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ON THE ADJACENT VERTEX-DISTINGUISHING TOTAL CHROMATIC NUMBER OF $S_m \vee S_n$

Donghan Zhang[1 §], Pengxiang Qiu[2], Liang Bian[3]
[1,2,3]Institute of Applied Mathematics
Lanzhou Jiaotong University
Lanzhou, 730070, P.R. CHINA
[1]e-mail: zhanghan227@sohu.com

Abstract: In this paper, the adjacent vertex-distinguishing total chromatic number of $S_m \vee S_n$was obtained.

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1. Introduction

The graph coloring is one of the chief topics in graph research. More and more experts begin to study adjacent vertex distinguishing total coloring. In this paper, the adjacent vertex-distinguishing total chromatic number of $S_m \vee S_n$ is obtained.

Definition 1. (see [3], [2]) If G is a connect graph, which order is at least, k is an positive integer and f is the mapping form $V(G) \cup E(G)$ to $\{1, 2, \cdots, k\}$, for any $u \in V(G)$, note down $C(u) = \{f(u)\} \cup \{f(uv)|uv \in E(G)\}$, if:

1. For any $uv, vw \in E(G)$, $u \neq w$, there is $f(uv) \neq f(vw)$;
2. For any $uv \in E(G)$, $u \neq v$, there is $f(u) \neq f(v)$, $f(u) \neq f(uv)$, $f(v) \neq f(uv)$;
3. For any $uv \in E(G)$, $u \neq v$, there is $C(u) \neq C(v)$.

Then, we call k of G adjacent vertex-distinguishing total coloring (in brief, it is note it as k-AVDTC). We can name $\min\{k|G has k-AVDTC\}$ as the

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§Correspondence author

number of adjacent vertex-distinguish total coloring, and note it as $\chi_{at}(G)$.

Definition 2. (see [3]) For graph G and H $(V(G) \cap V(H) = \phi E(G) \cap E(H) = \phi)$ a new graph denoted by $G \vee H$ is called the join of G and H. If $V(G \vee H) = V(G) \vee V(H)$, $E(G \vee H) = E(G) \cup E(H) \cup \{uv/u \in V(G)v \in V(H)\}$.

2. Main Results

Lemma 1. (see [3]) *If G has two adjacent maximum degree vertices, then*

$$\chi_{at} \geq \Delta(G) + 2\,.$$

Lemma 2. (see [3]) *Suppose K_n is complete graph with order n, then*

$$\chi_{at} = \begin{cases} n+1, & n \equiv 0(\ mod\ 2); \\ n+2, & n \equiv 1(\ mod\ 2) \text{ and } n \geq 3. \end{cases}$$

Lemma 3. *Suppose S_n is star with order $n+1$, then*

$$\Delta(S_m \vee S_n) = m + n + 1.$$

Theorem 1. *For $n = 0$, then:*

$$\chi_{at}(S_m \vee S_0) = \begin{cases} 3, & \text{if } m = 0; \\ 5, & \text{if } m = 1; \\ m+3, & \text{if } m \geq 2. \end{cases}$$

Proof. There are three cases to be considered:

Case 1. If $m = 0$, then: $S_0 \vee S_0 = K_2$, see Lemma 2.

Case 2. If $m = 1$, then: $S_1 \vee S_0 = K_3$, see Lemma 2.

Case 3. When $m \geq 2$, form Lemma 1, $\chi_{at} \geq m+3$ is right. So the only thing we need to prove is $S_m \vee S_0$ exists $(m+3)$-AVDTC.

Let, $V(S_m) = \{u_i | i = 0, 1, 2 \cdots m\}$, $E(S_m) = \{u_0 u_i | i = 1, 2 \cdots m\}$, $V(S_n) = \{v_i | i = 0, 1, 2 \cdots n\}$, $E(S_n) = \{v_0 v_i | i = 1, 2 \cdots n\}$.

A mapping f is defined as follows:

$f(u_0 u_i) = i,\ i = 1, 2 \cdots m\,, \quad f(u_0 v_0) = m+1;\ f(u_0) = m+2;$

$f(v_0 u_i) = i+1,\ i = 1, 2 \cdots m-1\,, \quad f(v_0 u_m) = m+2;\ f(v_0) = m+3;$

$f(u_i) = i+2, i = 1, 2 \cdots m-1\,, \quad f(u_m) = 1\,.$

It is easy to see that f is a $(m+3) - AVDTC$ of $(S_m \vee S_0)$. □

Theorem 2. *For $m \geq n \geq 1$, then*

$$\chi_{at}(S_m \vee S_n) = m + n + 3.$$

Proof. From Lemma 1 and Lemma 3, $\chi_{at}(S_m \vee S_n) \geq m + n + 3$ is right.

Let, $V(S_m) = \{u_i | i = 0, 1, 2 \cdots m\}$, $E(S_m) = \{u_0 u_i | i = 1, 2 \cdots m\}$ $V(S_n) = \{v_i | i = 0, 1, 2 \cdots n\}$, $E(S_n) = \{v_0 v_i | i = 1, 2 \cdots n\}$.

There are four cases to be considered:

Case 1. $m = n = 1$: $S_1 \vee S_1 = K_4$, see Lemma 2.

Case 2. $m = n \geq 2$: The only ting we need to prove is $S_n \vee S_n$ exists $(2n+3)$ of AVDTC.

A mapping f is defined as follows. Let $C = \{1, 2 \cdots 2n + 2\ , 0\}$,

$f(u_0 u_i) = i; i = 1, 2 \cdots n$; $f(u_0 v_i) = n + 1 + i; i = 1, 2 \cdots n$;

$f(u_0 v_0) = n + 1; f(u_0) = 2n + 2$;

$f(v_0) = 0; f(v_0 v_1) = 2n + 2$; $f(v_0 v_i) = i - 1; i = 2, 3 \cdots n$;

$f(v_0 u_1) = n$; $f(v_0 u_i) = n + i, i = 2, 3 \cdots n$;

$f(u_1 v_i) = n + i, i = 1, 2 \cdots n$;

$f(u_i v_j) = n + i + j(\bmod (2n + 3)), i = 2, 3, \cdots n, j = 1, 2, 3, \cdots n$;

$f(u_1) = 2n + 1, f(u_i) = i - 1, i = 2, 3 \cdots n$;

$f(v_i) = n - 1 + i, i = 1, 2 \cdots n$;

Obviously, f is a (2n+3)-AVDTC of $S_n \vee S_n$

Case 3. $m \geq 2$, $n = 1$: The only ting we need to prove is $S_m \vee S_1$ exists (m+4) of AVDTC.

We define a mapping f as follows;

$f(u_0 u_i) = i,\ i = 1, 2 \cdots m$; $f(u_0 v_0) = m + 2$.

$f(u_0 v_1) = m + 3$, $f(v_0 v_1) = m + 4$.

$f(v_0 u_i) = i + 1$, $i = 1, 2 \cdots m$.

$f(v_1 u_i) = i + 2$, $i = 1, 2 \cdots m$. $f(v_0) = 1$, $f(v_1) = 2$.

$f(u_0) = m + 4, f(u_i) = i + 3,\ i = 1, 2 \cdots m$.

Obviously, f is a (m+4)-AVDTC of $S_m \vee S_1$.

Case 4. $m > n \geq 2$: We define a mapping f as follows;

$f(u_0 u_i) = i + 2,\ i = 1, 2 \cdots m$;

$f(u_0 v_i) = m + 2;\ i = 1, 2 \cdots n$

$f(v_0 v_i) = m + i + 1,\ i = 1, 2 \cdots n$;

$f(v_0 u_i) = i + 1,\ i = 1, 2 \cdots m$;

$f(u_i v_j) = i + j + m + 2(\bmod (m + n + 3))$; $i = 1, 2 \cdots m,\ j = 1, 2 \cdots n$

$f(u_0 v_0) = 1$;

$f(u_0) = m + n + 3$, $f(v_0) = m + n + 2$;

$f(u_i) = i,\ i = 1, 2 \cdots m$;

$f(v_i) = m + i,\ i = 1, 2 \cdots n$;

Obviously, f is a mapping (m+n+3)-AVDTC of $S_m \vee S_n$.

So, Theorem 2 holds true. □

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