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(α, β) CONTRA CONTINUOUS FUNCTIONS AND $\alpha\beta$-IRRESOLUTE FUNCTIONS

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Abstract: The aim of this paper, is to investigate another characterizations of (α, β) contra continuous and (α, β) contra semi continuous functions. Also we introduce and investigate the $\alpha\beta$ irresolute functions and $\alpha\beta$ contra irresolute functions, some characterizations are obtained and some applications are shown.

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1. Introduction

The notion of contra continuity was introduced by Dontchev in [4]. He defined: a function $f : X \to Y$ is said to be contra continuous if the inverse image of any open set in Y is closed in X. Also Dontchev and Noiri in [5] introduced and investigated a new class of functions called contra semi continuous. A function $f : X \to Y$ is said to be contra semi continuous if the inverse image of any open set in Y is semi closed in X. In [15], E. Rosas C. Carpintero and

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J. Vielma gave the definitions of (α,β) contra continuous functions and (α,β) contra semi continuous functions as follows: $f : X \rightarrow Y$ is said to be (α,β) contra continuous ((α,β) contra semi continuous, respectively) if the inverse image of any β open set in Y is α closed (α semi closed, respectively) in X. In [9] Navagali gave the definition of irresolute functions as follows $f : X \rightarrow Y$ is said to be irresolute if the inverse image of any semi open set in Y is semi open in X. In this paper we introduce new characterizations of (α,β) contra continuous functions, and (α,β)contra semi continuous functions, also the $\alpha\beta$ irresolute functions, and the $\alpha\beta$ contra irresolute functions are defined and studied some characterizations of these class of functions using related notions among α topological kernel, α-semi topological kernel, α closure or the α semi closure. Also we gave some important results respect to these class of functions.

2. Preliminary

Let (X,τ) be a topological space. $\alpha : P(X) \rightarrow P(X)$ is said to be an operator associated to a topology τ, if it satisfies the following condition $U \subseteq \alpha(U)$ for all $U \in \tau$. A subset $A \subseteq X$ is said to be α-open set if for each $x \in A$ there exist $U \in \tau$ such that $x \in U$ and $\alpha(U) \subseteq A$ (a subset of X is said to be α-closed if its complement is an α-open set). We denote by Γ_α, the collection of all α-open sets. A subset $A \subseteq X$ is said to be α-semi open set if there exists $U \in \tau$ such that $U \subseteq A \subseteq \alpha(U)$ (a subset of X is said to be α-semi closed if its complement is an α-semi open set). We denote by α-SO(X,τ), the collection of all α-semi open sets. In general, the union of α-semi open sets is not necessarily α-semi open, but if we give an additional condition of monotony to the operator α (α is said to be a monotone operator if $\alpha(U) \subseteq \alpha(V)$ for all $U \subseteq V$), then the union of α-semi open sets is α-semi open.

Definition 2.1. Let (X,Γ,α) be a topological space and A be a subset of X:

1. The set $\bigcap\{U \in \Gamma_\alpha : A \subseteq U\}$ is called the α-kernel of A, and is denoted by α-ker(A).
2. The set $\bigcap\{U \in \alpha\text{-}SO(X,\Gamma) : A \subseteq U\}$ is called the α-semi kernel of A and is denoted by α-sker(A).

The following theorem, characterizes the α topological-kernel and the α-semi topological-kernel.

Theorem 2.1. *Let (X,Γ,α) be a topological space, A and B be a subsets of X, then*

1. $x \in \alpha$-ker(A) if and only if $A \cap F \neq \emptyset$ for any α-closed set F that contain x.

2. $A \subseteq \alpha$-ker(A) and $A = \alpha$-ker(A) if A is α-open.

3. If $A \subseteq B$, then α-ker(A)$\subseteq \alpha$-ker(B).

4. $x \in \alpha$-sker(A) if and only if $A \cap F \neq \emptyset$ for any α-semi closed set F that contain x.

5. $A \subseteq \alpha$-sker(A) and $A = \alpha$-sker(A) if A is α-semi open.

6. If $A \subseteq B$, then α-sker(A)$\subseteq \alpha$-sker(B).

Definition 2.2. Let (X,τ) be a topological space and $\alpha : P(X) \to P(X)$ be a monotone operator associated to a topology τ, we define the α semi closure of a subset A of X, denoted by α-scl(A) as the intersection of all α-semi closed sets containing A.

Note that, in the case that α is a monotone operator, for any subset $A \subseteq X$ the $\alpha - sCl(A)$ is an α semi-closed set.

The following lemma characterizes the points of the α semi-closure of a subset $A \subseteq X$.

Lemma 2.1. *Let (X, τ) be a topological space, $\alpha : P(X) \to P(X)$ be a monotone operator associated to τ and $A \subseteq X$. $x \in \alpha$ -scl(A) if and only if for all α-semi open set S of X containing x, $S \cap A \neq \emptyset$.*

Proof. Sufficiency. If there exists an α-semi open set $S \subseteq X$ for which $x \in S$ and $A \cap S = \emptyset$, then, $F = X \setminus S$ is an α-semi closed set, $A \subseteq F$ and $x \notin F$, this implies that $x \notin \alpha - sCl(A)$.

Necessity. Suppose that $x \notin \alpha - sCl(A)$. Then, there exists an α-semi closed set F in X, such that $A \subseteq F$ and $x \notin F$. Now the set $S = X \setminus F$, is an α-semi open set in X, $x \in S$ and $A \cap S = \emptyset$. □

3. (α, β) Contra Continuous and (α, β) Contra Semi-Continuous Functions

Now, we introduce the concepts of (α, β) contra continuous functions and (α, β) contra semi continuous functions and give a characterization of these functions using the notions of topological kernels and closures.

Definition 3.1. Let (X, Γ, α) and (Y, Φ, β) be topological spaces, a map $f : X \to Y$ is said to be (α, β) contra continuous if $f^{-1}(B)$ is α-closed in X for all β-open set B in Y.

Definition 3.2. Let (X, Γ, α) and (Y, Φ, β) be topological spaces, a map $f : X \to Y$ is said to be (α, β) contra semi continuous if $f^{-1}(B)$ is α-semi closed in X for all β-open set B in Y.

The following theorem gives a relation between (α, β)-contra continuous functions and (α, β)-contra semi continuous functions.

Theorem 3.1. *Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$. If f is an (α, β)-contra continuous function, then f is an (α, β)-contra semi continuous.*

The following theorem gives another characterization of (α, β)-contra continuous functions.

Theorem 3.2. *Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ be a function. The following propositions are equivalent:*

1. f is (α, β)-contra continuous.

2. For all β-closed set F in Y, $f^{-1}(F) \in \Gamma_\alpha$.

3. For each $x \in X$ and any β-closed set F in Y, such that $f(x) \in F$, there exists an α-open set U in X such that $x \in U$ and $f(U) \subset F$.

4. $f(\alpha\text{-cl}(A)) \subset \beta\text{-ker}(f(A))$ for all subsets A of X.

5. $\alpha\text{-cl}(f^{-1}(B)) \subset f^{-1}(\beta\text{-ker}(B))$ for all subsets B of Y.

Proof. The equivalence 1, 2 and 3 are proven in [15].

$(2 \Rightarrow 4)$ Let A be a subset of X and suppose that $y \notin \beta\text{-ker}(f(A))$, then there exists a β-closed set F in Y, such that $y \in F$ and $f(A) \cap F = \emptyset$, therefore $f^{-1}(f(A) \cap F) = \emptyset$. This implies that $A \cap f^{-1}(F) = \emptyset$, and therefore, $\alpha\text{-cl}(A) \subset (f^{-1}(F))^c$. It follows that $f(\alpha\text{-cl}(A)) \cap F = \emptyset$, which implies that $y \notin f(\alpha\text{-cl}(A))$. In consequence, we have proved that $f(\alpha\text{-cl}(A)) \subset \beta\text{-ker}(f(A))$ for all subset A of X.

$(4 \Rightarrow 5)$ Let B be any subset of Y. Then $f^{-1}(B) \subseteq X$. By hypothesis $f(\alpha\text{-cl}(f^{-1}(B))) \subset \beta\text{-ker}(f(f^{-1}(B)))$. It follows that $f(\alpha\text{-cl}(f^{-1}(B))) \subset \beta\text{-ker}(B)$, consequently, $\alpha\text{-cl}(f^{-1}(B) \subset f^{-1}(\beta\text{-ker}(B))$.

$(5 \Rightarrow 1)$ Let V be any β-open set in Y. By hypothesis $\alpha\text{-cl}(f^{-1}(V) \subset f^{-1}(\beta\text{-ker}(V))$, since V is a β-open set then, $\beta\text{-ker}(V) = V$. In consequence $\alpha\text{-cl}(f^{-1}(V) \subset f^{-1}(V)$. It follows that $\alpha\text{-cl}(f^{-1}(V) = f^{-1}(V)$. □

Theorem 3.3. *Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ be a function and α be a monotone operator. The following propositions are equivalent:*

1. f is (α, β)-contra semi continuous.

2. For all β-closed set F in Y, $f^{-1}(F) \in \alpha{-}SO(X, \Gamma)$.

3. For each $x \in X$ and any β-closed set F in Y, such that $f(x) \in F$, there exists an α-semi open set U in X such that $x \in U$ and $f(U) \subset F$.

4. *$f(\alpha$-scl$(A)) \subset \beta$-ker$(f(A))$ for all subsets A of X.*
5. *α-scl$(f^{-1}(B)) \subset f^{-1}(\beta$-ker$(B))$ for all subsets B of Y.*

Proof. Observe that the hypothesis of monotony of the operator α is needed in order to guarantee that the α-semi closure of any set is α-semi closed. Using this fact and proceeding in analogous form as the proof of the Theorem 3.2, we obtain all the results. □

We recall that the frontier of any set A in a topological space X, is defined as Fr(A) =cl$(A)\cap$cl$(X - A)$.

Definition 3.3. Let (X, Γ, α). We define the α-frontier of any set A of X as follows: α-Fr$(A) = \alpha$-cl$(A)\cap\alpha$-cl$(X - A)$. In the case that α is a monotone operator, we define the α-semi frontier of any set A of X as follows: α-sFr(A) $=\alpha$-scl$(A)\cap\alpha$-scl$(X - A)$.

Definition 3.4. Let (X, Γ, α) and α be a monotone operator. We define the α-semi frontier of any set A of X as follows: α-sFr(A) $=\alpha$-scl$(A)\cap\alpha$-scl$(X - A)$.

Observe that the α-frontier of any set A of X is α-closed and the α-semi frontier of any set A of X is α-semi closed. Thus, we have that α-Fr$(A) =$ α-cl$(A)-\alpha$-Int(A) and α-sFr$(A) = \alpha$-scl$(A)-\alpha$-sInt(A).

Using the above notions, we can describe, the set of points where any function $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ is not (α, β) contra semi continuous as follows.

Theorem 3.4. *Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ be a function and α be a monotone operator. The set of all points x in X such that $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ is not (α, β) contra semi continuous is exactly the union of the α-semi frontier of the inverse image of the β-closed set in Y that contains $f(x)$.*

Proof. Let us suppose that f is not (α, β) contra semi continuous at the point $x \in X$, then there exist a β-closed set F such that $f(x) \in F$ and $f(U) \cap (Y - F) \neq \emptyset$ for all α-semi open set U, such that $x \in U$. It follows that $U \cap f^{-1}(Y - F) \neq \emptyset$, but this means that $x \in \alpha$-scl$(f^{-1}(Y - F)) = \alpha$-scl$(X - f^{-1}(F))$.

Since $x \in f^{-1}(F)$, then $x \in \alpha$-scl$(f^{-1}(F)) \cap \alpha$-scl$(X - f^{-1}(F))$. Therefore $\{x \in X :\ f$ is not (α, β) contra semi continuous at the point $x\}$ is contained in α-sFr$(f^{-1}(F))$.

Conversely, let us suppose that $x \in \alpha$-sFr$(f^{-1}(F))$, where F is β-closed in Y, $f(x) \in F$ and f is (α, β) contra semi continuous. Then there exists a α-semi open U such that $x \in U$ and $f(U) \subset F$, therefore $x \in U \subset f^{-1}(F)$. From this, we obtain that $x \in \alpha$-semi int$(\ f^{-1}(F)) \subset X - \alpha$-sFr$(f^{-1}(F))$. In consequence, we obtain that $x \notin \alpha$-sFr$(f^{-1}(F))$, contradiction. Therefore f is not(α, β) contra semi continuous. □

In the same way, we have the following theorem and it can be proved in a similar form as the above theorem.

Theorem 3.5. *Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ be a function. The set of all points x in X such that $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ is not (α, β) contra continuous is exactly the union of all α-frontier of the inverse image of the β-closed set in Y that contains $f(x)$.*

Definition 3.5. A space (X, Γ, α) is said to be α-locally indiscrete if every α-open set in X is α-closed in X.

Definition 3.6. Let $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$. We say that f is (α, β)-continuous function if for each point $x \in X$ and every Φ-open neighborhood V of $f(x)$, there exists a Γ-open neighborhood U of x such that $f(\alpha(U)) \subseteq \beta(V)$.

The following lemma claims that for functions that are (α, id)-continuous, the inverse image of any open set in Y is an α-open in X.

Lemma 3.1. *If $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ is (α, id)-continuous then $f^{-1}(V)$ is α-open for each $V \in \Phi$.*

Proof. Let $V \in \Phi$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. Using the hypothesis, there exists a set $U_x \in \Gamma$ such that $x \in U_x$, $f(\alpha(U_x)) \subseteq id(V) = V$, therefore $U_x \subset \alpha(U_x) \subseteq f^{-1}(V)$ and the result follows. □

Theorem 3.6. *If $f : (X, \Gamma, \alpha) \to (Y, \Phi, \beta)$ is a (α, id) continuous function and X is α-locally indiscrete space, then f is an (α, β)-contra continuous function.*

In the case that α is a monotone operator, we obtain a similar result to the above theorem in the case of (α, β)-contra semi continuous functions.

Observation 3.1. *Recall that $A \subset X$ is a pre-open set if $A \subset$int(cl(A)). Then, we can easily see, if $\alpha : P(X) \to P(X)$ is defined as $\alpha(U) =$int(cl(U)), then α is an operator associated with the topology of X. In this case, we have that α-SO(X) $\subseteq PO(X, \Gamma)$, where $PO(X, \Gamma)$ is the set of all pre-open sets. It is easy to see that there exists pre-open sets that are not α-semi open, where $\alpha(U) =$int(cl(U)).*

Observation 3.2. *If f is an (α, β) contra semi continuous functions, where $\alpha(U) = int(cl(U))$ and β is the identity map, then all (α, β) contra semi continuous functions are pre-continuous. But in case that we are working* *with arbitrary operators α and β, there are not known relations between (α, β) contra semi continuous functions and pre-continuous functions.*

4. Characterization of $\alpha\beta$ Irresolute and $\alpha\beta$ Contra Irresolute Functions

In this section, we obtain some characterizations of the $\alpha\beta$ irresolute and $\alpha\beta$ contra irresolute Functions using α-semi closure and the α-semi kernel.

Definition 4.1. Let (X, τ, α), (Y, ψ, β) be two topological spaces α, β associated operators to τ, β, respectively. A function $f : (X, \tau) \to (Y, \psi)$ is said to be:

(a) (α, β) irresolute if the inverse image of any β-semi open set $V \subseteq Y$, is an α-semi open set in X.

(b) (α, β) contra irresolute if the inverse image of any β-semi open set $V \subseteq Y$, is an α-semi closed set in X.

(c) (α, β)-scontinuous if for any point $x \in X$ and any β-semi open set $V \subseteq Y$, such that $f(x) \in V$, there exists an α-semi open set U in X such that $x \in U$ and $f(\alpha(U)) \subseteq \beta(V)$.

Definition 4.2. Let (X,τ,α),(Y,ψ,β) be two topological spaces with associated operators α, β, respectively. A function $f : (X, \tau) \to (Y, \psi)$ is said to be (α, β)-sclosed if the image of any α-semi closed set V in X is β-semi closed in Y.

Lemma 4.1. *Let (X,τ,α),(Y,ψ,β) be two topological spaces with α, β monotone operators, respectively. The following conditions are equivalent:*

(1) $f : (X, \tau) \to (Y, \psi)$ is (α, β) irresolute function.
(2) For each subset $A \subseteq X$, $f(\alpha - sCl(A)) \subseteq \beta - sCl(f(A)$.
(3) For each β semi closed subset $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an α semi closed in X.
(4) For all $B \subseteq Y$, $\alpha - sCl(f^{-1}(B)) \subseteq f^{-1}(\beta - sCl(B))$.

Proof. $(3 \Rightarrow 2)$ Let A be a subset of X and suppose that $y \notin \beta$-sCl$(f(A))$, then there exists a β-semi open set G in Y, such that $y \in G$ and $f(A) \cap G = \emptyset$, therefore, $f^{-1}(f(A) \cap G) = \emptyset$, it says that $A \cap f^{-1}(G) = \emptyset$. In consequence, α-cl$(A) \subset (f^{-1}(G))^c$, follows that $f(\alpha\text{-cl}(A)) \cap G = \emptyset$; and therefore, $y \notin f(\alpha$-cl$(A))$. But it is said that $f(\alpha\text{-cl}(A)) \subset \beta$-sCl$(f(A))$ for all subset A of X.

$(2 \Rightarrow 3)$ Let V any β-semi closed subset in Y, then $f^{-1}(V) \subseteq X$. By hypothesis $f(\alpha\text{-scl}(f^{-1}(V))) \subset \beta\text{-sCl}(f(f^{-1}(V)))$, it follows that $f(\alpha\text{-scl}(f^{-1}(V))) \subset$

β-sCl(V). In consequence, $f(\alpha\text{-scl}(f^{-1}(V))) \subset V$, follows that α-sCl$(f^{-1}(V) \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is an α-semi closed set.

$(2 \Rightarrow 4)$ Let B be a subset of Y, then $f^{-1}(B) \subseteq X$. Using the hypothesis, that

$$f(\alpha - scl(f^{-1}(B))) \subseteq \beta - sCl(f(f^{-1}(B))) \subseteq \beta - sCl(B),$$

therefore, $\alpha - scl(f^{-1}(B)) \subseteq f^{-1}(\beta - sCl(B))$.

$(4 \Rightarrow 3)$ Suppose that V is any β-semi closed set in Y. Then $f^{-1}(V) \subseteq X$, by hypothesis, we obtain that

$$\alpha - sCl(f^{-1}(V)) \subseteq f^{-1}(\beta - sCl(V)).$$

But V is a β-semi closed set, then $\beta - scl(V) = V$. In consequence,

$$\alpha - sCl(f^{-1}(V)) \subseteq f^{-1}(V).$$

But this says that $f^{-1}(V)$ is an α-semi closed set in X.

The others implications $(1 \Rightarrow 3)$ and $(3 \Rightarrow 1)$, follow from the definition of (α, β) irresolute function and the complement of set. □

Lemma 4.2. *Let (X,τ,α), (Y,ψ,β) be two topological spaces with associated operators α, β and α be a monotone. The following conditions are equivalent:*

(1) $f : (X, \tau) \rightarrow (Y, \psi)$ is (α, β) contra irresolute function.
(2) For each $A \subseteq X$, $f(\alpha - sCl(A)) \subseteq \beta - sker(f(A))$.
(3) For each β semi closed set $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an α semi open set in X.
(4) For each $x \in X$ and F be a $\beta-$ semi closed set in Y, such that: $f(x) \in F$, there exists an α semi open set $U \subseteq X$ such that $x \in U$ and $f(U) \subseteq F$.
(5) For each $B \subseteq Y$, $\alpha - sCl(f^{-1}(B)) \subseteq f^{-1}(\beta - sker(B))$.

Proof. $(3 \Rightarrow 2)$ Let A be a subset of X and suppose that $y \notin \beta$-sker$(f(A))$, then there exists a β-semi closed set F in Y, such that $y \in F$ and $f(A) \cap F = \emptyset$, therefore $f^{-1}(f(A) \cap F) = \emptyset$, it said that $A \cap f^{-1}(F) = \emptyset$. In consequence, α-cl$(A) \subset (f^{-1}(F))^c$, follows that $f(\alpha\text{-cl}(A)) \cap F = \emptyset$, but, it said that $y \notin f(\alpha$-cl$(A))$. Therefore, $f(\alpha\text{-cl}(A)) \subset \beta$-sker$(f(A))$ for all subset A of X.

$(2 \Rightarrow 5)$ Let B be any subset in Y, then $f^{-1}(B) \subseteq X$. By the hypothesis $f(\alpha$-scl$(f^{-1}(B))) \subset \beta$-sker$(f(f^{-1}(B)))$, follows that $f(\alpha\text{-scl}(f^{-1}(B))) \subset \beta$-sker$(B)$. In consequence, $f(\alpha\text{-scl}(f^{-1}(B))) \subset \beta$-sker$(B)$, therefore, $\alpha - sCl(f^{-1}(B) \subset f^{-1}(\beta - sker(B))$.

$(5 \Rightarrow 1)$ Let V be any β-semi open set in Y, then $f^{-1}(V) \subseteq X$. By the hypothesis, $\alpha - scl(f^{-1}(V))) \subset f^{-1}(\beta - sker(V))$, but $\beta - sker(V) = V$, follows that $\alpha - scl(f^{-1}(V))) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is an α-semi closed set.

$(1 \Rightarrow 3)$ It follows using complement.

$(3 \Rightarrow 4)$ and $(4 \Rightarrow 3)$ are immediate. □

5. Applications

In this section, we will apply the above concepts in order to prove the invariance of certain properties of the domain and the range under the actions of the above functions.

Definition 5.1. Given the triplete (X, τ, α). X is said to be an α semi-connected space, if and only if the only subsets of X that are both α semi-open and α semi-closed in X are the $\emptyset$ and X itself.

Theorem 5.1. *Let $f : (X, \tau, \alpha) \to (Y, \sigma, \beta)$ be an (α, β) contra irresolute function. If X is an α semi-connected space, α, β monotone operators and Y is a β-semi T_1 space, then f is a constant map.*

Proof. Suppose that $f : (X, \tau, \alpha) \to (Y, \sigma, \beta)$ is an (α, β) contra irresolute function, X is an α semi-connected space, Y is a β-semi T_1 space and that f is not a constant map. Consequently, the collection $C = \{f^{-1}(\{y\}) : y \in Y\}$ is a partition of X, that have at least two elements. Since Y is a β-semi T_1 space, then each unitary set $\{y\}$, $y \in Y$, is β-semi closed (see [8], Corollary 1) and therefore each $f^{-1}(\{y\})$ is α-semi open. In consequence, $C = \{f^{-1}(\{y\}) : y \in Y\}$ is a partition of X by α-semi open sets. By the hypothesis, α is a monotone operator, then the union of α-semi open sets is α-semi open. Therefore, the collection C would contain a proper subset $A \neq \emptyset$ of X that is α-semi open and α-semi closed. Contradiction. □

Theorem 5.2. *If $f : (X, \tau, \alpha) \to (Y, \sigma, id)$ is an (α, id)-scontinuous function and α a monotone operator, then $f^{-1}(V)$ is an α semi-open set in X, for any open set $V \in \sigma$.*

Proof. Let $V \in \sigma$ and $x \in f^{-1}(V)$, since $f : (X, \tau, \alpha) \to (Y, \sigma, id)$, (α, id)-is a scontinuous function and $f(x) \in V$, there exists $U_x \in \alpha - SO(X, \tau)$ such that

$f(\alpha(U_x)) \subseteq V$, this implies that $\alpha(U_x) \subseteq f^{-1}(V)$. Therefore,

$$f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} \alpha(U_x).$$

But, each U_x, $x \in X$, is an α semi-open set and using the fact that α is a monotone operator, conclude that,

$$U_x \subseteq \alpha(Int(U_x)) \subseteq \alpha(U_x), \quad \forall x \in X.$$

In consequence,

$$f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x.$$

It follows that, $f^{-1}(V)$ is an α semi-open set in X. □

Theorem 5.3. *Let (X,τ,α) be a triplete with α a monotone operator, $A \subseteq X$ and $C_A : X \to \{0,1\}$ be the characteristic function of A. If C_A is an (α, id)-scontinuous function then A is an α semi-open set and α semi-closed set in X.*

Proof. Let us suppose that C_A is an (α, id)-scontinuous function, then $C_A^{-1}(\{1\}) = A$ and $C_A^{-1}(\{0\}) = X - A$. Using Theorem 5.2, A is an α-semi open set and α semi closed set. □

Corollary 5.1. *Let $f : (X, \tau, \alpha) \to (Y, \sigma, \beta)$ be surjective functions and the cardinality of σ is bigger than 2. If f is (α, β)-contra irresolute function and (α, β)-sclosed, then Y is not a β semi-connected space.*

Definition 5.2. Let (X, τ, α) be a triplete with α be a monotone operator. A subset B of X is said to be an α semi-generalized closed set, denoted by ($\alpha - sg$-closed) if the $\alpha - sCl(A) \subseteq O$, whenever $B \subseteq O$ and O is an α semi open set.

Definition 5.3. Let (X, τ, α) be a triplete with α be a monotone operator. X is said to be an α-semi $T_{1/2}$ space if all α-sg-closed set in X is an α semi closed set.

Theorem 5.4. *Let $f : (X, \tau, \alpha) \to (Y, \sigma, \beta)$ be an (α, β)-irresolute function and (α, β)-sclosed, then:*

(a) For all α-sg-closed set A in X, $f(A)$ is a β-sg-closed set in Y.

(b) $f^{-1}(B)$ is an α sg-closed set in X for all β semi closed set B in Y.

Proof. (a) Let V be a β-semi open set in Y such that $f(A) \subseteq V$, then $f^{-1}(V)$ is an α semi open set and $A \subseteq f^{-1}(V)$. It follows that, $\alpha - scl(A) \subseteq f^{-1}(V)$, since f is an (α,β)-sclosed, then $f(\alpha - scl(A))$ is a β-semi closed set. In consequence, $\beta - scl(f(A)) \subseteq \beta - scl(f(\alpha - scl(A))) = f(\alpha - scl(A)) \subseteq V$. It follows that $f(A)$ is a β-semi generalized closed set.

(b) Let B be a β-semi closed set in Y, and suppose that U is an α-semi open set in X such that $f^{-1}(B) \subseteq U$. Consider $F = \alpha - sCl(f^{-1}(B)) \cap U^c$. Then using the fact that α is a monotone operator, we may conclude that F is an α-semi closed set, therefore $f(F)$ is a β-semi closed set and

$$\begin{aligned} f(F) &= f(\alpha - sCl(f^{-1}(B)) \cap U^c) \\ &\subseteq f(\alpha - sCl(f^{-1}(B))) \cap f(U^c)) \\ &\subseteq \beta - sClf^{-1}(f(B)) \cap f(U^c) \subseteq \beta - sCl(B) \cap B^c = \emptyset. \end{aligned}$$

In consequence, $f(F) = \emptyset$, and $F = \emptyset$, therefore, $f^{-1}(B)$ is an α semi generalized set. □

Corollary 5.2. *Let $f : (X,\tau,\alpha) \to (Y,\sigma,\beta)$ be an (α,β)-irresolute function and (α,β)-sclosed, then:*

(a) If f is an injective function and (Y,σ) is a β-semi $T_{1/2}$ space, then (X,τ) is an α-semi $T_{1/2}$ space.

(b) If f is a bijective function and (X,τ) is an α-semi $T_{1/2}$ space, then (Y,σ) is a β-semi $T_{1/2}$ space.

Proof. (a). Suppose that A is an α-sg-closed set in X. Using Theorem 3.5 and the hypothesis, then $f(A)$ is a β-sg-closed set in Y, but Y is a β-semi $T_{1/2}$ space, then $f(A)$ is a β-semi closed set in Y. Now, using the fact that f is an injective function and (α,β) irresolute, it follows that $A = f^{-1}(f(A))$ is an α-semi closed set in X. In consequence, (X,τ) is an α-semi $T_{1/2}$ space.

(b) Given $y \in Y$, there exists a unique point $x \in X$ such that $y = f(x)$. It follows that each unitary set in Y is a β-semi open or β-semi closed and therefore Y is a β-semi $T_{1/2}$ space (see [12], Theorem 8). □

Theorem 5.5. *Let $f : (X,\tau,\alpha) \to (Y,\sigma,\beta)$ be an (α,β)-irresolute function and (α,β)-sclosed, then:*

(a) If f is an injective function and (Y,σ) is a β-semi T_1 space, then (X,τ) is an α-semi T_1 space.

(b) If f is a bijective function and (X,τ) is an α-semi T_1 space, then (Y,σ) is a β-semi T_1 space.

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