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STUDY OF UNSTEADY DUSTY FLUID FLOW THROUGH RECTANGULAR CHANNEL IN FRENET FRAME FIELD SYSTEM

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Abstract: The geometry of an unsteady viscous incompressible fluid with uniform distribution of dust particles through a long rectangular channel under the influence of time dependent periodic pressure gradient has been studied. Initially the fluid and dust particles are at rest. The analytical expressions for velocities of fluid and dust particles are obtained by solving the partial differential equations using variable separable method. The changes in the velocity profiles at different times are shown graphically.

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1. Introduction

Interest in problems of mechanics systems with more than one phase has developed rapidly in recent years. Situations which occur frequently are concerned

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with the flow of a dusty liquid, i.e., the system of fluid and dust particles. The influence of dusty fluid flow has its importance in many applications like environmental pollution, smoke emission from vehicles, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactors and formation of raindrops, etc. Also it is useful in the study of lunar ash flow which explains many features of lunar soil.

Using the formulation of P.G. Saffman [14] who gave the governing equations of dusty viscous fluid flow. Liu [10] has studied the flow induced by an oscillating infinite plat plate in a dusty gas. Michael and Miller [11] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in the semi-infinite space above a rigid plane boundary. Samba Siva Rao [15] has obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient, using appropriate boundary conditions. Dalal, Datta and Mukherjea [5] have studied flow in rectangular channel. Recently, Parveen Sharma and Varshney [12] studied the flow in hexagonal channel.

To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [9], Truesdell [16], Indrasena [8], Purushotham [13], Bagewadi, Shantharajappa and Gireesha [1, 2, 3] have applied differential geometry techniques. Further, the authors [2, 3] have studied two-dimensional dusty fluid flow in Frenet frame field system. Recently the authors [6, 7] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential. The present paper deals with study of flow of an unsteady viscous fluid with uniform distribution of dust particles through a rectangular channel under the influence of periodic pressure gradient in anholonomic co-ordinate system. Further by considering the fluid and dust particles to be at rest initially, the analytical expressions are obtained for velocities of fluid and dust particles. The graphical representation of the velocity profiles at different times are given.

2. Equations of Motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are written in the following form, see [14]:

For fluid phase:

$$\nabla . \overrightarrow{u} = 0 \quad \text{(Continuity)}, \tag{2.1}$$

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u} . \nabla) \overrightarrow{u} \tag{2.2}$$

$$= -\rho^{-1}\nabla p + \upsilon\nabla^2\overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) \qquad \text{(Linear Momentum)}\,.$$

For dust phase:

$$\nabla.\overrightarrow{v} = 0, \qquad \text{(Continuity)}\,, \tag{2.3}$$

$$\frac{\partial\overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \qquad \text{(Linear Momentum)}\,. \tag{2.4}$$

We have following nomenclature: $\overrightarrow{u}-$ velocity of the fluid phase, $\overrightarrow{v}-$ velocity of dust phase, $\rho-$ density of the gas, $p-$ pressure of the fluid, $N-$ number of density of dust particles, $\upsilon-$ kinematic viscosity, $k = 6\pi a\mu-$ Stoke's resistance (drag coefficient), $a-$ spherical radius of dust particle, $m-$ mass of the dust particle, $\mu-$ the coefficient of viscosity of fluid particles, $t-$ time.

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively, geometrical relations are given by Frenet formulae, [4]:

$$\begin{aligned}
&i) && \frac{\partial\overrightarrow{s}}{\partial s} = k_s\overrightarrow{n},\ \frac{\partial\overrightarrow{n}}{\partial s} = \tau_s\overrightarrow{b} - k_s\overrightarrow{s},\ \frac{\partial\overrightarrow{b}}{\partial s} = -\tau_s\overrightarrow{n}\,,\\
&ii) && \frac{\partial\overrightarrow{n}}{\partial n} = k'_n\overrightarrow{s},\ \frac{\partial\overrightarrow{b}}{\partial n} = -\sigma'_n\overrightarrow{s},\ \frac{\partial\overrightarrow{s}}{\partial n} = \sigma'_n\overrightarrow{b} - k'_n\overrightarrow{n}\,,\\
&iii) && \frac{\partial\overrightarrow{b}}{\partial b} = k''_b\overrightarrow{s},\ \frac{\partial\overrightarrow{n}}{\partial b} = -\sigma''_b\overrightarrow{s},\ \frac{\partial\overrightarrow{s}}{\partial b} = \sigma''_b\overrightarrow{n} - k''_b\overrightarrow{b}\,,\\
&iv) && \nabla.\overrightarrow{s} = \theta_{ns} + \theta_{bs};\ \nabla.\overrightarrow{n} = \theta_{bn} - k_s;\ \nabla.\overrightarrow{b} = \theta_{nb}\,,
\end{aligned} \tag{2.5}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. Formulation and Solution of the Problem

We consider a dusty fluid initially at rest within a closed retangular channel. The binormal direction is taken as axis of the channel as shown in Figure 1. The flow is due to the influence of time dependent Periodic pressure gradient. The

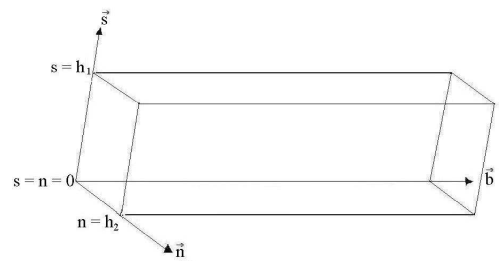

Figure 1: Schematic diagram of the dusty fluid flow

dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid. Also the number density of the dust particles is taken as a constant throughout the flow.

For this described flow the velocity components of both fluid and dust particles are respectively given by:

$$u_s = 0; \quad u_n = 0; \quad v_s = 0; \quad v_n = 0\,, \tag{3.1}$$

where (u_s, u_n, u_b) and (v_s, v_n, v_b) are velocity components of fluid and dust particles respectively.

By virtue of system of equations (2.5) the intrinsic decomposition of equations (2.2) and (2.4) using equation (3.1) give the following forms;

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial s} - 2{\sigma'}_n\frac{\partial u_b}{\partial n} + \tau_s k_s u_b\,, \tag{3.2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial n} - 2\tau_s\frac{\partial u_b}{\partial s} + {\sigma'}_n k'_n u_b\,, \tag{3.3}$$

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu\left[\frac{\partial^2 u_b}{\partial s^2} + \frac{\partial^2 u_b}{\partial n^2} - C_r u_b\right] + \frac{kN}{\rho}(v_b - u_b)\,, \tag{3.4}$$

$$\frac{\partial v_b}{\partial t} = \frac{k}{m}(u_b - v_b)\,, \tag{3.5}$$

$$v_b^2 k''_b = 0\,, \tag{3.6}$$

where $C_r = (\tau_s^2 + {\sigma'}_n^2)$ is called curvature number, see [3].

From equation (3.6) we see that $v_b^2 k''_b = 0$ which implies either $v_b = 0$ or $k''_b = 0$. The choice $v_b = 0$ is impossible, since if it happens then $u_b = 0$, which shows that the flow does not exist. Hence $k''_b = 0$, it suggests that the curvature of the streamline along binormal direction is zero. Thus no radial flow exists.

Assuming the pressure gradient to be periodic with period $2\pi/\beta^2$ to be impressed on the system for $t > 0$, we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial b} = Rp\{\alpha e^{i\beta^2 t}\}\,, \tag{3.7}$$

where α and β are reals and Rp denotes real part.

Using above relation the velocity components u_b, v_b can be written as

$$u_b(s,n,t) = Rp\{w_1(s,n)e^{i\beta^2 t}\}\,, \tag{3.8}$$
$$v_b(s,n,t) = Rp\{w_2(s,n)e^{i\beta^2 t}\}\,. \tag{3.9}$$

Using relations (3.7), (3.8) and (3.9) in equations (3.4) and (3.5) one can obtain the following differential equations;

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} - w_1\left(\frac{i\beta^2}{\nu} + C_r\right) + \frac{l}{\tau\nu}(w_2 - w_1) + \frac{\alpha}{\nu} = 0\,, \tag{3.10}$$
$$i\beta^2 w_2 = \frac{1}{\tau}(w_1 - w_2)\,, \tag{3.11}$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (3.11) implies

$$w_2 = \frac{w_1}{(1 + i\beta^2\tau)}\,. \tag{3.12}$$

Eliminating w_2 from (3.10) and (3.12) we obtain the following equation

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} + Q^2 w_1 + R = 0\,, \tag{3.13}$$

where

$$Q^2 = -(x+iy), \quad x = \left(\frac{C_r\nu\lambda^2 + \beta^4\tau l}{\lambda^2\nu}\right), \quad y = \left(\frac{\beta^2(l+\lambda^2)}{\lambda^2\nu}\right),$$
$$\lambda^2 = 1 + \beta^4\tau^2, \quad \text{and} \quad R = \frac{\alpha}{\nu}\,.$$

Equation (3.13) can be subject to the following no slip boundary conditions;

$$w_1(0,n) = 0, \quad w_1(h_1,n) = 0, \quad w_1(s,0) = 0, \quad w_1(s,h_2) = 0\,. \tag{3.14}$$

To solve equation (3.13) we assume the solution in the following form, [17]

$$w_1(s,n) = U(s,n) + V(s)\,. \tag{3.15}$$

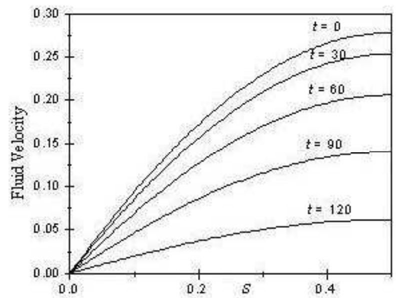

Figure 2: Variation of fluid phase velocity with s

Substitution of $w_1(s,n)$ in equation (3.13) yields

$$\frac{\partial^2 U}{\partial s^2} + \frac{\partial^2 V}{\partial s^2} + \frac{\partial^2 U}{\partial n^2} + Q^2(U+V) + R = 0\,,$$

so that if V satisfies

$$\frac{\partial^2 V}{\partial s^2} + Q^2 V + R = 0\,,$$

then

$$\frac{\partial^2 U}{\partial s^2} + \frac{\partial^2 U}{\partial n^2} + Q^2 U = 0\,. \tag{3.16}$$

In similar manner if $w_1(s,n)$ is inserted in no slip boundary conditions, one can obtain

$$\begin{cases} w_1(0,n) = U(0,n) + V(0) = 0, w_1(h_1,n) = U(h_1,n) + V(h_1) = 0, \\ w_1(s,0) = U(s,0) + V(s) = 0, w_1(s,h_2) = U(s,h_2) + V(s) = 0\,. \end{cases}$$

By solving the problem

$$\frac{\partial^2 V}{\partial s^2} + Q^2 V + R = 0\,,$$
$$V(0) = 0, \quad V(h_1) = 0\,,$$

one can obtain the solution in the form

$$V(s) = \frac{R}{Q^2}\left(\frac{\sin(Q(h_1 - s)) + \sin(Qs)}{\sin(Qh_1)} - 1\right)\,. \tag{3.17}$$

Using variable separable method, the solution of the problem (3.16) with the conditions

$$U(0,n) \;=\; 0, \qquad U(h_1,n)) = 0,$$

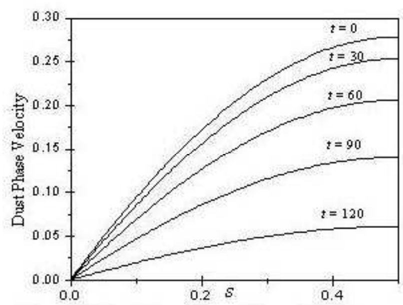

Figure 3: Variation of dust phase velocity with s

$$U(s,0) = -V(s), \quad U(s,h_2) = -V(s),$$

is obtained in the form

$$\begin{aligned} U(s,n) &= \frac{2h_1^2R}{\pi}\sum_{r=1}^{\infty}\frac{((-1)^r-1)}{r}\frac{1}{r^2\pi^2-h_1^2\beta^2} \\ &\quad\times\left(\frac{\sin(E(h_2-n))-\sin(En)}{\sin(Eh_2)}\right)\sin\left(\frac{r\pi}{h_1}s\right), \end{aligned} \tag{3.18}$$

where $E^2 = Q^2 - F^2$ and $F = \frac{r\pi}{h_1}$ Now by substituting (3.17) and (3.18) in (3.15) we have

$$\begin{aligned} w_1(s,n) &= \frac{2h_1^2R}{\pi}\sum_{r=1}^{\infty}\frac{((-1)^r-1)}{r}\frac{1}{r^2\pi^2-h_1^2\beta^2} \\ &\quad\times\left(\frac{\sin(E(h_2-n))-\sin(En)}{\sin(Eh_2)}\right)\sin\left(\frac{r\pi}{h_1}s\right) \\ &\quad+\frac{R}{Q^2}\left(\frac{\sin(Q(h_1-s))+\sin(Qs)}{\sin(Qh_1)}-1\right). \end{aligned}$$

Using w_1 in equation (3.12) one can see that

$$\begin{aligned} w_2(s,n) &= \frac{2h_1^2R}{(1+i\beta^2\tau)\pi}\sum_{r=1}^{\infty}\frac{((-1)^r-1)}{r}\frac{1}{r^2\pi^2-h_1^2\beta^2} \\ &\quad\times\left(\frac{\sin(E(h_2-n))-\sin(En)}{\sin(Eh_2)}\right)\sin\left(\frac{r\pi}{h_1}s\right) \\ &\quad+\frac{R}{Q^2(1+i\beta^2\tau)}\left(\frac{\sin(Q(h_1-s))+\sin(Qs)}{\sin(Qh_1)}-1\right). \end{aligned}$$

Using w_1 and w_2 in (3.8) and (3.9)respectively we obtain the following relations

$$\begin{aligned} u_b(s,n,t) &= \frac{2h_1^2 R}{\pi(C_1^2+D_1^2)} \sum_{r=1}^{\infty} \frac{((-1)^r-1)}{r} \frac{1}{r^2\pi^2 - h_1^2\beta^2} \\ &\times \left(m_1\cos(\beta^2 t) - m_2 \sin(\beta^2 t)\right) \sin\left(\frac{r\pi}{h_1}s\right) \\ &- \frac{R}{x^2+y^2}\left(m_3\cos(\beta^2 t) - m_4\sin(\beta^2 t)\right) \end{aligned}$$

Using w_1 in equation (3.12) one can see that

$$\begin{aligned} v_b(s,n,t) = &\frac{2h_1^2 R}{\pi\lambda^2(C_1^2+D_1^2)} \sum_{r=1}^{\infty} \frac{((-1)^r-1)}{r} \frac{1}{r^2\pi^2 - h_1^2\beta^2} \\ &\times \left((m_1+m_2\beta^2\tau)\cos(\beta^2 t) - (m_2 - m_1\beta^2\tau)\sin(\beta^2 t)\right) \sin\left(\frac{r\pi}{h_1}s\right) \\ &- \frac{R}{(x^2+y^2)\lambda^2}\left((m_3+m_4\beta^2\tau)\cos(\beta^2 t) - (m_4+m_3\beta^2\tau)\sin(\beta^2 t)\right)\ , \end{aligned}$$

where

$$\begin{aligned} m_1 &= (CC_1 + DD_1), \quad m_2 = (C_1 D - CD_1)\,, \\ & \quad m_3 = (AA_1 + BB_1), \quad m_4 = (A_1 B - AB_1)\,, \\ A &= \sin(z_1(h_1 - s))\cosh(z_2(h_1 - s)) + \sin(z_1 s)\cosh(z_2 s) \\ & \quad - \sin(z_1 h_1)\cosh(z_2 h_1)\,, \\ B &= \cos(z_1(h_1 - s))\sinh(z_2(h_1 - s)) + \cos(z_1 s)\sinh(z_2 s) \\ & \quad - \cos(z_1 h_1)\sinh(z_2 h_1)\,, \\ A_1 &= \sin(z_1 h_1)\cosh(z_2 h_1), \quad B_1 = \cos(z_1 h_1)\sinh(z_2 h_1)\,, \\ C &= \sin(z_3(h_2 - n))\cosh(z_4(h_2 - n)) - \sin(z_3 n)\cosh(z_4 n)\,, \\ D &= \cos(z_3(h_2 - n))\sinh(z_4(h_2 - n)) - \cos(z_3 n)\sinh(z_4 n)\,, \end{aligned}$$

$$\begin{aligned} C_1 &= \sin(z_3 h_2)\cosh(z_4 h_2), \quad D_1 = \cos(z_3 h_2)\sinh(z_4 h_2)\,, \\ z_1 &= -\sqrt{\frac{-x+\sqrt{x^2+y^2}}{2}}, \quad z_2 = \sqrt{\frac{x+\sqrt{x^2+y^2}}{2}}\,, \\ z_3 &= -\sqrt{\frac{-(x+F^2)+\sqrt{(x+F^2)^2+y^2}}{2}}\,, \end{aligned}$$

$$z_4 = \sqrt{\frac{(x+F^2)+\sqrt{(x+F^2)^2+y^2}}{2}}.$$

4. Conclusion

Figure 2 and Figure 3 show the distribution of velocity profiles for the fluid and dust particles, which are parabolic in nature. From these it is observed that velocity of fluid particles is parallel to that of dust. Also the velocity of both fluid and dust particles, which are nearer to the axis of flow, move with the greater velocity. Further one can observe that if the dust is very fine, i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same.

Graphs are drawn for the following values $h_1 = 1$, $h_2 = 2$, $n = 2$, $\tau = 0.5$, $\beta = 0.1$, $r = 1$, $l = 1$, $\nu = 1$, $R = 1$.

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