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# A NOTE ON MELLIN TRANSFORM AND PARTIAL DIFFERENTIAL EQUATIONS

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**Abstract:** In this study we consider Mellin transform to solve particular partial differential equations by using the double convolution in Mellin sense.

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## 1. Introduction

Integral transform are extensively used in solving several kinds of boundary value problems and integral equation. The double Mellin transform introduced by P.L. Kropivsky and E. Ben in the study of fragmentation in two dimensions, see [4] and also used by Yu.A. Brychkov et al, see [2]. The double Mellin transform is defined as

$$M_{xy}[f(x,y);p,q] = F(p,q) = \int_0^\infty \int_0^\infty f(x,y)x^{p-1}y^{q-1}dxdy\,, \tag{1}$$

where $p$ and $q$ are complex numbers, the symbol $M_{xy}[f(x,y);p,q]$ is used in order to show double Mellin transform respect to $x$ and $y$ consequently and arises in a natural way in the solution of boundary value problems concerning an infinite wedge. We also give definition of double convolution in the Mellin

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sense by using the generalized functions.

## 2. Elementary Properties of the Mellin transform

The elementary properties of Mellin transform can be found in several books in the literature for single variable, for example, see [5] by Ian N. Sneddon. In this study we shall try to extend some of the properties.

By making the simple change of variables $x = \dfrac{u}{a}$ and $y = \dfrac{v}{b}$ in definition of integral we have

$$\int_0^\infty \int_0^\infty f(ax, by) x^{p-1} y^{q-1} dx\, dy = a^{-p} b^{-q} \int_0^\infty \int_0^\infty f(u, v) u^{p-1} v^{q-1} du dv\,,$$

and it follows that

$$M_{xy}[f(ax, by); p, q] = a^{-p} b^{-q} F(p, q)\,, \tag{2}$$

since multiplying $f(x, y)$ by $x^a y^b$ merely results in change $p, q$ to $p + a, q + b$, thus the relation

$$M_{xy}[x^a y^b f(x, y); p, q] = F(p + a, q + b)\,, \quad a > 0, b > 0\,. \tag{3}$$

Similarly

$$M_{xy}\left[f(x^a, y^b); p, q\right] = \frac{1}{ab} F\left(\frac{p}{a}, \frac{q}{b}\right)\,, \quad a > 0, b > 0\,. \tag{4}$$

If we use the well-known formula

$$\frac{d^2}{dp\, dq}\left(x^{p-1} y^{\,p-1}\right) = \log(x) \log(y) x^{p-1} y^{p-1}\,,$$

we get

$$M_{xy}\left[\log(x) \log(y) f(x, y); p, q\right] = \frac{d^2}{dp\, dq} F(p, q)\,. \tag{5}$$

Now if we consider $f(x, y) = e^{-(x+y)}$ then by taking multiple Mellin transform we obtain well known result

$$F(p, q) = \Gamma(p)\Gamma(q)\,,$$

and by taking the second partial derivatives with respect to $p$ and $q$ consequently yields

$$\frac{d^2}{dpdq} F(p, q) = \Psi(p)\Gamma(p)\Psi(q)\Gamma(q)\,.$$

This is the right hand side of equation (5), where the symbol $\Psi(p)$ is known as digamma

$$\Psi(p) = \left(\frac{d}{dp}\Gamma(p)\right)/\Gamma(p)\,,$$

and similarly for $\Psi(q)$. Now if we take multiple Mellin transform for the function $f(x,y)$ multiply by $\log(x)$ and $\log(y)$ then we can obtain

$$M_{xy}\left[\log(x)\log(y)f(x,y);p,q\right] = \Psi(p)\Gamma(p)\Psi(q)\Gamma(q)\,.$$

It is also easy to make a statement that the multiple Mellin transform of $f(x,y) = e^{-(\lambda x+\mu y)}$ times any polynomials in the same variables $x$ and $y$ of the same degree in $p$ and $q$, multiplied by the gamma function $\Gamma(p)\Gamma(q)$. If we let

$$h_{nm}(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}x^i y^j$$

be some polynomials, then we obtain:

$$\int_0^\infty\int_0^\infty h_{nm}(x,y)e^{-(\lambda x+\mu y)}x^{p-1}y^{q-1}dx\,dy = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}$$
$$\times\int_0^\infty\int_0^\infty e^{-(\lambda x+\mu y)}x^{i+p-1}y^{j+q-1}dx\,dy = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\frac{\Gamma(p)\Gamma(q)}{\lambda^{i+p}\mu^{j+q}}(p)_i(q)_j\,,$$

where

$$(p)_i = \frac{\Gamma(p+i)}{\Gamma(p)},\quad (q)_i = \frac{\Gamma(q+i)}{\Gamma(q)}\,,$$

and it is known as the Pochhammer symbol, see [1]. In particular if we let $n = 1$ and $m = 1$, obtain

$$\sum_{i=0}^{1}\sum_{j=0}^{1} a_{ij}\frac{\Gamma(p+i)\Gamma(q+j)}{\lambda^{i+p}\mu^{j+q}}$$
$$= \frac{a_{00}\Gamma(p)\Gamma(q)}{\lambda^p\mu^q} + \frac{a_{10}\Gamma(p+1)\Gamma(q)}{\lambda^{1+p}\mu^q} + \frac{a_{01}\Gamma(p)\Gamma(q+1)}{\lambda^p\mu^{1+q}} + \frac{a_{11}\Gamma(p+1)\Gamma(q+1)}{\lambda^{1+p}\mu^{q+1}}\,.$$

Another useful result can be obtained by changing the variables of integration in the integral on the left of the formula

$$\int_0^1\int_0^1 (1-u)^{m-1}(1-v)^{n-1}u^{p-1}v^{q-1}du\,dv = \frac{\Gamma(m)\Gamma(n)\Gamma(p)\Gamma(q)}{\Gamma(m+p)\Gamma(n+q)},$$

$$\operatorname{Re} p, q > 0, \quad \operatorname{Re} m, n > 0 \,.$$

Let $u = \frac{x}{1+x}$, $v = \frac{y}{1+y}$, then it follows that

$$\int_0^\infty \int_0^\infty \frac{x^{p-1} y^{q-1}}{(1+x)^{m+p} (1+y)^{n+q}} dx\, dy = \frac{\Gamma(m)\Gamma(n)\Gamma(p)\Gamma(q)}{\Gamma(m+p)\Gamma(n+q)} \,, \tag{6}$$

which can be interpreted in the form

$$M_{xy}\left[(1+x)^{-m-p} (1+y)^{-n-q} ; p, q\right] = \frac{\Gamma(m)\Gamma(n)\Gamma(p)\Gamma(q)}{\Gamma(m+p)\Gamma(n+q)} \,. \tag{7}$$

If we let $a = m + p$ and $b = n + q$ substitute in equation (7) we get

$$M_{xy}[(1+x)^{-a} (1+y)^{-b} ; p, q] = \frac{\Gamma(a-p)\Gamma(b-q)\Gamma(p)\Gamma(q)}{\Gamma(a)\Gamma(b)} \,. \tag{8}$$

In particular if we take $a = 1$ and $b = 1$ and make use of the relation

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin \pi p}, \quad \Gamma(q)\Gamma(1-q) = \frac{\pi}{\sin \pi q} \,, \quad 0 < \operatorname{Re} p, q < 1 \,,$$

then it follows that

$$M_{xy}[(1+x)^{-1} (1+y)^{-1} ; p, q] = \pi^2 \csc(\pi p) \csc(\pi q), \quad 0 < \operatorname{Re} p,\, q < 1 \,. \tag{9}$$

## 3. Mellin Transform of Derivatives and Integral

In the applications of Mellin traransform we might need to express the Mellin transform of the derivatives. By the definition of the double Mellin traransform we have

$$M_{xy}\left[\frac{\partial^2}{\partial x \partial y} f(x,y); p, q\right] = \int_0^\infty \int_0^\infty \frac{\partial^2}{\partial x \partial y} f(x,y) x^{p-1} y^{q-1} dx\, dy \,, \tag{10}$$

$$M_{xy}\left[\frac{\partial^2}{\partial x \partial y} f(x,y); p, q\right] = \int_0^\infty \frac{\partial}{\partial x} \left( \int_0^\infty \frac{\partial}{\partial y} f(x,y) y^{q-1} dy \right) x^{p-1} dx \,, \tag{11}$$

then first we get:

$$\int_0^\infty \frac{\partial}{\partial y} f(x,y) y^{q-1} dy = [f(x,y)]_0^\infty - (q-1) \int_0^\infty f(x,y) y^{q-2} dy \,. \tag{12}$$

Hence if there exist $\sigma_1, \sigma_2$ such that

$$\lim_{y\to 0} y^{q-1} f(x,y) = 0, \quad \lim_{y\to\infty} y^{q-1} f(x,y) = 0\,,$$

where $\sigma_1 < \operatorname{Re} q < \sigma_2$ and if $F(x, q-1)$ exists in the band, then

$$\int_0^\infty \frac{\partial}{\partial y} f(x,y) y^{q-1} dy = -(q-1) F(x, q-1)\,, \tag{13}$$

then substitute (13) in (12) we get

$$\begin{aligned} M_{xy}\left[\frac{\partial^2}{\partial x \partial y} f(x,y); p, q\right] &= -(q-1) \int_0^\infty \frac{\partial}{\partial x} F(x, q-1) x^{p-1} dx \\ &= -(q-1) \left[F(x, q-1)\right]_0^\infty + (q-1)(p-1) \int_0^\infty F(x, q-1) x^{p-1} dx\,. \end{aligned}$$

Similarly, if there exist $\alpha_1, \alpha_2$ such that

$$\lim_{x\to 0} x^{p-1} f(x,y) = 0, \quad \lim_{x\to\infty} x^{p-1} f(x,y) = 0\,,$$

when $\alpha_1 < \operatorname{Re} p < \alpha_2$ and if $F(p-1, q-1)$ exists in the band, then

$$M_{xy}\left[\frac{\partial^2}{\partial x \partial y} f(x,y); p, q\right] = (p-1)(q-1)\ F(p-1, q-1)$$
$$\sigma_1 < \operatorname{Re} q < \sigma_2, \quad \alpha_1 < \operatorname{Re} p < \alpha_2\,. \tag{14}$$

By applying multiple Mellin transform for second derivative with respect to $x$ we have

$$\begin{aligned} M_{xy}\left[\frac{\partial^2}{\partial x^2} f(x,y); p, q\right] &= \int_0^\infty \int_0^\infty x^{p-1} y^{q-1} \frac{\partial^2}{\partial x^2} f(x,y) dx\, dy \\ &= \int_0^\infty \left(\int_0^\infty x^{p-1} \frac{\partial^2}{\partial x^2} f(x,y) dx\right) y^{q-1} dy\,. \end{aligned}$$

Then the integral with respect to $x$ follows that

$$\begin{aligned} &\int_0^\infty x^{p-1} \frac{\partial^2}{\partial x^2} f(x,y) dx \\ &\qquad = \left[x^{p-1} \frac{\partial}{\partial x} f(x,y)\right]_0^\infty - (p-1) \int_0^\infty x^{p-2} \frac{\partial}{\partial x} f(x,y) dx\,. \end{aligned} \tag{15}$$

Similarly,

$$\lim_{x\to 0} x^{p-1}\frac{\partial}{\partial x}f(x,y)=0, \quad \lim_{x\to\infty} x^{p-1}\frac{\partial}{\partial x}f(x,y)=0\,.$$

Now integrating the last term in equation (15) to obtain

$$-(p-1)\int_0^\infty x^{p-2}\frac{\partial}{\partial x}f(x,y)dx=(p-1)(p-2)F(p-2,y)\,. \tag{16}$$

Taking Mellin transform for equation (16) with respect to $y$ obtain

$$\begin{aligned} M_{xy}\left[\frac{\partial^2}{\partial x^2}f(x,y);p,q\right] &= (p-1)(p-2)\int_0^\infty y^{q-1}F(p-2,y)dy \\ &= (p-1)(p-2)F(p-2,q)\,. \end{aligned}$$

We can obviously prove by induction over $n$ that for all values of $p$ for which $F(p-n,q)$ exists and

$$M_{xy}\left[\frac{\partial^n}{\partial x^n}f(x,y);p,q\right]=(-1)^n\frac{\Gamma(p)}{\Gamma(p-n)}\ F(p-n,q)\,,$$

then

$$\lim_{x\to 0} x^{p-r-1}\frac{\partial^r}{\partial x^r}f(x,y)=0, \quad r=0,1,2...,n-1\,.$$

Similarly as above we can get multiple Mellin transform for second derivative with respect to $y$

$$M_{xy}\left[\frac{\partial^2}{\partial y^2}f(x,y);p,q\right]=(q-1)(q-2)F(p,q-2)\,.$$

Also we can prove by induction over $m$ multiple Mellin transform for all values of $q$, for which $F(p,q-m)$ exists and

$$M_{xy}\left[\frac{\partial^m}{\partial y^m}f(x,y);p,q\right]=(-1)^m\frac{\Gamma(q)}{\Gamma(q-m)}\ F(p,q-m)$$

then

$$\lim_{y\to 0} y^{\,p-h-1}\frac{\partial^h}{\partial y^h}f(x,y)=0, \quad h=0,1,2...,n-1\,.$$

Moreover multiple Mellin transform for second derivative with respect to $x$ and $y$ exist:

$$M_{xy}\left[xy\frac{\partial^2}{\partial x\partial y}f(x,y);p,q\right]=\int_0^\infty\int_0^\infty \frac{\partial^2}{\partial x\partial y}f(x,y)x^p y^q dxdy\,,$$

$$\int_0^\infty \int_0^\infty \frac{\partial^2}{\partial x \partial y} f(x,y) x^p y^q dx dy = \int_0^\infty \frac{\partial}{\partial y} \left( \int_0^\infty \frac{\partial}{\partial x} f(x,y) x^p dx \right) dy \,. \quad (17)$$

We integrate the integral inside equation (17) in the right hand and obtain:

$$\int_0^\infty \frac{\partial}{\partial x} f(x,y) x^p dx = [f(x,y)x^p]\,|_0^\infty - p \int_0^\infty f(x,y) x^{p-1} dx \,. \quad (18)$$

Similarly as above, if there exist $\alpha_1, \alpha_2$ such that

$$\lim_{x \to 0} x^p f(x,y) = 0, \quad \lim_{x \to \infty} x^p f(x,y) = 0 \,,$$

where $\alpha_1 < \operatorname{Re} p < \alpha_2$ and if $F(p,y)$ exists in the band, then

$$-p \int_0^\infty f(x,y) x^{p-1} dx = pF(p,y) \,. \quad (19)$$

By taking Mellin transform with respect to $y$ in the equation (19) we get

$$M_{xy} \left[ xy \frac{\partial^2}{\partial x \partial y} f(x,y); p, q \right] = -p \left[ F(p,y) y^q \right] |_0^\infty - pq \int_0^\infty F(p,y) y^{q-1} dy \,.$$

Similarly as above, if there exist $\sigma_1, \sigma_2$ such that

$$\lim_{y \to 0} y^q f(x,y) = 0, \quad \lim_{y \to \infty} y^q f(x,y) = 0$$

and if $F(p,q)$ exists in the band, then

$$M_{xy} \left[ xy \frac{\partial^2}{\partial x \partial y} f(x,y); p, q \right] = pqF(p,q) \,,$$

$$\sigma_1 < \operatorname{Re} q < \sigma_2, \;\; \alpha_1 < \operatorname{Re} p < \alpha_2 \,.$$

There exist multiple Mellin transform for first derivative with respect to $x$:

$$M_{xy} \left[ x \frac{\partial}{\partial x} f(x,y); p, q \right] = pF(p,q) \quad (20)$$

In general, multiple Mellin transform for derivatives of order $n$ with respect to $x$ are given by

$$M_{xy} \left[ \left( x \frac{\partial}{\partial x} \right)^n f(x,y); p, q \right] = (-p)^n F(p,q) \,. \quad (21)$$

In particular we deduce by putting $n = 2$ in equation that

$$M\left[x^2\frac{\partial^2}{\partial x^2}f(x,y) + x\frac{\partial}{\partial x}f(x,y);p,q\right] = p^2F(p,q)\,. \tag{23}$$

Similarly in general multiple Mellin transform for derivatives of order $m$ with respect to $y$ given by

$$M_{xy}\left[\left(y\frac{\partial}{\partial y}\right)^m f(x,y);p,q\right] = (-q)^m F(p,q)\,. \tag{24}$$

In particular we deduce by putting $m = 2$ in equation that

$$M_{xy}\left[y^2\frac{\partial^2}{\partial y^2}f(x,y) + y\frac{\partial}{\partial y}f(x,y);p,q\right] = q^2F(p,q)\,. \tag{25}$$

We note that some of these results can be written conveniently in the in term of integrals rather than derivatives. For instance we can write equation (14) in the form

$$M_{xy}\left[\frac{\partial^2}{\partial x\partial y}f(x,y);p,q\right] = (p-1)(q-1)M\left[\int_0^x\int_0^y f(u,v)dudv;p-1,q-1\right]\,. \tag{26}$$

Replace $p$ and $q$ by $p+1$ and $q+1$ we can now write this as

$$M_{xy}\left[\int_0^x\int_0^y f(u,v)dudv;p,q\right] = \frac{1}{pq}F(p+1,q+1)\,. \tag{27}$$

## 4. Convolution in Mellin Sense

Before we define the convolution in Mellin sense we shall make use of the Mellin transform of certain integral expressions. For instance,

$$\begin{aligned}
M_{xy}&\left[\int_0^\infty\int_0^\infty u^m v^n f(ux,vy)g(u,v)dudv;p,q\right] \\
&= \int_0^\infty\int_0^\infty x^{p-1}y^{q-1}dxdy\int_0^\infty\int_0^\infty u^m v^n f(ux,vy)g(u,v)dudv \\
&= \int_0^\infty\int_0^\infty u^m v^n g(u,v)dudv\int_0^\infty\int_0^\infty x^{p-1}y^{q-1}f(ux,vy)dxdy
\end{aligned}$$

$$= \int_0^\infty \int_0^\infty u^{m-p} v^{n-q} g(u,v) du dv \int_0^\infty \int_0^\infty h^{p-1} k^{q-1} f(h,k) dh dk \,.$$

Thus

$$M_{xy} \left[ \int_0^\infty \int_0^\infty u^m v^n f(ux, vy) g(u,v) du dv; p, q \right] \\ = F(p,q) G(m-p+1, n-q+1) \,. \quad (28)$$

If we replace the variables $ux$ and $vy$ by $\frac{x}{u}$ and $\frac{y}{v}$ respectively in equation (28) we have

$$M_{xy} \left[ \int_0^\infty \int_0^\infty u^m v^n f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v) du dv; p, q \right] \\ = G(m+p+1, n+q+1) F(p,q) \,. \quad (29)$$

In particular if we take $m = -1$ and $n = -1$ in equation (29) we obtained

$$M_{xy} \left[ \int_0^\infty \int_0^\infty u^{-1} v^{-1} f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v) du dv; p, q \right] = G(p,q) F(p,q) \,. \quad (30)$$

The equation (30) is known as double Mellin transform for double convolution in Mellin sense. Therefore we have the following definition.

**Definition 1.** Let $f$ and $g$ be function from $R_+^n$ into $\mathbb{C}$. The function $f(x,y) * *g(x,y)$ defined by mean of

$$f(x,y) *_m *g(x,y) = \int_0^\infty \int_0^\infty u^{-1} v^{-1} f(\frac{x}{u}, \frac{y}{v}) g(u,v) du dv \quad (31)$$

is called double Mellin convolution of $f$ and $g$. The symbol $*_m*$ is used for double convolution with respects to $x$ and $y$. The inverse of equation (30) can be given by the relation

$$M_{pq}^{-1} \left[ G(p,q) F(p,q); x, y \right] = \int_0^\infty \int_0^\infty u^{-1} v^{-1} f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v) du dv$$

and it is known as the inverse Mellin double convolution.

**Definition 2.** The space $H_{a_1,a_2}^{b_1,b_2}$, where $a_1, a_2, b_1, b_2 \in R^n$ and $a_1 \leq b_1$ $a_2 \leq b_2$ is the linear space of $R_+^n \to \mathbb{C}$ function, such that $x^{p-1} y^{q-1} f(x,y) \in L_1\left(R_+^n\right)$ for all $p, q \in C_{a_1,a_2}^{b_1,b_2}$, where

$$C_{a_1,a_2}^{b_1,b_2} = \left\{ p, q : p, q \in \mathbb{C}^n, a_1 \leq \operatorname{Re} p \leq b_1 \text{ and } a_2 \leq \operatorname{Re} q \leq b_2 \right\} .$$

**Theorem 1.** *Let* $f, g \in H^{b_1,b_2}_{a_1,a_2}$. *Then* $f(x,y) *_m *g(x,y) \in H^{b_1,b_2}_{a_1,a_2}$ *and double Mellin transform for double convolution exists and*

$$M_{xy}\left[\int_0^\infty \int_0^\infty u^{-1}v^{-1} f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v)dudv; p, q\right] = G(p,q)F(p,q) \qquad (32)$$

*Proof.* Since

$$\begin{aligned}
&\int_0^\infty \int_0^\infty |f(x,y) *_m *g(x,y)|\, dxdy \\
&\quad = \int_0^\infty \int_0^\infty x^{p-1}y^{q-1} \left| \int_0^\infty \int_0^\infty u^{-1}v^{-1} f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v)dudv \right| dxdy \\
&\quad \leq \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty x^{p-1}y^{q-1} \left| f\left(\frac{x}{u}, \frac{y}{v}\right) g(u,v) \right| u^{-1}v^{-1} dxdydudv \\
&\qquad = \int_0^\infty \int_0^\infty |f(h,k)|\, h^{p-1}k^{q-1} dhdk \int_0^\infty \int_0^\infty |g(u,v)|\, u^{p-1}v^{q-1},
\end{aligned}$$

the integrals at the right-hand side exist if $a_1 \leq p \leq b_1$and $a_2 \leq q \leq b_2$ . We conclude the existence of the left-hand side, $f(x,y) *_m *g(x,y) \in H^b_a$ and the equation (32) follows

$$\begin{aligned}
&M_{xy}\left[f(x,y) *_m *g(x,y); p, q\right] \\
&\quad = \int_0^\infty \int_0^\infty u^{p-1}v^{q-1} g(u,v)dudv \int_0^\infty \int_0^\infty h^{p-1}k^{q-1} f(h,k)dhdk \\
&\quad = G(p,q)F(p,q) . \quad \square
\end{aligned}$$

## 5. An Application of Mellin Transform in Partial Differential Equation

**Example 1.** Now if we want to solve the partial differential equation with non constant coefficient $x^2u_{xx}(x,y) - 2xyu_{xy}(x,y) + y^2u_{yy}(x,y) = e^{-(x+y)}$. We take multiple Mellin transform for both sides of the equation

$$M_{xy}\left[x^2u_{xx}(x,y) - 2xyu_{xy}(x,y) + y^2u_{yy}(x,y)\right] = M_{xy}\left[e^{-(x+y)}\right] .$$

Thus we get

$$p^2U(p,q) - 2pqU(p,q) + q^2U(p,q) = \Gamma(p)\Gamma(q) .$$

After simplifying we obtain

$$U(p,q) = \frac{\Gamma(p)\Gamma(q)}{(p-q)^2}\,.$$

If the inverse multiple Mellin transform exists then we write it as

$$u(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\Gamma(p)\Gamma(q)}{(p-q)^2}x^{-p}y^{-q}dpdq\,,$$

where $p \neq q$.

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