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VALIDITY OF THE PURCHASING POWER PARITY IN THE V4 COUNTRIES

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Abstract: This paper deals with the analysis of the monthly values of the V4 countries (Visegrád countries) national currencies against the euro in order to test the validity of the purchasing power parity (PPP). The analysis was done in two ways – through testing of the real exchange rates stationarity and through the cointegration theory. According to the results it is possible to conclude that the validity of the PPP is in cases of all the analysed exchange rates problematic.

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Key Words: exchange rates, purchasing power parity, V4 countries, stationarity, ADF test, cointegration theory, Engle-Granger procedure

1. Introduction

1 May 2004 was a date of the biggest enlargement of the European Union (EU) in its history – 10 candidate countries (Cyprus, Czech Republic, Estonia, Lithuania, Latvia, Hungary, Malta, Poland, Slovak Republic and Slovenia) became new member states of the EU. As a result of the above mentioned enlargement, the EU area grew by 23 per cent and population by 75 million people.

The process of the EU entrance is for the 10 mentioned countries already successfully finished, timing of the entrance into the European Monetary Union

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(EMU) has been in fact still relevant. The precondition of the EMU entrance is the fulfilment of the Maastricht criteria called also the nominal convergence criteria. One of these criteria is the criterion of the exchange rate stability. In order to verify the fulfilment of this criterion at least two-year membership in the Exchange Rate Mechanism (ERM II) is necessary – during this period the exchange rate of the national currency against the euro must not get over the ± 15 % band around the agreed parity. Seven out of the ten new member countries (Cyprus, Estonia, Lithuania, Latvia, Malta, Slovak Republic and Slovenia) have already become members of the ERM II and fixed the parity of their national currencies against the euro in order to accept the euro as soon as possible (Slovenia will join the euro area on 1 January 2007). The Czech Republic, Hungary and Poland have been still waiting for the right moment to enter the ERM II.

Examination, analysis and maybe also forecasting of the exchange rates of national currencies of the above mentioned countries against the euro seems to be quite interesting. The three still non-member countries of the ERM II together with the Slovak Republic are often called the V4 (Visegrád) countries. From these four contries, only Slovakia plans to adopt the euro in 2009, in the other three countries' monetary policy, early ERM II membership and adoption of the euro before 2010 are options than rather strategic targets. As a result of the similar political and economic development in the V4 countries will this paper be concentrated on the analysis of the exchange rates of these countries' national currencies, i.e. Czech koruna (CZK), Hungarian forint (HUF), Polish zloty (PLN) and Slovak koruna (SKK) against the euro (EUR).

The aim of this paper is to verify the validity of the purchasing power parity (PPP) for the monthly values (January 1999 – April 2006) of the exchange rates CZK/EUR, HUF/EUR, PLN/EUR and SKK/EUR based on testing of the stationarity of the real exchange rates and cointegration theory.

2. Purchasing Power Parity

Purchasing power parity (PPP) is one of the theories, which try to describe the long-run behaviour of exchange rates. PPP explains movements in the exchange rates between two countries' currencies by changes in the countries' price levels. The concept of PPP is generally attributable to the Swedish economist Gustav Cassel, who formulated the approach in 1920s. Cassel's theory represents a synthesis of the work of the nineteenth-century British economists, among them David Ricardo (the originator of the theory of comparative advantage).

The PPP theory extends the law of one price (i.e. identical goods must be sold in different countries for the same price, on condition that the prices are expressed in terms of the same currency) on the general price level, the development of which is usually described by changes in prices of the product basket measured by consumer price index (CPI), but it is also possible to use the producer price index (PPI) or GDP deflator (Chocholatá [7], Ivaničová and Rublíková [12]).

Two versions of PPP are commonly used – the absolute version and the relative version. According to the absolute version of PPP the exchange rate between two countries' currencies must equal the ratio of the countries' price levels, i.e. the price levels must be in all countries the same, on condition that they are measured in the same currency.

Let the P^* be the price of the product basket sold in a foreign country (expressed in a foreign currency) and P the corresponding price of the same product basket in a domestic country (expressed in a domestic currency). Then the corresponding exchange rate S according to the absolute PPP is as follows:

$$S = \frac{P}{P^*}, \quad \text{resp.} \quad P = S.P^*, \tag{1}$$

More often used is the relative PPP (it makes actually no sense to use the absolute PPP until the analysed product baskets and weights of goods and services in product baskets are not the same) analysing the percentage changes in exchange rates between two countries over any period in dependence of inflation rate changes in analysed countries

$$\frac{S_t - S_{t-1}}{S_{t-1}} = \frac{\pi_t - \pi_t^*}{1 + \pi_t^*}\,, \tag{2}$$

where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ denotes the percentage change of the price level during the period $(t-1, t)$, i.e. the inflation rate in a domestic country and π_t^* the inflation rate in a foreign country.

Quite often used is also the formulation of the PPP in terms of the real exchange rate R according to the following formula

$$R = S.\frac{P^*}{P}\,. \tag{3}$$

From (3) it is evident that the real exchange rate R is similarly as nominal exchange rate defined by formula (1), time-invariant and when the absolute PPP holds, it should be equal to 1.

3. Tests for Long-Run Equilibrium Relationships Using the Augmented Dickey-Fuller (ADF) Test of Stationarity

Verifying of the PPP validity is usually performed on logarithmic transformations (denoted by small letters of variables) of (1) and (3). The mentioned logarithmic transformations are for time t as follows

$$s_t = p_t - p_t^* , \tag{4}$$

or

$$r_t = s_t + p_t^* - p_t . \tag{5}$$

s_t denotes the logarithm of the exchange rate S_t, p_t a p_t^* logarithms of the price levels in domestic (P_t) and foreign country (P_t^*) and r_t logarithm of the real exchange rate R_t.

Historically, various methods were used to test the validity of the PPP, before the mid-1980s the most popular methods were ordinary and generalized least squares. Since mid-1980s, as a result of advances in econometrics for nonstationary time series the unit root tests and cointegration theory have become rather widely used.

One of the most popular procedures is to test the real exchange rate defined by formula (5), and it is necessary to say that the long-run PPP is valid only if the time series r_t (r_t can be characterised also as a short-run deviation from PPP in time t– see e.g. Enders [4]) is stationary. After all, if the deviations from PPP are nonstationary, i.e. are permanent in nature, the theory of PPP can be rejected.

Stationarity of the real exchange rate r_t can be tested by various unit root tests. The most often used is the Augmented Dickey-Fuller (ADF) test using the regression (6) with an intercept (ϕ_0) and trend (t), from which will be firstly excluded the time trend (in case of its statistical insignificance) and then also the intercept (again in case of its statistical insignificance):

$$\Delta r_t = \phi_0 + \delta r_{t-1} + \phi_2 t + \sum_{i=2}^{p} \beta_i \Delta r_{t-i+1} + \varepsilon_t , \tag{6}$$

where δ, ϕ_2 and β_i are parameters.

In case of ADF test the tested hypotheses are $H_0 : \delta = 0$ against $H_1 : \delta < 0$, in order to find out, if the analysed time series of the real exchange rate has a character of the random walk (i.e. is nonstationary) or is stationary. If it is not possible to reject the H_0 by testing the level of the analysed time series, the first differences of this time series should be tested etc. until it is possible to reject the H_0 and determine the order of integration (number of unit roots).

The above mentioned ADF test also seeks to determine whether the real exchange rate behaves as a unit root series. Confirmation of the unit root behaviour in the real exchange rate emerges if the real exchange rate appears to wander aimlessly (i.e. is nonstationary) with no obvious mean. In this case, PPP is rejected, since at a minimum it requires the real exchange rate to fluctuate about some constant. Evidence against unit root behaviour emerges when the real exchange rate fluctuates about a fixed mean (constant) with a tendency to return to it (i.e. is stationary). In this case, PPP is confirmed (Breuer [2]).

4. Cointegration and Purchasing Power Parity

Suppose, that x_t and y_t are nonstationary variables integrated of the same order d, i.e. $I(d)$ and it would be interesting to find out wheter there exist a long-run equilibrium between these variables. According to the Engle and Granger's theory of cointegration (Engle and Granger [5]) the variables x_t and y_t are cointegrated of order (d, b), i.e. *CI(d,b)*, where $d \geq b \geq 0$, if both variables are integrated of the order d and there exist a linear combination of these two variables $\gamma_1 y_t + \gamma_2 x_t$ which is integrated of order $(d - b)$. Vector γ is called cointegrating vector and for any nonzero value of λ, $(\lambda\gamma_1, \lambda\gamma_2)$ is also a cointegrating vector. Typically, one of the variables is used to normalize the cointegrating vector by fixing its coefficient at unity. To normalize the cointegrating vector, e.g. with respect to y_t, simply select $\lambda = 1/\gamma_1$.

The conception of cointegration can be generalized for the situation of more than two, e.g. k, variables. Then these k variables $x_{1t}, x_{2t}, ..., x_{kt}$ are *CI(d,b)*, where $d \geq b \geq 0$, if all variables are integrated of order d and there exist a linear combination of these variables which is integrated of order $(d - b)$. The cointegrating vector is in case of k variables not unique – it can exist as many as $k-1$ linearly independent cointegrating vectors which is called the cointegration rank.

Many authors use the term cointegration to refer to the case in which variables are *CI(1,1)*, so that there exist a linear combination that is $I(0)$, i.e. stationary. In this context we can now suppose, that two variables - x_t and y_t are integrated of order 1 and to test whether these $I(1)$ variables are cointegrated of order (1,1), i.e. *CI(1,1)*. The Engle-Granger procedure (see e.g. Enders [4]) can be performed in several steps.

In the first step it is necessary to test the variables for their order of integration using e.g. Dickey-Fuller (DF) test, ADF test or another test for existence of the unit root. If the variables are integrated of the same order (with exception of stationarity) the procedure continues, in case of different orders, it is possible to conclude, that the variables are not cointegrated.

After finding that both variables are $I(1)$, the long-run equilibrium relationship will be estimated in the form $y_t = \beta_0 + \beta_1 x_t + e_t$ using the ordinary least squares (OLS) method. In order to determine if the variables are actually cointegrated, the residual sequence from this equation can be denoted by $\hat{e}_t$. Then it is necessary to use the ADF test taking into account that the time series residual sequence $\hat{e}_t$ comes from a regression equation, so there is no need to include an intercept term, i.e.

$$\Delta\hat{e}_t = \delta\hat{e}_{t-1} + \sum_{i=2}^{n} \delta_i \Delta\hat{e}_{t-i+1} + \varepsilon_t \,. \tag{7}$$

In this context it is useful to note that there is not possible to use the “classic” Dickey – Fuller tables, because the $\hat{e}_t$ sequence is generated from a regression equation. Fortunately, Engle and Granger provide test statistics that can be used to test the hypothesis $\delta = 0$. When more than two variables appear in the equilibrium relationship, the appropriate tables are provided by Engle and Yoo [6] (for more information see also Enders [4]).

The acceptance of the null hypothesis about the existence of a unit root means that the variables x_t and y_t are not cointegrated. The rejection of the null hypothesis also implies that the variables are cointegrated (i.e. confirmation of the long-run equilibrium relationship between these variables).

If the variables x_t and y_t are cointegrated, i.e. *CI(1,1)*, the residuals from the equilibrium regression (7) can be used to estimate the following error-correction model (ECM) expressing the dynamics of the equilibrium relationship between the two variables:

$$\Delta y_t = \alpha + \alpha_x \hat{e}_{t-1} + \sum_{i=1} \alpha_1(i)\Delta y_{t-i} + \sum_{i=1} \alpha_2(i)\Delta x_{t-i} + \varepsilon_t, \tag{8}$$

where $\hat{e}_{t-1}$ is a deviation from the long-run equilibrium in period $(t-1)$, α_x is a speed of adjustment coefficient and $\alpha, \alpha_1(i)$and $\alpha_2(i)$ are parameters.

In the last step it is necessary to evaluate the model adequacy. Enders [4] presents several procedures that can be used in order to determine whether the estimated ECM is appropriate.

The alternative way to test the PPP validity offers also the introduced Engle – Granger methodology. Based on formula (5), in case that the PPP holds, the sequence formed by the sum $s_t + p_t^*$ should be cointegrated with the p_t sequence. In this context we can call the sum $s_t + p_t^*$ the foreign price level expressed in a domestic currency f_t, i.e. $f_t = s_t + p_t^*$. In case of the PPP validity it should exist a linear combination of the form

$$f_t = \beta_0 + \beta_1 p_t + e_t \tag{9}$$

such that $\hat{e}_t$ is stationary and the cointegrating vector is such that $\beta_1 = 1$ (it is clear that the the sequences $s_t + p_t^*$ and p_t must be integrated of the same order, with differing orders of integration, it would have been possible to conclude that PPP failed).

5. Testing of the PPP Validity – Application Part

As it was already mentioned in the introduction, the aim of this paper is to test the validity of the PPP through testing of the real exchange rates stationarity and cointegration theory. The analysis was done for monthly values (January 1999 – April 2006) of exchange rates CZK/EUR, HUF/EUR, PLN/EUR and SKK/EUR using the modern econometric software *EViews 5.1.* The data about exchange rates and inflation rates were obtained from web – pages of individual national banks (see [16]).

5.1. Testing of the Real Exchange Rates Stationarity

The aim of this part of analysis was to test the stationarity of the logarithmic transformation of the above mentioned real exchange rates the calculation of which was based on validity of the formula (5).

The individual real exchange rates (R_t) were calculated using the formula (3) the logarithmic transformation of which (i.e. formula (5)) was a final subject of testing in this part of analysis. The national price levels (P_t and P_t^*) were defined through the monthly inflation rates on a year-on-year basis measured by consumer price indices (V4 countries) or harmonized indices of consumer prices (EMU countries) and the nominal exchange rates (S_t) were also expressed in form of indices (i.e. as a change in comparison to the same month of the previous year).

Although the whole analysis was carried out on the logarithmic transformation of the above mentioned real exchage rates, the following graphs depict the indices of the nominal and real exchange rates (not the logarithmic transformation of them): Figure 1.

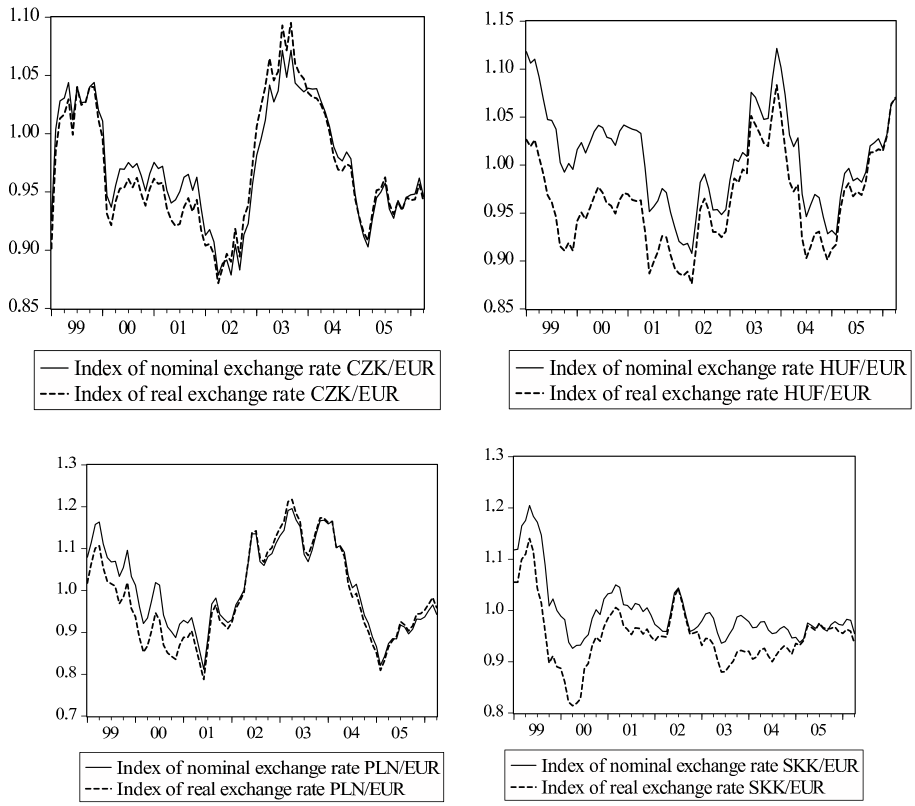

Figure 1: Graphs 1-4

To verify the PPP validity the ADF test was used to test the existence of the unit root in the time series of logarithmic transformations of individual real exchange rates, although from graphs 1 – 4 it seems to be clear that the depicted time series are obviously not stationary (logarithmic transformation is a monotonous transformation). The ADF test was carried out according to the formula (6) and the results are summarized in Table 1 (letter "i" together with the exchange rate denotes the logarithmic transformation of the real exchange rate index), where symbols ***a, b, c*** in this table refer to the tests applied gradually on regression (6) with an intercept and trend; then after excluding of the trend, i.e. only with an intercept; and finally without both intercept and trend. Letter ***L*** denotes testing for the unit root in level of the individual time series and letter ***D*** testing for the unit root in first differencies of the concrete time series. Rejection of the null hypothesis on the significance level 0,01 is denoted by an asterisk (*).

	i_czk/eur	***i_huf/eur***	***i_pln/eur***	***i_skk/eur***
La	-2,081	-2,750	-1,794	-2,947
Lb	-2,046	-2,410	-1,768	-3,003
Lc	-1,880	-1,949	-1,679	-1,894
Da	-9,196*	-6,772*	-6,833*	-5,988*

Table 1:

According to the results presented in Table 1 it is clear that all the analysed time series are on the significance level 0,01 nonstationary - integrated of order 1, i.e. I(1). It means that the validity of PPP is in all analysed cases problematic.

5.2. Testing of the PPP through the cointegration theory

Validity of the PPP requires the sum $s_t + p_t^* = f_t$ from formula (5) to be cointegrated with the p_t sequence. First of all it is necessary to test for the order of integration of the time series f_t and p_t through e.g. the ADF test – the results are summarized in Table 2.

	Czech Rep.		***Hungary***		***Poland***		***Slovak Rep.***	
	$\boldsymbol{F_t}$	$\boldsymbol{p_t}$	$\boldsymbol{f_t}$	$\boldsymbol{p_t}$	$\boldsymbol{f_t}$	$\boldsymbol{p_t}$	$\boldsymbol{f_t}$	$\boldsymbol{p_t}$
La	-2,271	-3,185	-2,629	-2,170	-1,868	-1,854	-7,050*	-2,310
Lb	-2,130	-3,276	-2,818	-0,708	-1,912	-1,302	-	-1,453
Lc	-2,149	-1,993	-2,481	-1,439	-2,015	-1,470	-	-0,731
Da	-9,339*	-7,044*	-6,911*	-6,774*	-7,199*	-5,542*	-	-7,751*

Table 2:

Sequences f_t and p_t are integrated of the same order – I(1) in all cases with exception of the Slovak Republic. According to the results, it can be also concluded that in case of the SKK/EUR exchange rate the PPP failed.

The next step is to estimate the long-run equilibrium relation of the form (9) for each out of the three exchange rates (CZK/EUR, HUF/EUR and PLN/EUR) and the results are as follows (with t – statistics in parenthesis):

Czech Republic: $\hat{f}_t = 0,012 - 0,792 p_t$
$(1,211)\ (-2,358)$

Hungary: $\hat{f}_t = 0,002 + 0,382 p_t$
$(0,144)\ \ (1,880)$

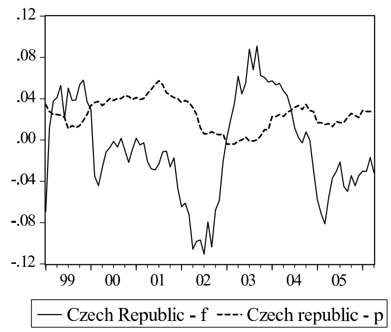

Figure 2: Graphs 5-6

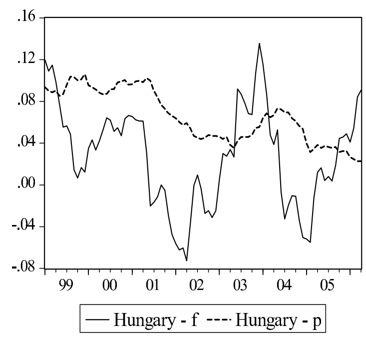

Figure 3: Graphs 7-8

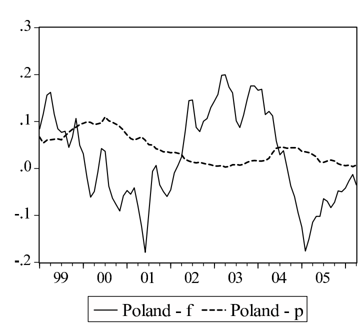

Figure 4: Graphs 9-10

Poland: $\hat{f}_t = 0,061 - 0,901 p_t$
$(3,628)\,(-2,793)$

Sequences f_t, p_t and residuals $\hat{e}_t$ from these equations are depicted in graphs $5 - 10$.

	Czech Republic	***Hungary***	***Poland***
	$\hat{e}_t$	$\hat{e}_t$	$\hat{e}_t$
Lc	-2,138	-2,765	-1,949
Dc	-9,462*	-6,808*	-7,483*

Table 3:

Although it seems to be evident from graphs 5 – 10 that the individual pairs of time series f_t and p_t are not cointegrated and that residuals are not stationary, we tested the stationarity of residuals also through the ADF test. Results of testing residuals for unit roots are in Table 3. Rejection of the null hypothesis on the significance level 0,01 taking into account the modified critical values is again denoted by an asterisk (*). Symbol ***Lc*** denotes testing for the unit root in levels without both trend and intercept and symbol ***Dc*** testing for the unit root in first differencies of the concrete time series again without both trend and intercept.

Taking into account graphs 5 – 10 and also the results of the ADF tests in table 3 it can be concluded that all residual sequences are nonstationary integrated of order 1, i.e. I(1) which means that the PPP theory does not hold in any of these cases.

6. Conclusion

The analysis of the exchange rates CZK/EUR, HUF/EUR, PLN/EUR and SKK/EUR based on the PPP describing the relationship between the exchange rates and inflation rates was done in two ways. First part was concentrated on examination of stationarity of the above mentioned real exchange rates through the ADF test. All analysed individual real exchange rates were identified to be non-stationary, which means that the PPP could not hold.

In the second part, the analysis was based on cointegration theory, where the cointegration between sequences of foreign price level expressed in a domestic currency and domestic price level was tested. The validity of the PPP in Slovakia could be rejected immediately after using the ADF test on the corresponding time series which were identified as integrated of different orders. Although in cases of the remaining three exchange rates were the corresponding time series integrated of the same order (order 1), the residuals from the individual regression equations were not stationary, i.e. the PPP could not be valid.

The analysis also proved the problematic validity of the PPP which could be caused by different factors, e.g. unstable development of the inflation rates in analysed countries (mainly as a result of changes in energy and transportation prices).

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