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$(\alpha,\beta)-(m_X,m_Y)$-CONTRA SEMI CONTINUOUS FUNCTIONS AND $(\alpha,\beta)-(m_X,m_Y)$-IRRESOLUTE FUNCTIONS

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Abstract: The aim of this paper, is to investigate and characterizes the $(\alpha,\beta)-(m_X,m_Y)$-contra continuous and $(\alpha,\beta)-(m_X,m_Y)$ contra semi continuous functions. Also we introduce and investigate the $\alpha\beta-(m_X,m_Y)$ irresolute functions and $\alpha\beta-(m_X,m_Y)$ contra irresolute functions, some characterizations are obtained and some applications are shown.

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1. Introduction

The notion of contra continuity was introduced by Dontchev in [4]. He defined a function $f : X \to Y$ is said to be contra continuous if the inverse image of any open set in Y is closed in X. Also Dontchev and Noiri in [5] introduced and investigated a new class of functions called contra semi continuous. A function $f : X \to Y$ is said to be contra semi continuous if the inverse image of any open set in Y is semi closed in X. In [13]. E. Rosas, C. Carpintero, gave the definitions of (α,β)-contra continuous functions and (α,β)-contra semi continuous

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functions as follows: $f : X \to Y$ is said to be (α, β) contra continuous ((α, β) contra semi continuous respectively) if the inverse image of any β open set in Y is α closed (α semi closed set respectively) in X. In the same way, Jafari and Noiri in [9] defined the notion of contra precontinuous functions as follows: $f : X \to Y$ is said to be contra precontinuous if the inverse image of any open set in Y is preclosed in X. Also Erdal Ekici in [6] defined the notion of almost contra precontinuous functions as follows $f : X \to Y$ is said to be almost contra precontinuous if the inverse image of any regular open set in Y is preclosed in X. In [11]. Navagali, gave the definition of irresolute functions as follows $f : X \to Y$ is said to be irresolute if the inverse image of any semi open set in Y is semi open in X. In this paper, given an m-space (X, m_X) without topological Structure. We introduce and study the notion of $(\alpha, \beta) - (m_X, m_Y)$- contra continuous functions and $(\alpha, \beta) - (m_X, m_Y)$-contra semi continuous functions, also the $\alpha\beta - (m_X, m_Y)$-irresolute functions and the $\alpha\beta - (m_X, m_Y)$ contra irresolute functions are defined and studied some characterizations of these classes of functions using related notions among $\alpha - m_X$ kernel, $\alpha - m_X$-semi kernel, $\alpha - m_X$ closure or the $\alpha - m_X$ semi closure. Also we gave some important results with respect to these class of functions, where (α, β) are m-operators on m_X and m_Y respectively. We can see that this work generalize the results given in [5], [10], [13] and [16].

2. Preliminary

Let X be a nonempty set without topological structure. A collection m_X of subsets of X is said to be an m structure if $X, \emptyset \in m_X$. The elements of m_X are called m_X-open sets. The pair (X, m_X) is called an m-space. $\alpha : P(X) \to P(X)$ is said to be an m-operator on m_X if α, is expansive on m_X (that is $U \subseteq \alpha(U)$ for all $U \in m_X$). The triple (X, m_X, α) is called an $\alpha - m$-space. A subset $A \subseteq X$ is said to be $\alpha - m_X$-open set if for each $x \in A$ there exists $U \in m_X$ such that $x \in U$ and $\alpha(U) \subseteq A$ (a subset of X is said to be $\alpha - m_X$-closed if its complement is an $\alpha - m_X$-open set). We denote by $O(X, m_X, \alpha)$, the collection of all $\alpha - m_X$-open sets. A subset $A \subseteq X$ is said to be $\alpha - m_X$-semi open set if there exists $U \in m_X$ such that $U \subseteq A \subseteq \alpha(U)$ (a subset of X is said to be $\alpha - m_X$-semi closed if its complement is an $\alpha - m_X$-semi open set). We denote by $SO(X, m_X, \alpha)$, the collection of all $\alpha - m_X$-semi open sets. In general, the union of $\alpha - m_X$-open sets is $\alpha - m_X$-open, but the union of $\alpha - m_X$-semi open sets is not necessarily $\alpha - m_X$-semi open, but if we give an additional condition of monotony to the operator α (α is said to be a monotone operator if $\alpha(U)\subseteq\alpha(V)$ for all $U\subseteq V$) and m_X satisfies the property (B) of Maki (that is, the union of any collection of elements of m_X is an element of m_X), then the union of $\alpha-m_X$-semi open sets is $\alpha-m_X$-semi open. In the case that (X,m_X) is an m-space and m_X satisfy the property (B) of Maki, we said that m_X is a minimal structure on X.

Definition 2.1. Let (X,m_X,α) be an $\alpha-m$-space and A be a subset of X:

1. The set $\bigcap\{U\in O(X,m_X,\alpha): A\subseteq U\}$ is called the $\alpha-m_X$-kernel of A, and is denoted by $\alpha-m_X$-ker(A).

2. The set $\bigcap\{U\in m_X-SO(X,m_X,\alpha): A\subseteq U\}$ is called the $\alpha-m_X$-semi kernel of A and is denoted by $\alpha-m_X$-sker(A).

Observation 2.1. In the case that, m_X is a topology on X, we obtain the notions of $\alpha-$ kernel and $\alpha-$semi kernel defined in [18].

The following theorem, characterizes the m_X-kernel and the $\alpha-m_X$-semi kernel.

Theorem 2.1. *Let (X,m_X,α) be an $\alpha-m$-space. A and B subsets of X. Then:*

1. $x\in\alpha-m_X$-ker (A) if and only if $A\cap F\neq\emptyset$ for any $\alpha-m_X$-closed set F that contain x.

2. $A\subseteq\alpha-m_X$-ker (A) and $A=\alpha-m_X$-ker (A) if A is $\alpha-m_X$ -open.

3. If $A\subseteq B$, then $\alpha-m_X$-ker $(A)\subseteq\alpha-m_X$-ker (B).

4. $x\in\alpha-m_X$-sker (A) if and only if $A\cap F\neq\emptyset$ for any $\alpha-m_X$-semi closed set F that contain x.

5. $A\subseteq\alpha-m_X$-sker (A) and $A=\alpha-m_X$-sker (A) if A is $\alpha-m_X$-semi open.

6. If $A\subseteq B$, then $\alpha-m_X$-sker $(A)\subseteq\alpha-m_X$-sker (B).

Definition 2.2. Let (X,m_X,α) be an $\alpha-m$-space. We define the $\alpha-m_X$ semi clausure of a subset A of X, denoted by $\alpha-m_X$-scl(A) as the intersection of all $\alpha-m_X$-semi closed sets containing A.

Observation 2.2. In the case that, m_X is a topology on X, we obtain the notions of $\alpha-$ semi closure [14].

Note that, in the case that m_X is a minimal structure and α is a monotone operator, then for any subset $A\subseteq X$ the $\alpha-m_X-\text{scl}(A)$ is an $\alpha-m_X$ semi-closed set.

The following lemma characterizes the points of the $\alpha-m_X$ semi-closure of a subset $A\subseteq X$.

Lemma 2.1. *Let (X, m_X, α) be an $\alpha - m$-space and $A \subseteq X$. $x \in \alpha - m_X$-*scl *(A) if and only if for all $\alpha - m_X$-semi open set S of X containing X, $S \cap A \neq \emptyset$.*

Proof. Sufficiency. If there exists an $\alpha - m_X$-semi open set $S \subseteq X$ such that $x \in S$ and $A \cap S = \emptyset$, then, $F = X \setminus S$ is an $\alpha - m_X$-semi closed set, $A \subseteq F$ and $x \notin F$, this implies that $x \notin \alpha - m_X - \operatorname{scl}(A)$.

Necessity. Suppose that $x \notin \alpha - m_X - \operatorname{scl}(A)$. Then, there exists an $\alpha - m_X$-semi closed set F in X, such that $A \subseteq F$ and $x \notin F$. Now the set $S = X \setminus F$, is an $\alpha - m_X$-semi open set in X, $x \in S$ and $A \cap S = \emptyset$. □

3. $(\alpha, \beta) - (m_X, m_Y)$-Contra Continuous Functions and $(\alpha, \beta) - (m_X, m_Y)$-Contra Semi-Continuous Functions

Now, we introduce the concepts of $(\alpha, \beta) - (m_X, m_Y)$ contra continuous functions and $(\alpha, \beta) - (m_X, m_Y)$ contra semi continuous functions and give a characterization of these functions using notions of ker nels and closures.

Definition 3.1. Let (X, m_X, α) and (Y, m_Y, β) be m-spaces, a map $f : X \to Y$ is said to be $(\alpha, \beta) - (m_X, m_Y)$ contra continuous if $f^{-1}(B)$ is $\alpha - m_X$-closed set in X for all $\beta - m_Y$-open set B in Y.

Observation 3.1. Observe that in the above definition:

1. If m_X, (resp. m_Y) is a topology on X, (resp. Y,) $\alpha, (\beta)$ the identities operators on X, (resp. Y,) we obtain the notions of contra continuous functions defined in [4].

2. If $m_X, (resp. m_Y)$ is the collections of $\alpha - (resp. \beta)$ open sets in X, (resp. Y,) we obtain the notion of (α, β)-contra continuous functions defined in [18].

3. If m_Y is the collections of closed sets in Y, m_X is the collections of preopen sets in X and α, (resp. β) is the identities in X (resp. Y). We obtain the notion of contra precontinuous functions defined in [9].

4. If m_Y is the collections of regular open set in Y, m_X is the collections of preclosed sets in X, β the identities in Y and α is the identities in X. We obtain the notion of almost contra precontinuous functions defined in [6].

5. If m_Y is the collections of regular open sets in Y, that is the collection of all $S \in P(Y)$ such that $S = \operatorname{int}(\operatorname{cl}(S))$, $m_X = \delta - PO(X)$ that is the collections of all $S \in P(Y)$ such that $S = \operatorname{int}(\delta - \operatorname{cl}(S))$, where $\delta - \operatorname{cl}(S)$ is the set of all $x \in X$ such that $S \cap (\operatorname{int}(\operatorname{cl}(U))) \neq \emptyset$, β the identity in Y and α the identity in X. We obtain the notion of almost δ precontinuous functions defined in [7].

6. If m_Y is the collections of regular open sets in Y, $m_X = \delta - PC(X)$ is the collections of δ-precloset sets in X and α, (resp. β) is the identities in X (resp. Y). We obtain the notion of $(\delta - pre, s)$ continuous functions defined in [8].

Definition 3.2. Let (X, m_X, α) and (Y, m_Y, β) be m spaces, a map $f : X \to Y$ is said to be $(\alpha, \beta) - (m_X, m_Y)$ contra semi continuous if $f^{-1}(B)$ is $\alpha - m_X$-semi closed in X for all $\beta - m_Y$ -open set B in Y.

Observation 3.2. Observe that in the above definition:

1. If m_X, m_Y are topology on X, Y respectively, α, β the identities operators on X, Y respectively, we obtain the notions of contra continuous functions defined in [4].

2. If m_X is a collection of α-semi open set and m_Y is the collection of β semi open sets in Y, we obtain the notion of (α, β)-contra semi continuous functions defined in [17]

The following theorem gives a relation between $(\alpha, \beta) - (m_X, m_Y)$-contra continuous functions and $(\alpha, \beta) - (m_X, m_Y)$ -contra semi continuous functions.

Theorem 3.1. *Let $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$. If f is an $(\alpha, \beta) - (m_X, m_Y)$-contra continuous function, m_X is a minimal structure and α is a monotone operator, then f is an $(\alpha, \beta) - (m_X, m_Y)$-contra semi continuous.*

The following theorem gives another characterization of $(\alpha, \beta) - (m_X, m_Y)$-contra continuous functions.

Theorem 3.2. *Let $f : (X, m_X) \to (Y, m_Y, \beta)$ be a function and m_X a minimal structure. The following propositions are equivalent:*

1. f is $(\alpha, \beta) - (m_X, m_Y)$-contra continuous.

2. For all $\beta - m_Y$-closed set F in Y, $f^{-1}(F) \in O(X, m_X, \alpha)$.

3. $f(\alpha - m_X - \text{cl}(A)) \subset \beta - m_Y\text{-ker}(f(A))$ for all subset A of X.

4. $\alpha - m_X - \text{cl}(f^{-1}(B)) \subset f^{-1}(\beta - m_Y\text{-ker}(B))$ for all subset B of Y.

5. For each $x \in X$ and any $\beta - m_Y$-closed set F in Y, such that $f(x) \in F$, there exists an $\alpha - m_X$-open set U in X such that $x \in U$ and $f(U) \subset F$.

Proof. The equivalence $1 \Leftrightarrow 2$ follows easily.

$(2 \Rightarrow 3)$ Let A be a subset of X and suppose that $y \notin \beta - m_Y\text{-ker}(f(A))$, then there exists a $\beta - m_Y$-closed set F in Y, such that $y \in F$ and $f(A) \cap F = \emptyset$, therefore $f^{-1}(f(A) \cap F) = \emptyset$. This implies that $A \cap f^{-1}(F) = \emptyset$, and therefore, $\alpha - m_X - \text{cl}(A) \subset (f^{-1}(F))^c$. It follows that $f(\alpha - m_X - \text{cl}(A)) \cap F = \emptyset$, which implies that $y \notin f(\alpha - m_X - \text{cl}(A))$. In consequence, we have proved that $f(\alpha - m_X - \text{cl}(A)) \subset \beta - m_Y\text{-ker}(f(A))$ for all subset A of X.

($3 \Rightarrow 4$) Let B be any subset of Y. Then $f^{-1}(B) \subseteq X$. By hypothesis $f(\alpha - m_X{-}\mathrm{cl}\,(f^{-1}(B))) \subset \beta - m_Y\text{-ker}\,(f(f^{-1}(B)))$. It follows that $f(\alpha - m_X{-}\mathrm{cl}\,(f^{-1}(B))) \subset \beta{-}m_Y\text{-ker}\,(B)$, consequently, $\alpha{-}m_X\text{-cl}\,(f^{-1}(B) \subset f^{-1}(\beta{-}m_Y\text{-ker}\,(B))$.

($4 \Rightarrow 1$) Let V be any $\beta - m_Y$-open set. By hypothesis

$$\alpha - m_X - \mathrm{cl}\,(f^{-1}(V) \subset f^{-1}(\beta - m_Y - \ker\,(V)),$$

since V is a $\beta{-}m_Y$-open set then, $\beta{-}m_Y\text{-ker}\,(V) = V$. In consequence $\alpha{-}m_X$-cl $(f^{-1}(V) \subset f^{-1}(V)$. It follows that $\alpha - m_X\text{-cl}\,(f^{-1}(V) = f^{-1}(V)$.

($1 \Rightarrow 5$) Let $x \in X$ and F be any $\beta - m_Y$-closed set in Y, such that $f(x) \in F$. By hypothesis f is $(\alpha, \beta) - (m_X, m_Y)$-contra continuous, then $f^{-1}(B) \in O(X, m_X, \alpha)$ and $x \in f^{-1}(B)$. Taking $U = f^{-1}(B)$ the result follows.

($5 \Rightarrow 1$) Let F be any $\beta{-}m_Y$-closed set in Y. If $f^{-1}(F) = \emptyset$, then $f^{-1}(F) \in O(X, m_X, \alpha)$, if $f^{-1}(F) \neq \emptyset$, then for each $x \in X$ such that $f(x) \in F$, we can find sets $U_x \in O(X, m_X, \alpha)$ such that $x \in U_x$ and $f(U_x) \subseteq F$. It follows that $U_x \subseteq f^{-1}(F)$ for each $x \in U_x$ and $f^{-1}(F)$ is an m_X-open set in X. □

Theorem 3.3. *Let $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$ be a function and α be a monotone operator. If m_X is a minimal structure, the following properties are equivalent:*

1. *f is $(\alpha, \beta) - (m_X, m_Y)$-contra semi continuous.*
2. *For all $\beta - m_Y$-closed set F in Y, $f^{-1}(F) \in{-}SO(X, m_X, \alpha)$.*
3. *For each $x \in X$ and any $\beta - m_Y$-closed set F in Y, such that $f(x) \in F$, there exists an $\alpha - m_X$-semi open set U in X such that $x \in U$ and $f(U) \subset F$.*
4. *$f(\alpha - m_X\text{-scl}\,(A)) \subset \beta - m_Y\text{-ker}\,(f(A))$ for all subset A of X.*
5. *$\alpha - m_X\text{-scl}\,(f^{-1}(B)) \subset f^{-1}(\beta - m_Y\text{-ker}\,(B)$ for all subset B of Y.*

Proof. Observe that the hypothesis of monotony of the operator α is needed in order to guarantee that the $\alpha - m_X$-semi closure of any set is $\alpha - m_X$-semi closed. Using this fact and proceeding in analogous form as the proof of Theorem 3.2, we obtain all the results. □

We recall that the frontier of any set A in a topological space X, is defined as $\mathrm{Fr}(A) = \mathrm{cl}\,(A) \cap \mathrm{cl}\,(X - A)$.

Definition 3.3. Let (X, m_X, α) be an $\alpha{-}m$ space. We define the $\alpha{-}m_X$-frontier of any set A of X as follows: $\alpha - m_X\text{-Fr}(A) = \alpha - m_X\text{-cl}\,(A) \cap \alpha - m_X\text{-cl}\,(X - A)$.

Definition 3.4. Let (X,m_X,α) be an $\alpha-m$ space, where m_X is a minimal structure and α be a monotone operator. We define the $\alpha-m_X$-semi frontier of any set A of X as follows:

$$\alpha-m_X-sFr(A)=(\alpha-m_X-\mathrm{scl}\,(A))\cap(\alpha-m_X-\mathrm{scl}\,(X-A)).$$

Observe that the $\alpha-m_X$-frontier of any set A of X is an $\alpha-m_X$-closed set and the $\alpha-m_X$-semi frontier of any set A of X is an $\alpha-m_X$ -semi closed set. Thus, we have that

$$\alpha-m_X-Fr(A)=(\alpha-m_X-\mathrm{cl}\,(A))-(\alpha-m_X-\mathrm{Int}\,(A)),$$

and

$$\alpha-m_X-sFr(A)=(\alpha-m_X-\mathrm{scl}\,(A))-(\alpha-m_X-\mathrm{sInt}\,(A)).$$

Using the above notions, we can describe, the set of points where any function $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ is not $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous as follows.

Theorem 3.4. *Let $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ be a function and α be a monotone operator. The set of all points x in X such that $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ is not $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous is exactly the union of the $\alpha-m_X$-semi frontier of the inverse image of the $\beta-m_Y$-closed set in Y that contains $f(x)$.*

Proof. Suppose that f is not $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous at the point $x\in X$, then there exists a $\beta-m_Y$-closed set F such that $f(x)\in F$ and $f(U)\cap(Y-F)\neq\emptyset$ for all $\alpha-m_X$-semi open set U such that $x\in U$. It follows that $U\cap f^{-1}(Y-F)\neq\emptyset$, but this means that $x\in\alpha-m_X\text{-scl}\,(f^{-1}(Y-F))=\alpha-m_X$-scl $(X-f^{-1}(F))$. Since $x\in f^{-1}(F)$, then $x\in\alpha-m_X\text{-scl}\,(f^{-1}(F))\cap\alpha-m_X\text{-scl}\,(X-f^{-1}(F))$. Therefore, the set of all $x\in X$ such that f is not an $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous at the point x is contained in $\alpha-m_X$-sFr$(f^{-1}(F))$.

Conversely, suppose that $x\in\alpha-m_X\text{-sFr}(f^{-1}(F))$, where F is $\beta-m_Y$-closed set in Y , $f(x)\in F$ and f is $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous. Then there exists an $\alpha-m_X$-semi open set U such that $x\in U$ and $f(U)\subset F$, therefore $x\in U\subset f^{-1}(F)$. From this, we obtain that $x\in\alpha-m_X$-semi int($f^{-1}(F))\subset X-\alpha-m_X\text{-sFr}(f^{-1}(F))$. In consequence, we obtain that $x\notin\alpha-m_X\text{-sFr}(f^{-1}(F))$, contradiction. Therefore f is not $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous. □

In the same way, we have a similar result for functions $f:(X,m_X,\alpha) \rightarrow (Y,m_Y,\beta)$ that are not $(\alpha,\beta)-(m_X,m_Y)$ contra continuous and can be proved in a similar form as the above theorem but without condition on the operator α.

Theorem 3.5. *Let $f:(X,m_X,\alpha)\rightarrow(Y,m_Y,\beta)$ be a function. The set of all points x in X such that $f:(X,m_X,\alpha)\rightarrow(Y,m_Y,\beta)$ is not $(\alpha,\beta)-(m_X,m_Y)$ contra continuous is exactly the union of all $\alpha-m_X$-frontier of the inverse image of the $\beta-m_Y$-closed set in Y that contains $f(x)$.*

Definition 3.5. An $\alpha-m$ space (X,m_X,α) is said to be $\alpha-m_X$-locally indiscrete if every $\alpha-m_X$-open set in X is $\alpha-m_X$-closed in X.

Observation 3.3. Observe that the above definition does not have a topological structure, but if m_X is a topology then we obtain the notion of α-locally indiscrete space given in [18].

Definition 3.6. Let $f:(X,m_X,\alpha)\rightarrow(Y,m_Y,\beta)$ be a map between α (resp. β)$-m$-spaces. We said that f is $(\alpha,\beta)-(m_X,m_Y)$-continuous function if for each point $x\in X$ and every m_Y-open set V containing $f(x)$, there exists an m_X-open set U containing x such that $f(\alpha(U))\subseteq\beta(V)$.

Observation 3.4. If in the above definition m_X and m_Y are topologies on X, Y respectively, we obtain the notion of (α,β)-continuous function given in [17].

In the following lemma, we indicate that the inverse image of any m_Y-open set in Y is an $\alpha-m_X$-open in X under functions that are $(\alpha,id)-(m_X,m_Y)$-continuous.

Lemma 3.1. *If $f:(X,m_X,\alpha)\rightarrow(Y,m_Y,id)$ is $(\alpha,id)-(m_X,m_Y)$-continuous then $f^{-1}(V)$ is $\alpha-m_X$-open for each $V\in m_Y$.*

Proof. Let $V\in m_Y$ and $x\in f^{-1}(V)$. Then $f(x)\in V$. By hypothesis, there exists a set $U_x\in m_X$, such that $x\in U_x$, and $f(\alpha(U_x))\subseteq id(V)=V$, therefore $U_x\subset\alpha(U_x)\subseteq f^{-1}(V)$ and the result follows. □

Theorem 3.6. *If $f:(X,m_X,\alpha)\rightarrow(Y,m_Y,id)$ is an $(\alpha,id)-(m_X,m_Y)$ continuous function and X is an $\alpha-m_X$-locally indiscrete space, then f is $(\alpha,id)-(m_X,m_Y)$-contra continuous function.*

In the case that α is a monotone operator, we obtain a similar result to the above theorem for $(\alpha,id)-(m_X,m_Y)$-contra semi continuous functions.

Observation 3.5. Recall that $A\subset X$ is an m_X pre-open set if $A\subset m_X$-$\mathrm{int}(m_X\text{-}\mathrm{cl}(A))$. Then, we can easily see, if $\alpha:P(X)\rightarrow P(X)$ is defined as

$\alpha(U)=m_X$-int $(m_X-\mathrm{cl}\,(U))$, then α is an operator associated with m_X. In this case, we have that $SO(X,m_X,\alpha)\subseteq PO(X,m_X)$, where $PO(X,m_X)$ is the set of all m_X- pre-open sets. It is easy to see that there exists m_X-pre-open sets that are not $\alpha-m_X$-semi open, where $\alpha(U)=m_X$-int $(m_X-\mathrm{cl}\,(U))$.

Observation 3.6. If f is an $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous functions, where $\alpha(U)=m_X$-int $(m_X-\mathrm{cl}\,(U))$ and β is the identity map, then all $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous functions are $\alpha-m_X$ pre-continuous. But in case that we are working with arbitrary operators α there are not known relations between $(\alpha,\beta)-(m_X,m_Y)$-contra semi continuous functions and $\alpha-(m_X,m_Y)$-pre-continuous functions.

4. Characterizations of $\alpha\beta-(m_X,m_Y)$-Irresolute and $\alpha\beta-(m_X,m_Y)$-Contra Irresolute Functions

In this section, we obtain some characterizations of the $\alpha\beta$ irresolute and $\alpha\beta$-contra irresolute functions using α-semi closure and α-semi ker nel.

Definition 4.1. Let (X,m_X,α), (Y,m_Y,β) be two α (respectively β)$-m$-spaces. A function $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ is said to be:

1. $(\alpha,\beta)-(m_X,m_Y)$-irresolute if the inverse image of any $\beta-m_Y$-semi open set $V\subseteq Y$, is an $\alpha-m_X$-semi open set in X.
2. $(\alpha,\beta)-(m_X,m_Y)$-contra irresolute if the inverse image of any $\beta-m_Y$-semi open set $V\subseteq Y$, is an $\alpha-m_X$-semi closed set in X.
3. $(\alpha,\beta)-(m_X,m_Y)$-scontinuous if for any point $x\in X$ and any $\beta-m_Y$-semi open set $V\subseteq Y$, such that $f(x)\in V$, there exists an $\alpha-m_X$-semi open set U in X such that $x\in U$ and $f(\alpha(U))\subseteq\beta(V)$.

Definition 4.2. Let (X,m_X,α), (Y,m_Y,β) be two $\alpha-(\beta-)m$-spaces. A function $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ is said to be $(\alpha,\beta)-(m_X,m_Y)$-sclosed if the image of any $\alpha-m_X$-semi closed set V in X is $\beta-m_Y$-semi closed in Y.

Lemma 4.1. *Let (X,m_X,α), (Y,m_Y,β) be two $\alpha-$(resp. $\beta-$)m-spaces where α,β are monotone operators and m_X a minimal structure. The following conditions are equivalent:*

1. *$f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ is an $(\alpha,\beta)-(m_X,m_Y)$-irresolute function.*
2. *For each subset $A\subseteq X$ $f(\alpha-m_X-\mathrm{scl}\,(A))\subseteq\beta-m_Y-\mathrm{scl}\,(f(A)$.*
3. *For each $\beta-m_Y$-semi closed subset $V\subseteq Y$ the inverse image $f^{-1}(V)$ is an $\alpha-m_X$-semi closed in X.*
4. *For all $B\subseteq Y$ the $\alpha-m_X-\mathrm{scl}\,(f^{-1}(B))\subseteq f^{-1}(\beta-m_Y-\mathrm{scl}\,(B))$.*

Proof. (3 ⇒ 2) Let A be a subset of X and suppose that $y \notin \beta - m_Y$-sCl$(f(A))$, then there exists a $\beta - m_Y$-semi open set G in Y, such that $y \in G$ and $f(A) \cap G = \emptyset$, therefore, $f^{-1}(f(A) \cap G) = \emptyset$. It follows that $A \cap f^{-1}(G) = \emptyset$. In consequence, $\alpha - m_X - \mathrm{cl}(A) \subset (f^{-1}(G))^c$, follows that $f(\alpha - m_X - \mathrm{cl}(A)) \cap G = \emptyset$; and therefore, $y \notin f(\alpha - m_X - \mathrm{cl}(A))$. It follows that $f(\alpha - m_X - \mathrm{cl}(A)) \subset \beta - m_Y$-sCl$(f(A))$ for all subset A of X.

(2 ⇒ 3) Let V any $\beta - m_Y$-semi closed subset in Y, then $f^{-1}(V) \subseteq X$. By hypothesis $f(\alpha - m_X\text{-scl}(f^{-1}(V))) \subset \beta - m_Y\text{-sCl}(f(f^{-1}(V)))$. It follows that $f(\alpha - m_X\text{-scl}(f^{-1}(V))) \subset \beta - m_Y\text{-sCl}(V)$. In consequence, $f(\alpha - m_X\text{-scl}(f^{-1}(V))) \subset V$, then $\alpha - m_X\text{-sCl}(f^{-1}(V) \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is an $\alpha - m_X$-semi closed set.

(2 ⇒ 4) Let B be a subset of Y, then $f^{-1}(B) \subseteq X$. By hypothesis,

$$f(\alpha - m_X - \mathrm{scl}(f^{-1}(B))) \subseteq \beta - m_Y - \mathrm{scl}(f(f^{-1}(B))) \subseteq \beta - m_Y - \mathrm{scl}(B),$$

therefore, $\alpha - m_X - \mathrm{scl}(f^{-1}(B)) \subseteq f^{-1}(\beta - m_Y - \mathrm{scl}(B))$.

(4 ⇒ 3) Suppose that V is any $\beta - m_Y$-semi closed set in Y. Then $f^{-1}(V) \subseteq X$. By hypothesis, we obtain that

$$\alpha - m_X - \mathrm{scl}(f^{-1}(V)) \subseteq f^{-1}(\beta - m_Y - \mathrm{scl}(V)).$$

But V is a $\beta - m_Y$-semi closed set, then $\beta - m_Y - \mathrm{scl}(V) = V$. In consequence,

$$\alpha - m_X - \mathrm{scl}(f^{-1}(V)) \subseteq f^{-1}(V).$$

But this says that $f^{-1}(V)$ is an $\alpha - m_X$-semi closed set in X.

The others implications (1 ⇒ 3) and (3 ⇒ 1), it follows from the definition of $(\alpha, \beta) - (m_X, m_Y)$ irresolute function and the complement of set. □

Lemma 4.2. *Let (X, m_X, α), (Y, m_Y, β) be two α $(\beta) - m$-spaces and α be a monotone. The following conditions are equivalents:*

1. $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$ is an $(\alpha, \beta) - (m_X, m_Y)$-contra irresolute function.

2. For each $A \subseteq X$, $f(\alpha - m_X - \mathrm{scl}(A)) \subseteq \beta - m_Y - \mathrm{sker}(f(A))$.

3. For each, $\beta - m_Y$ semi closed set $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an $\alpha - m_X$-semi open set in X.

4. For each $x \in X$ and F be a $\beta - m_Y$-semi closed set in Y, such that $f(x) \in F$, there exists an $\alpha - m_X$-semi open set $U \subseteq X$ such that $x \in U$ and $f(U) \subseteq F$.

5. For each $B \subseteq Y$, $\alpha - m_X - \mathrm{scl}(f^{-1}(B)) \subseteq f^{-1}(\beta - m_Y - \mathrm{sker}(B))$.

Proof. (3 $\Rightarrow$ 2) Let A be a subset of X and suppose that $y \notin \beta - m_Y$-sker $(f(A))$, then there exists a $\beta - m_Y$-semi closed set F in Y, such that $y \in F$ and $f(A) \cap F = \emptyset$, therefore $f^{-1}(f(A) \cap F) = \emptyset$, it said that $A \cap f^{-1}(F) = \emptyset$. In consequence, $\alpha - m_X - \mathrm{cl}\,(A) \subset (f^{-1}(F))^c$, follows that $f(\alpha - m_X - \mathrm{cl}\,(A)) \cap F = \emptyset$, but, it said that $y \notin f(\alpha - m_X - \mathrm{cl}\,(A))$. Therefore, $f(\alpha - m_X - \mathrm{cl}\,(A)) \subset \beta - m_Y$-sker $(f(A))$ for all subset A of X.

(2 $\Rightarrow$ 5) Let B be any subset in Y, then $f^{-1}(B) \subseteq X$. By hypothesis $f(\alpha - m_X\text{-scl}\,(f^{-1}(B))) \subset \beta - m_Y\text{-sker}\,(f(f^{-1}(B)))$. It follows that $f(\alpha - m_X\text{-scl}\,(f^{-1}(B))) \subset \beta - m_Y\text{-sker}\,(B)$. In consequence, $f(\alpha - m_X\text{-scl}\,(f^{-1}(B))) \subset \beta - m_Y\text{-sker}\,(B)$, therefore, $\alpha - m_X - \mathrm{scl}\,(f^{-1}(B) \subset f^{-1}(\beta - m_Y - \mathrm{sker}\,(B))$.

(5 $\Rightarrow$ 1) Let V be any $\beta - m_Y$-semi open set in Y, then $f^{-1}(V) \subseteq X$. By hypothesis, $\alpha - m_X - \mathrm{scl}\,(f^{-1}(V))) \subset f^{-1}(\beta - m_Y - \mathrm{sker}\,(V))$, but $\beta - m_Y - \mathrm{sker}\,(V) = V$, follows that $\alpha - m_X - \mathrm{scl}\,(f^{-1}(V))) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is an $\alpha - m_X$-semi closed set.

(1 $\Rightarrow$ 3) Follows using complement.

(3 $\Rightarrow$ 4) and (4 $\Rightarrow$ 3) are immediate. □

5. Applications

In this section, we will apply the above concepts in order to prove the invariance of certain properties of the domain and the range under the actions of the above functions.

Definition 5.1. Given (X, m_X, α) an $\alpha - m$ space, X is said to be an $\alpha - m_X$ semi-connected space, if the only subsets of X that are both $\alpha - m_X$ semi-open and $\alpha - m_X$ semi-closed in X are the $\emptyset$ and X itself.

Observation 5.1. 1. If α is the identity map and m_X is a topology on X, then we obtain the notion of connected space.

2. If α is any operator and m_X is a topology on X, then we obtain the notion of α-semi connected space given in [19].

3. If α is the identity map and m_X is a collection of all (β,γ) semi open sets in X, where β,γ are operators, then we obtain the notion of (β,γ)-semi connected space given in [20].

Definition 5.2. We say that (X, m_X, α) is an $\alpha - m_X$-semi T_1, if for each pair of distinct points $x, y \in X$, there exist m_X-open sets U and V of X containing x and y respectively, such that $y \notin \alpha(U)$ and $x \notin \alpha(V)$.

Theorem 5.1. *Let $f : (X, m_X, \alpha) \rightarrow (Y, m_Y, \beta)$ be an $(\alpha, \beta) - (m_X, m_Y)$-contra irresolute function. m_X, m_Y are minimal structures. If X is an $\alpha - m_X$ semi-connected space, α, β are monotone operators and Y is a $\beta - m_X$-semi T_1 space, then f is a constant map.*

Proof. Suppose that $f : (X, m_X, \alpha) \rightarrow (Y, m_Y, \beta)$ is an $(\alpha, \beta) - (m_X, m_Y)$-contra irresolute function, X is an $\alpha - m_X$ semi-connected space, Y is a $\beta - m_Y$-semi T_1 space and that f is not a constant map. Consequently, the collection $C = \{f^{-1}(\{y\}) : y \in Y\}$ is a partition of X, that have at least two elements. Since Y is a $\beta - m_Y$-semi T_1 space, then each unitary set $\{y\}$, $y \in Y$, is $\beta - m_Y$-semi closed (see [3]) and therefore each $f^{-1}(\{y\})$ is $\alpha - m_X$-semi open. In consequence, $C = \{f^{-1}(\{y\}) : y \in Y\}$ is a partition of X by $\alpha - m_X$-semi open sets. By hypothesis, α is a monotone operator, then the union of $\alpha - m_X$-semi open sets is $\alpha - m_X$-semi open. Therefore, the collection C would contain a proper subset $A \neq \emptyset$ of X that is $\alpha - m_X$-semi open and $\alpha - m_X$-semi closed. Contradiction. □

Theorem 5.2. *If $f : (X, m_X, \alpha) \rightarrow (Y, m_Y, id)$ is an $(\alpha, id) - (m_X, m_Y)$-scontinuous function, where m_X is a minimal structure and α is a monotone operator, then $f^{-1}(V)$ is an $\alpha - m_X$ semi-open set in X, for any m_Y-open set $V \in m_Y$.*

Proof. Let $V \in m_Y$ and $x \in f^{-1}(V)$, since $f : (X, m_X, \alpha) \rightarrow (Y, m_Y, id)$ is $(\alpha, id) - (m_X, m_Y)$-is a scontinuous function and $f(x) \in V$, there exists $U_x \in SO(X, m_X, \alpha)$ such that $f(\alpha(U_x)) \subseteq V$, this implies that $\alpha(U_x) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} \alpha(U_x)$. But, each U_x, $x \in X$, is an $\alpha - m_X$-semi-open set and using the fact that α is a monotone operator, conclude that,

$$U_x \subseteq \alpha(m_X - \text{Int}\,(U_x)) \subseteq \alpha(U_x), \quad \forall x \in X.$$

In consequence, $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. It follows that, $f^{-1}(V)$ is an $\alpha - m_X$ semi-open set in X. □

Theorem 5.3. *Let (X, m_X, α) be an $\alpha - m$ space where m_X is a minimal structure and α is a monotone operator. Let $A \subseteq X$ and $C_A : X \rightarrow \{0, 1\}$ be the characteristic function of A. If C_A is an $(\alpha, id) - (m_X, m_Y)$-scontinuous function then A is an $\alpha - m_X$ semi-open set and $\alpha - m_X$-semi-closed set in X.*

Proof. Suppose that C_A is an $(\alpha, id) - (m_X, m_Y)$-scontinuous function, then $C_A^{-1}(\{1\}) = A$ and $C_A^{-1}(\{0\}) = X - A$. Using Theorem 5.2, A is an $\alpha - m_X$-semi open set and $\alpha - m_X$ semi closed set. □

Corollary 5.1. *Let $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ be surjective functions and the cardinality of m_Y is bigger than 2. If f is $(\alpha,\beta)-(m_X,m_Y)$-contra irresolute function and $(\alpha,\beta)-(m_X,m_Y)$-sclosed, then Y is not a $\beta-m_Y$ semi-connected space.*

Definition 5.3. Let (X,m_X,α) be an $\alpha-m$ space with α be a monotone operator. A subset B of X is said to be an $\alpha-m_X$ semi-generalized closed set, denoted by ($\alpha-m_X-sg$-closed) if the $\alpha-m_X-\mathrm{scl}\,(A)\subseteq O$, whenever $B\subseteq O$ and O is an $\alpha-m_X$ semi open set.

Definition 5.4. Let (X,m_X,α) be an $\alpha-m$ space with α be a monotone operator. X is said to be an $\alpha-m_X$-semi $T_{1/2}$ space if all $\alpha-m_X$-sg-closed set in X are $\alpha-m_X$ semi closed set.

Theorem 5.4. *Let $f:(X,m_X,\alpha)\to(Y,m_Y,\beta)$ be an $(\alpha,\beta)-(m_X,m_Y)$-irresolute function and $(\alpha,\beta)-(m_X,m_Y)$-sclosed, where m_X is a minimal structure, then:*

(a) For all $\alpha-m_X$-sg-closed set A in X, $f(A)$ is a $\beta-m_Y$-sg-closed set in Y.

(b) $f^{-1}(B)$ is an $\alpha-m_X$ sg-closed set in X for all $\beta-m_Y$ semi closed set B in Y.

Proof. (a) Let V be a $\beta-m_Y$-semi open set in Y such that $f(A)\subseteq V$, then $f^{-1}(V)$ is an $\alpha-m_X$ semi open set and $A\subseteq f^{-1}(V)$. It follows that, $\alpha-m_X-\mathrm{scl}\,(A)\subseteq f^{-1}(V)$, since f is an $(\alpha,\beta)-(m_X,m_Y)$-sclosed, then $f(\alpha-m_X-\mathrm{scl}\,(A))$ is a $\beta-m_X$-semi closed set. In consequence, $\beta-m_X-\mathrm{scl}\,(f(A))\subseteq\beta-m_Y-\mathrm{scl}\,(f(\alpha-m_X-\mathrm{scl}\,(A)))=f(\alpha-m_X-\mathrm{scl}\,(A))\subseteq V$. Therefore, $f(A)$ is a $\beta-m_X$-semi generalized closed set.

(b) Let B be a $\beta-m_Y$-semi closed set in Y, and suppose that U is an $\alpha-m_X$-semi open set in X such that $f^{-1}(B)\subseteq U$. Consider $F=\alpha-m_X-\mathrm{scl}\,(f^{-1}(B))\cap U^c$. Then using the fact that α is a monotone operator, we may conclude that F is an $\alpha-m_X$-semi closed set, therefore $f(F)$ is a $\beta-m_Y$-semi closed set and

$$\begin{aligned} f(F) &= f(\alpha-m_X-\mathrm{scl}\,(f^{-1}(B))\cap U^c)\\ &\subseteq f(\alpha-m_X-\mathrm{scl}\,(f^{-1}(B)))\cap f(U^c))\\ &\subseteq \beta-m_Y-\mathrm{sCl}\,f^{-1}(f(B))\cap f(U^c)\subseteq\beta-m_Y-\mathrm{scl}\,(B)\cap B^c=\emptyset. \end{aligned}$$

In consequence, $f(F)=\emptyset$, and $F=\emptyset$, therefore, $f^{-1}(B)$ is an $\alpha-m_X$ semi generalized set. □

Corollary 5.2. *Let $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$ be an $(\alpha, \beta) - (m_X, m_Y)$-irresolute function and $(\alpha, \beta) - (m_X, m_Y)$-sclosed, where m_X, m_Y are minimal structures and (α, β) monotone operators, then:*

(a) If f is an injective function and (Y, m_Y) is a $\beta - m_Y$-semi $T_{1/2}$ space, then (X, m_X) is an $\alpha - m_X$-semi $T_{1/2}$ space.

(b) If f is a bijective function and (X, m_X) is an $\alpha - m_X$-semi $T_{1/2}$ space, then (Y, m_Y) is a $\beta - m_Y$-semi $T_{1/2}$ space.

Proof. (a) Suppose that A is an $\alpha - m_X$-sg-closed set in X. Using Theorem 5.5 and the hypothesis, then $f(A)$ is a $\beta - m_Y$-sg-closed set in Y, but Y is a $\beta - m_Y$-semi $T_{1/2}$ space, then $f(A)$ is a $\beta - m_Y$-semi closed set in Y. Now, using the fact that f is an injective function and $(\alpha, \beta) - (m_X, m_Y)$ irresolute, it follows that $A = f^{-1}(f(A))$ is an $\alpha - m_X$-semi closed set in X. In consequence, (X, m_X) is an $\alpha - m_X$-semi $T_{1/2}$ space.

(b) Given $y \in Y$, there exists a unique point $x \in X$ such that $y = f(x)$. It follows that each unitary set in Y is a $\beta - m_Y$-semi open set or $\beta - m_Y$-semi closed set and therefore, Y is a $\beta - m_Y$-semi $T_{1/2}$ space (see [18]). □

Theorem 5.5. *Let $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$ be an $(\alpha, \beta) - (m_X, m_Y)$-irresolute function and $(\alpha, \beta) - (m_X, m_Y)$-sclosed where m_X, m_Y are minimal structures and (α, β) monotone operators, then:*

(a) If f is an injective function and (Y, m_Y) is a $\beta - m_Y$-semi T_1, then (X, m_X) is an $\alpha - m_X$-semi T_1.

(b) If f is a bijective function and (X, m_X) is an $\alpha - m_X$-semi T_1, then (Y, m_Y) is a $\beta - m_Y$-semi T_1.

Another application of the $(\alpha, \beta) - (m_X, m_Y)$-irresolute function is related with the notion of $\alpha - m_X$ semi compactness.

Definition 5.5. Let (X, m_X, α) be an $\alpha - m$ space. A subset A of X is said to be an $\alpha - m_X$-semi compact if given $\{U_i : i \in I\}$ a covering of A by $\alpha - m_X$ semi open sets. There exist a finite subcollection $\{U_{i_0} : i_0 \in I_0\}$ of $\{U_i : i \in I\}$ such that $A \subseteq \bigcup_{i \in I_0} U_i$.

Theorem 5.6. *Let $f : (X, m_X, \alpha) \to (Y, m_Y, \beta)$ be an $(\alpha, \beta) - (m_X, m_Y)$-irresolute function and surjective. If X is $\alpha - m_X$ semi compact, then Y is $\beta - m_Y$ semi compact.*

Observation 5.2. In the above definition if:

1. X is a topological space $\alpha =$ closure and m_X is the collection of semi open sets of X, then we obtain the definition of semi compact space given in [10].

2. $\alpha = id$ and m_X any m structure, then we obtain the definition of m-compact space given in [13].

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