991-2004, have also been graphed, but only in the cases where cointegration is assured through all the tests. Then, the cointegration factors β_i (6) of the normed cointegration vectors represent the *purchasing power* of the *MTW* with respect to the prices of some *basic food products* or the *CPI* for the selected subperiods. These cointegration factors β_i increase gradually for some *basic food products*, showing that the *purchasing power* of the *MTW*, expressed in terms of the prices of these *basic food products*, respectively the *CPI*, improved in Ukraine from 1993 to 2004.

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Appendix: Unit Root Tests

A battery of unit root tests is realised for the 21 time series. This analysis belongs to the actual standards and is therefore put in this Appendix. In accordance with the discussion on misspecification tests presented in Section 4, Spanos' property (2) of *linearity* of an initial $AR(p)$-model (1) is questioned, applying the Schmidt-Phillips test to investigate the presence of *trend polynomials* in the series.

1. Schmidt-Phillips Unit Root Test

Schmidt and Phillips have developed an LM test to determine a unit root in presence of a deterministic trend. The unit root test (see [39]) has been realised consequently to the empirical work of [34]. A common motivation for testing for a unit root is to test the hypothesis that a series is *difference stationary* against the alternative that it is *trend stationary.* Schmidt and Phillips have developed such a test, whereas the Dickey-Fuller test is not convenient for this purpose.

The special interest to apply this tests is that *deterministic trend* can be modelled by *polynomials* of degree p. Following tests have been performed:

a) The Schmidt-Phillips test is realised with trend polynomials of degree $p = 1, 2, 3$ on the levels of the series with significance level $\alpha = 0.05$. The result

Indexes pol. deg. crit. val. (1)	y_t $p=1$ -3.04 (2)	y_t $p=2$ -3.55 (3)	y_t $p=3$ -3.99 (4)	Δy_t $p=1$ -3.04 (5)	Δy_t $p=2$ -3.55 (6)	Δy_t $p=3$ -3.99 (7)
Bread	-.86	-1.18	-1.28	-3.38	-5.13	-5.12
Flour	-1.26	-1.43	-1.40	-7.07	-7.11	-7.10
Macaroni	-1.08	-1.17	-1.19	-8.59	-8.54	-8.53
Semolina	-1.00	-1.10	-1.22	-6.25	-6.31	-6.33
Beef	-0.88	-0.99	-1.00	-5.13	-5.29	-5.47
Buckwheat	-1.53	-1.70	-1.76	-7.10	-7.12	-7.13
Butter	-2.28	-2.28	-2.75	-6.40	-6.71	-6.89
Eggs	-3.32	-2.89	-3.52	-7.98	-8.81	-9.33
Rice	-1.05	-1.11	-1.33	-5.69	-5.94	-6.19
Pork	-1.10	-1.18	-1.22	-5.98	-6.12	-6.25
Salo	-1.19	-1.07	-1.14	-5.28	-5.31	-5.36
Oil	-1.65	-1.78	-2.14	-6.38	-6.42	-6.45
Milk	-2.05	-1.51	-1.98	-3.00*	-3.52*	-4.08
Sugar	-1.61	-1.95	-2.18	-7.38	-7.46	-7.49
Tea	-1.05	-1.41	-1.93	-8.06	-8.27	-8.26
Salt	-0.82	-0.86	-1.04	-8.89	-10.13	-11.63
Vodka	-1.40	-1.86	-2.35	-6.85	-6.94	-6.97
Cognac	-1.23	-1.96	-2.25	-5.76	-5.98	-5.83
Champagne	-1.06	-1.13	-1.46	-4.24	-4.35	-4.22
CPI	-0.92	-1.63	-1.67	-3.67	-4.12	-4.71
MTW	-1.83	-2.95	-4.70*	-17.93	-17.79	-17.97

Table 7: Schmidt-Phillips root tests with polynomial degrees $p = 1, 2, 3$

is that all the 21 series exhibit to be non stationary, see col. (2), (3), (4) in Table 7, all the statistics being smaller than the corresponding critical values.

b) Then, the Schmidt-Phillips unit root test is realised for trend polynomials of degree $p = 1$, 2, 3 on the differenced series. The result is that all the 21 series exhibit to be stationary, col. (5), (6), (7) in Table 7, all the statistics being greater than the corresponding critical values, except that of *milk* for polynomial degrees $p = 1$, 2.

Indexes crit. v. (1)	y_t R_1 0.26 (2)	y_t R_2 0.35 (3)	Δy_t R_1 0.26 (4)	Δy_t R_2 0.35 (5)	$\Delta^2 y_t$ R_1 0.26 (6)	$\Delta^2 y_t$ R_2 0.35 (7)
Bread	0.0044	0.0335	0.6049	0.049	1.9808	1.9808
Flour	0.0062	0.0403	0.9757	0.9757	2.5534	2.3213
Macaroni	0.0031	0.0254	1.2974	1.2881	2.9379	2.9047
Semolina	0.0029	0.0254	0.8068	0.8041	2.5914	2.1843
Beef	0.0022	0.0194	0.5906	0.5789	2.3054	1.3417
Buckwheat	0.0180	0.0595	0.9943	0.9824	2.2837	2.2736
Butter	0.0117	0.1290	0.9555	0.8341	2.3302	2.3250
Eggs	0.0436	0.2622	1.5402	1.1606	2.6448	2.4622
Rice	0.0057	0.0280	0.7583	0.6916	2.1170	2.0389
Pork	0.0035	0.0299	0.7353	0.7500	2.3616	1.4199
Salo	0.0071	0.0353	0.5885	0.6099	2.1586	2.0796
Oil	0.0045	0.0686	0.8424	0.8319	1.9926	1.9185
Milk	0.0040	0.1028	0.4830	0.2169*	1.7506	1.3166
Sugar	0.0078	0.0658	1.0754	1.0393	2.2829	2.1184
Tea	0.0030	0.0303	1.2481	1.2481	2.5160	2.5142
Salt	0.0018	0.0172	1.6675	1.3508	3.0836	2.7446
Vodka	0.0026	0.0495	0.9412	0.9301	2.2112	2.1953
Cognac	0.0017	0.0470	1.9794	1.9299	3.1940	3.1663
Champ.	0.0071	0.0363	1.5556	1.4643	2.7034	2.6832
CPI	0.0018	0.0233	1.1947	0.3406*	2.1309	0.4837
TotWages	0.0093	0.0901	2.7195	2.7862	3.1709	3.1682

Table 8: Sargan-Bhargava unit root tests

2. Bhargava-Type Formulation of the Unit Root Test

In the present subsection, the Bhargava approach to the discussion of unit roots is applied. The question to be solved is to know if the series have to be detrended or to be differenced prior to further analysis. This depends on the fact whether the series are *trend-stationary* (TS) or *difference-stationary* (DS) (see [31], p. 37-38). A first glance at the graphs of Figure 1 and Figure 2 shows that the time series can either be TS with drift, and with or without dummy seasonals, or DS with or without drift, and with or without dummy seasonals. This question is crucial for the further cointegration analysis.

If series are trend-stationary the data generating process of y_t can be written

Series (1)	γ (2)	σ_γ (3)	$p-$ *value* (4)	p (5)	DW (6)	Augment-tation (7)
Bread	-0.18	0.06	0.001	2	1.794	$\gamma_2 \neq 0$
Flour	-0.32	0.08	0.000	2	2.003	$\gamma_1, \gamma_2 \neq 0$
Macaroni	-0.36	0.06	0.000	0	2.184	$\gamma_1, \gamma_2 \neq 0$
Semolina	-0.30	0.87	0.001	2	2.045	
Beef	-0.10	0.56	0.056	2	1.906	$\gamma_1, \gamma_2, \Psi \neq 0$
Buckwheat	-0.42	0.07	0.000	2	1.951	$\gamma_2 \neq 0$
Butter	-0.64	0.07	0.000	0	1.988	$\alpha, \Psi \neq 0$
Eggs	-0.99	0.08	0.000	0	2.046	$\alpha, \Psi \neq 0$
Rice	-0.38	0.06	0.000	0	1.799	$\alpha \neq 0$
Pork	-0.23	0.07	0.001	2	1.974	$\gamma_1, \gamma_2 \neq 0$
Salo	-0.17	0.06	0.005	2	1.908	$\gamma_2 \neq 0$
Oil	-0.51	0.07	0.000	1	1.996	$\alpha, \gamma_1, \Psi \neq 0$
Milk	-0.45	0.05	0.000	2	1.748	$\gamma_2, \Psi \neq 0$
Sugar	-0.51	0.07	0.000	0	1.888	$\Psi \neq 0$
Tea	-0.56	0.08	0.000	0	1.976	
Salt	-0.65	0.10	0.000	1	2.036	$\alpha, \gamma_1 \neq 0$
Vodka	-0.55	0.08	0.000	1	2.004	$\alpha, \gamma_1 \neq 0$
Cognac	-0.79	0.11	0.000	1	2.012	$\alpha, \gamma_1 \neq 0$
Champagne	-0.73	0.08	0.000	0	1.985	
CPI	-0.64	0.09	0.000	0	1.662	$\alpha, \Psi \neq 0$
TotWages	-1.26	0.06	0.000	2	2.013	$\mu, \gamma_2, \Psi \neq 0$

Table 9: Joint F. Wald-tests leading to parsimonious $AR(p)$ (11)

as

$$y_t = \gamma_0 + \gamma_1 t + e_t\,, \tag{7}$$

where e_t is a stationary ARMA process.

If it is difference-stationary, the y_t can be written as a random walk plus a stationary part

$$y_t = \alpha_0 + y_{t-1} + e_t\,, \tag{8}$$

where e_t is again a stationary ARMA process.

[2] has developed a method nesting the two models together, setting $u_t = \rho u_{t-1} + e_t$ for the error term e_t in (7), getting

$$y_t = \gamma_0 + \gamma_1 t + u_t\,, \quad u_t = \rho u_{t-1} + e_t, \tag{9}$$

Indexes trend crit. val. (1)	y_t no τ_μ (2)	y_t yes τ_τ (3)	Δy_t no τ_μ (4)	Δy_t yes τ_τ (5)	degree of integr. (6)
Bread	-0.510	-1.933	-4.018	-3.987	I(1)
Flour	-0.967	-2.726	-3.908	-3.889	I(1)
Macar.	-0.870	-1.961	-3.773	-3.766	I(1)
Semolina	-0.809	-2.582	-4.123	-4.103	I(1)
Beef	0.972	-1.737	-2.795*	-3.053*	$\geq$ I(2)
Buckwh.	-1.439	-2.537	-5.125	-5.115	I(1)
Butter	-1.047	-4.140*	-5.999	-5.976	$\leq$ I(1)
Eggs	-1.039	-4.274*	-5.554	-5.541	$\leq$ I(1)
Rice	-0.999	-1.874	-5.265	-5.252	I(1)
Pork	0.927	-1.825	-2.817*	-4.062	I(0)
Salo	0.168	-2.411	-2.746*	-2.911*	$\geq$ I(2)
Oil	-0.361	-3.067*	-5.782	-5.768	$\leq$ I(1)
Milk	0.479	-4.957*	-6.613	-6.714	$\leq$ I(1)
Sugar	-0.845	-2.809	-5.784	-5.767	I(1)
Tea	-0.295	-2.147	-6.776	-6.753	I(1)
Salt	-0.624	-1.189	-5.101	-5.097	I(1)
Vodka	-0.097	-3.412*	-6.229	-6.223	$\leq$ I(1)
Cognac	0.615	-3.310*	-5.729	-5.798	$\leq$ I(1)
Champ.	-1.551	-1.379	-6.398	-6.498	I(1)
CPI	-1.721	-0.004	-4.480	-4.724	I(1)
TotWages	4.118	1.394	-7.054	-7.971	I(1)

Table 10: ADF unit root tests, 2 lags, $\tau_\mu = -2.89$, $\tau_\tau = -3.45$

so that (7) becomes $y_t = \gamma_0 + \gamma_1 t + \rho[y_{t-1} - \gamma_0 - \gamma_1(t-1)] + e_t$ which can be written as

$$y_t = \beta_0 + \beta_1 t + \rho y_{t-1} + e_t, \tag{10}$$

where $\beta_0 = \gamma_0(1-\rho) + \gamma_1\rho$ and $\beta_1 = \gamma_1(1-\rho)$. Note that if $\rho = 1$, then $\beta_1 = 0$. This means, that under the null hypothesis $\rho = 1$, the parameters β_0 and β_1 represent the coefficients of linear trend t and of quadratic trend t^2, whereas under the alternative hypothesis they represent the level and the coefficient of t in linear trend (see [31], p. 39).

The corresponding Sargan-Bhargava test[5] is applied. It has been developed

[5] The Sargan-Bhargava test is described in Maddala (1998), p. 82-86 and in Bhargava

for the null hypothesis that the observations available on a variable or their deviations from some appropriate constants follow a random walk against stationary and non-stationary alternatives. The Sargan-Bhargava unit root test is computed with a RATS routine that computes the statistics R_1 and R_2 (see [2], p. 378) for a significance level $\alpha = 5\%$. The results are as follows, presented in Table 8.

The simple random walk hypothesis, R_1-statistics, col. (2), and the random walk with constant drift hypothesis, R_2-statistics, col. (3), are both not rejected for all the 21 series in levels. On the other hand, the simple random walk hypothesis, R_1-statistics, col. (4), (6) and the random walk with constant drift hypothesis, R_2-statistics, col. (5), (7) are both rejected for all the 21 series in the first and second differences, except for *milk* and *CPI*, where the constant drift hypothesis, R_2-statistics, is not rejected for the first differences.

3. Investigating Seasonality: Attaining a Parsimonious Model

The fundamental idea of the present analysis is guided by the method of Spanos (2003) and to investigate the presence of unit roots. Modelisation starts with a generalisation of (7), but for the differenced series Δy_{ti}, setting the general ARIMA model (1), including an intercept, a trend component and dummy seasonals for the the differenced series Δy_{ti}. Moreover, simple algebra permits to convert the $AR(p)$ models to the equivalent "augmented Dickey-Fuller test" form (see [15], p. 225) and ([20], p. 517)

$$\Delta^2 y_t = \alpha + \mu t + \Psi D_t + \gamma \Delta y_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta^2 y_{t-j} + \varepsilon_t. \tag{11}$$

An F. Wald test is applied to the 21 series. The initial parameters are: a *constant*, a *linear trend*, *deterministic seasonals* and a linear combination of lagged differences $\Delta^2 y_{t-j}$, $j = 1, ..., p-1$. The most parsimonious models (11), describing each of these 21 series Δy_t, are estimated through a linear regression. The reduction of parameters is conducted with a cascade of two or three consecutive OLS F. Wald tests, applied to each of the differenced time series.

The result is that all the 21 series Δy_{ti} exhibit significant γ's at a significance level of $\alpha = 0.05\%$, all the p-values being much smaller, see col. (4), Table 9. This means that there is no longer a unit root in the differences Δy_{ti}. Indeed, the number of lags can be lowered from $p = 12$ to $p \leq 2$ for all the series. The

(1986), p. 369-384.

deterministic intercepts α, the trends μ and the seasonal matrices Ψ could not be eliminated from all the models, see Table 9, col. (7).

Concluding, only differenced prices of *beef*, *butter*, *eggs*, *oil*, *milk*, *sugar*, *CPI* and *MTW* exhibit seasonality among the 21 series.

The fact that all the series can be presented by models with a number of lags $p \leq 2$ will be used for setting the vector autoregressive (VAR) models for cointegration analysis. The seasonals also cannot be eliminated in the VARs, as is shown, see Section 5.2.

Moreover, the *Durbin-Watson (DW)* test values fluctuate comfortably around 2, showing absence of first oder serial correlation in the residuals of models (11), see Table 9, col. (6).

4. Augmented Dickey-Fuller Tests

Finally, the augmented Dickey-Fuller unit root tests are here realised with a constant and alternatively with and without linear trend for lags = 0, 1, 2.

a) First, in Table 10, the ADF test statistics are reported for unit root tests of $lag = 2$ with or without *trend*, see col. (2) and (3). The results of the test are not reported for $lags = 0, 1$ with and without trend, but they are analogous. Namely, without trend, for all the 21 series in the levels, the null hypothesis of existence of a unit root, $\rho = 1$, cannot be rejected against the hypothesis of no unit root, $|\rho| < 1$[6]. With trend, for the 15 series in the levels, the null hypothesis of existence of a unit root, $\rho = 1$, cannot be rejected against the hypothesis of no unit root, $|\rho| < 1$, except for *butter*, *eggs*, *oil*, *milk*, *vodka*, *cognac*.

b) Second, for 19 series in the first differences, the null hypothesis of existence of a unit root, $\rho = 1$, is clearly rejected, with or without trend, meaning covariance stationarity for all these differenced series. For *beef* and *salo*, the null hypothesis of existence of a unit root, $\rho = 1$, is not rejected, with or without trend, meaning that intergratedness $I(2)$ is not excluded for these series. The result for *pork* is only concluding for the test with trend.

5. Integratedness I(1)

The result of this analysis of the unit roots with different approaches (Schmidt-Phillips, Sargan-Bargava, joint F. Wald test to attain parsimonious

[6]Following Maddala (1998), p. 38, this means covariance stationarity: That is, mean and variance are constant and the covariances γ_τ only depend on the differences $\tau = t_1 - t_2$, Maddala (1998), p. 10.

models, augmented Dickey-Fuller) tend to show that all the 21 series are tentatively integrated $I(1)$. Thus, cointegration analyses can go on, based on these results.