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A REVERSE HILBERT'S TYPE INTEGRAL INEQUALITY

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Abstract: By introducing the weight function, a reverse Hilbert's type integral inequality with some parameters and a best constant factor is given. As applications, two equivalent forms of the reverse inequality and some special results are obtained.

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1. Introduction

If $p > 1, \frac{1}{p} + \frac{1}{q} = 1, f, g \geq 0$, such that $0 < \int_0^\infty f^p(t)dt < \infty$ and $0 < \int_0^\infty g^q(t)dt < \infty$, then the famous Hardy-Hilbert's integral inequality and its equivalent form are given by

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{x+y} dxdy < \frac{\pi}{\sin(\frac{\pi}{p})} (\int_0^\infty f^p(x)dx)^{\frac{1}{p}} (\int_0^\infty g^q(x)dx)^{\frac{1}{q}}; \tag{1.1}$$

$$\int_0^\infty [\int_0^\infty \frac{f(x)}{x+y} dx]^p dy < [\frac{\pi}{\sin(\frac{\pi}{p})}]^p \int_0^\infty f^p(x)dx, \tag{1.2}$$

where the constant factors $\frac{\pi}{\sin(\frac{\pi}{p})}$ and $[\frac{\pi}{\sin(\frac{\pi}{p})}]^p$ are the best possible (cf. Hardy et al [1]). Both of them are important in analysis and its applications (cf. Mitrinovic et al [3]).

In recent years, Zhao et al [8] consider a reverse Hilbert-Pachpatte's inequality; Yang [4, 5] got some reverse inequalities concerning some best extensions of (1.1). In 2004, by introducing a independent parameter λ, Yang

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[4] gave a reverse integral inequality of (1.1) and its equivalent inequality as follows: If $0 < p < 1, \frac{1}{p} + \frac{1}{q} = 1, 2 - p < \lambda < 2 - q$, such that $f, g \geq 0$ and $0 < \int_0^\infty t^{1-\lambda} f^p(t)dt < \infty$, $0 < \int_0^\infty t^{1-\lambda} g^q(t)dt < \infty$, then one has the following equivalent inequalities

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dxdy > B(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q})[\int_0^\infty x^{1-\lambda} f^p(x)dx]^{\frac{1}{p}} \times [\int_0^\infty y^{1-\lambda} g^q(y)dy]^{\frac{1}{q}}, \quad (1.3)$$

$$\int_0^\infty x^{(q-1)(\lambda-1)} [\int_0^\infty \frac{g(y)}{(x+y)^\lambda} dy]^q dx < [B(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q})]^q \int_0^\infty y^{1-\lambda} g^q(y)dy, \quad (1.4)$$

$$\int_0^\infty y^{(p-1)(\lambda-1)} [\int_0^\infty \frac{f(x)}{(x+y)^\lambda} dx]^p dy > [B(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q})]^p \int_0^\infty x^{1-\lambda} f^p(x)dx, \quad (1.5)$$

where the constant factors $k_\lambda(p) = B(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q})$, $[k_\lambda(p)]^q$ and $[k_\lambda(p)]^p$ are all the best possible. $B(u, v)$ is the Bate function defined by

$$B(u, v) = \int_0^\infty \frac{t^{u-1}}{(1+t)^{u+v}} dt \quad (u, v > 0). \quad (1.6)$$

By introducing some parameters that have been used first in [5], the main objective of this paper is to build a new reverse Hilbert's type integral inequality with a best constant factor. As applications, two equivalent forms of the reverse inequality and some particular results are considered.

2. Lemmas

Lemma 2.1. *If $p < 0$ or $0 < p < 1, \frac{1}{p} + \frac{1}{q} = 1, f, g \geq 0$, such that $f \in L^p(E), g \in L^q(E)$, then*

$$\int_E f(t)g(t)dt \geq (\int_E f^p(t)dt)^{\frac{1}{p}} (\int_E g^q(t)dt)^{\frac{1}{p}}, \quad (2.1)$$

where the equality holds if and only if there exists non-negative real numbers A and B such that they are not all zero and $Af^p(t) = Bg^q(t)$ a.e., see Kuang [2].

Lemma 2.2. *If $p < 0$ or $0 < p < 1, r > 1, \frac{1}{r} + \frac{1}{s} = 1, \lambda > 0$. Define weighting function $\omega_\lambda(p, r, t)$ as follows*

$$\omega_\lambda(p, r, t) := \int_0^\infty \frac{t^{(p-1)(1-\frac{\lambda}{r})}}{(t+y)^\lambda y^{1-\frac{\lambda}{s}}} dy, t \in (0, \infty). \tag{2.2}$$

Then it yields

$$\omega_\lambda(p, r, t) = t^{p(1-\frac{\lambda}{r})-1} B(\frac{\lambda}{s}, \frac{\lambda}{r}), \tag{2.3}$$

where, $B(u, v)$ is the Beta function defined by (1.6).

Proof. For $t \in (0, \infty)$, setting $u = \frac{y}{t}$ in the integral (2.2), one finds

$$\omega_\lambda(p, r, t) = t^{p(1-\frac{\lambda}{r})-1} \int_0^\infty \frac{u^{\frac{\lambda}{s}-1}}{(1+u)^\lambda} du.$$

By (1.6), one has (2.3). The lemma is proved. □

Lemma 2.3. *If $0 < \varepsilon < \frac{-\lambda q}{r} (\lambda > 0, r > 1, q < 0), a > 0$, setting $f_\varepsilon(x)$ and $g_\varepsilon(y)$ as*

$$f_\varepsilon(x) = \begin{cases} 0 & x \in (0, a), \\ x^{\frac{\lambda}{r}-\frac{\varepsilon}{p}-1} & x \in [a, \infty), \end{cases} \tag{2.4}$$

$$g_\varepsilon(y) = \begin{cases} 0 & x \in (0, a), \\ y^{\frac{\lambda}{s}-\frac{\varepsilon}{q}-1} & y \in [a, \infty), \end{cases} \tag{2.5}$$

one has

$$\int_a^\infty [\int_0^\infty \frac{f_\varepsilon(x) g_\varepsilon(y)}{(x+y)^\lambda} dy] dx \le \frac{1}{a^\varepsilon \varepsilon} B(\frac{\lambda}{s} - \frac{\varepsilon}{q}, \frac{\lambda}{r} + \frac{\varepsilon}{q}). \tag{2.6}$$

Proof. For $a > 0$, $x \in [a, \infty)$, setting $u = \frac{y}{x}$ in the integral (2.6), by (2.4) and (2.5), one has

$$\int_a^\infty [\int_0^\infty \frac{f_\varepsilon(x) g_\varepsilon(y)}{(x+y)^\lambda} dy] dx \le \int_a^\infty [\int_0^\infty \frac{x^{\frac{\lambda}{r}-\frac{\varepsilon}{p}-1} y^{\frac{\lambda}{s}-\frac{\varepsilon}{q}-1}}{(x+y)^\lambda} dy] dx$$
$$= \int_a^\infty x^{-\varepsilon-1} [\int_0^\infty \frac{u^{\frac{\lambda}{s}-\frac{\varepsilon}{q}-1}}{(1+u)^\lambda} du] dx = \frac{1}{a^\varepsilon \varepsilon} \int_0^\infty \frac{u^{\frac{\lambda}{s}-\frac{\varepsilon}{q}-1}}{(1+u)^\lambda} du.$$

By using (1.6), one has (2.6). The lemma is proved. □

3. Main Results

Theorem 3.1. *If* $0 < p < 1, \frac{1}{p} + \frac{1}{q} = 1, r > 1, \frac{1}{r} + \frac{1}{s} = 1, \lambda > 0$ *and* $f, g \geq 0$, *such that* $0 < \int_0^\infty t^{p(1-\frac{\lambda}{r})-1} f^p(t)dt < \infty$, $0 < \int_0^\infty t^{q(1-\frac{\lambda}{s})-1} g^q(t)dt < \infty$, *then*

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dxdy > k_\lambda(r)[\int_0^\infty x^{p(1-\frac{\lambda}{r})-1} f^p(x)dx]^{\frac{1}{p}} [\int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy]^{\frac{1}{q}}, \quad (3.1)$$

where the constant factor $k_\lambda(r) = B(\frac{\lambda}{s}, \frac{\lambda}{r})$ *is the best possible.*

Proof. By (2.1) and (2.2), one has

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dxdy$$

$$= \int_0^\infty \int_0^\infty [\frac{f(x)}{(x+y)^{\frac{\lambda}{p}}} \frac{x^{(1-\frac{\lambda}{r})/q}}{y^{(1-\frac{\lambda}{s})/p}}][\frac{g(y)}{(x+y)^{\frac{\lambda}{q}}} \frac{y^{(1-\frac{\lambda}{s})/p}}{x^{(1-\frac{\lambda}{r})/q}}]dxdy$$

$$\geq [\int_0^\infty \int_0^\infty \frac{x^{(p-1)(1-\frac{\lambda}{r})} f^p(x)}{(x+y)^\lambda y^{1-\frac{\lambda}{s}}} dydx]^{\frac{1}{p}} [\int_0^\infty \int_0^\infty \frac{y^{(q-1)(1-\frac{\lambda}{s})} g^q(y)}{(x+y)^\lambda x^{1-\frac{\lambda}{r}}} dxdy]^{\frac{1}{q}}$$

$$= [\int_0^\infty \omega_\lambda(p, r, x) f^p(x)dx]^{\frac{1}{p}} [\int_0^\infty \omega_\lambda(q, s, y) g^q(y)dy]^{\frac{1}{q}}. \quad (3.2)$$

If (3.2) takes the form of equality, then by Lemma 2.1, there exist real numbers A and B, such that they are not all zero, and

$$A \frac{x^{(p-1)(1-\frac{\lambda}{r})} f^p(x)}{(x+y)^\lambda y^{1-\frac{\lambda}{s}}} = B \frac{y^{(q-1)(1-\frac{\lambda}{s})} g^q(y)}{(x+y)^\lambda x^{1-\frac{\lambda}{r}}} \text{ a.e. in } (0, \infty) \times (0, \infty).$$

Hence one finds

$$Ax^{p(1-\frac{\lambda}{r})} f^p(x) = By^{q(1-\frac{\lambda}{s})} g^q(y) \text{ a.e. in } (0, \infty) \times (0, \infty).$$

It follows that there exists a constant C, such that

$$Ax^{p(1-\frac{\lambda}{r})} f^p(x) = By^{q(1-\frac{\lambda}{s})} g^q(y) = C \text{ a.e. in } (0, \infty) \times (0, \infty).$$

Without loss of generality, suppose that $A \neq 0$, one has

$$x^{p(1-\frac{\lambda}{r})-1} f^p(x) = \frac{C}{Ax}, \text{ a.e. in } (0, \infty),$$

which contradicts the face that $0 < \int_0^\infty t^{p(1-\frac{\lambda}{r})-1} f^p(t)dt < \infty$. Hence, (3.2) takes the form of strict inequality. Then by (2.3), it follows that (3.1) is valid.

Setting $f_\varepsilon(x)$ and $g_\varepsilon(y)$ as (2.4) and (2.5), since

$$[\int_a^\infty x^{p(1-\frac{\lambda}{r})-1} f_\varepsilon^p(x)dx]^{\frac{1}{p}} [\int_a^\infty y^{q(1-\frac{\lambda}{s})-1} g_\varepsilon^q(y)dy]^{\frac{1}{q}} = \frac{1}{a^\varepsilon \varepsilon}, \tag{3.3}$$

if the constant factor $k_\lambda(r)$ in (3.1) is not the best possible, then by the preservation of limit, there exists a positive constant K ($K > k_\lambda(r)$) and $\alpha > 0$, such that $\forall a \in (0, \alpha)$,

$$\int_a^\infty [\int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dy]dx > K[\int_a^\infty x^{p(1-\frac{\lambda}{r})-1} f^p(x)dx]^{\frac{1}{p}} [\int_a^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy]^{\frac{1}{q}}, \tag{3.4}$$

is still valid. In particular, replace $f(x)$ and $g(y)$ by $f_\varepsilon(x)$ and $g_\varepsilon(y)$ in (3.4), for $0 < \varepsilon < \frac{-\lambda q}{r}$ ($q < 0$), considering (2.6) and (3.3), one has

$$\frac{1}{a^\varepsilon} B(\frac{\lambda}{s} - \frac{\varepsilon}{q}, \frac{\lambda}{r} + \frac{\varepsilon}{q}) \geq \varepsilon \int_a^\infty [\int_0^\infty \frac{f_\varepsilon(x)g_\varepsilon(y)}{(x+y)^\lambda} dy]dx$$

$$> \varepsilon K[\int_a^\infty x^{p(1-\frac{\lambda}{r})-1} f_\varepsilon^p(x)dx]^{\frac{1}{p}} [\int_a^\infty y^{q(1-\frac{\lambda}{s})-1} g_\varepsilon^q(y)dy]^{\frac{1}{q}} = \frac{K}{a^\varepsilon},$$

and then $k_\lambda(r) \geq K(\varepsilon \to 0^+)$. This contradicts the fact that $K > k_\lambda(r)$. It follows that the constant factor in (3.1) is the best possible. The theorem is proved. □

Theorem 3.2. *If* $0 < p < 1$, $\frac{1}{p} + \frac{1}{q} = 1, r > 1, \frac{1}{r} + \frac{1}{s} = 1, \lambda > 0$ *and* $g \geq 0$, *such that* $0 < \int_0^\infty t^{q(1-\frac{\lambda}{s})-1} g^q(t)dt < \infty$, *then one has the following inequality equivalent to (3.1)*

$$\int_0^\infty x^{\frac{q\lambda}{r}-1} [\int_0^\infty \frac{g(y)}{(x+y)^\lambda} dy]^q dx < [k_\lambda(r)]^q \int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy, \tag{3.5}$$

where the constant factor $[k_\lambda(r)]^q$ *is the best possible.*

Proof. Since $0 < \int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy < \infty$, one sets

$$f(x) = x^{\frac{q\lambda}{r}-1} [\int_0^\infty \frac{g(y)}{(x+y)^\lambda} dy]^{q-1}, \quad x \in (0, \infty).$$

Then $f(x) \geq 0$, by using (3.2) and $q < 0$, one has

$$\int_0^\infty x^{p(1-\frac{\lambda}{r})-1} f^p(x)dx = \int_0^\infty x^{\frac{q\lambda}{r}-1} [\int_0^\infty \frac{g(y)}{(x+y)^\lambda} dy]^q dx = \int_0^\infty \int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dxdy$$

$$\geq k_\lambda(r)[\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx]^{\frac{1}{p}}[\int_0^\infty y^{q(1-\frac{\lambda}{s})-1}g^q(y)dy]^{\frac{1}{q}}, \tag{3.6}$$

and

$$0<\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx \leq [k_\lambda(r)]^q\int_0^\infty y^{q(1-\frac{\lambda}{s})-1}g^q(y)dy<\infty. \tag{3.7}$$

It follows that (3.6) and (3.7) are strict inequalities. Thus inequality (3.5) holds.

On the other hand, if (3.5) is valid, by using (3.5) and Hölder's inequality (2.1), in view of $q<0$, one has

$$\int_0^\infty\int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda}dxdy = \int_0^\infty [x^{\frac{\lambda}{r}-\frac{1}{q}}\int_0^\infty \frac{g(y)}{(x+y)^\lambda}dy][x^{\frac{1}{q}-\frac{\lambda}{r}}f(x)]dx$$
$$\geq \{\int_0^\infty x^{\frac{q\lambda}{r}-1}[\int_0^\infty \frac{g(y)}{(x+y)^\lambda}dy]^q dx\}^{\frac{1}{q}}\{\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx\}^{\frac{1}{p}}$$
$$> k_\lambda(r)\{\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx\}^{\frac{1}{p}}\{\int_0^\infty y^{q(1-\frac{\lambda}{s})-1}g^q(y)dy\}^{\frac{1}{q}}, \tag{3.8}$$

(3.1) is valid. It follows that (3.5) is equivalent to (3.1).

If the constant factor in (3.5) is not the best possible, one can get a contradiction that the constant factor in (3.1) is not the best possible by using (3.8). The theorem is proved. □

Theorem 3.3. *If $0<p<1$, $\frac{1}{p}+\frac{1}{q}=1, r>1, \frac{1}{r}+\frac{1}{s}=1, \lambda>0$ and $f\geq 0$, such that $0<\int_0^\infty t^{p(1-\frac{\lambda}{r})-1}f^p(t)dt<\infty$, then one has the following inequality equivalent to (3.1)*

$$\int_0^\infty y^{\frac{p\lambda}{s}-1}[\int_0^\infty \frac{f(x)}{(x+y)^\lambda}dx]^p dy > [k_\lambda(r)]^p\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx, \tag{3.9}$$

where the constant factor $[k_\lambda(r)]^p$ is still the best possible.

Proof. Since $0<\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx<\infty$, one sets

$$g(y)=y^{\frac{p\lambda}{s}-1}[\int_0^\infty \frac{f(x)}{(x+y)^\lambda}dx]^{p-1}, \quad y\in(0,\infty).$$

Then $g(y)\geq 0$, by (3.2), one has

$$\int_0^\infty y^{q(1-\frac{\lambda}{s})-1}g^q(y)dy$$
$$=\int_0^\infty y^{\frac{p\lambda}{s}-1}[\int_0^\infty \frac{f(x)}{(x+y)^\lambda}dx]^p dy = \int_0^\infty\int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda}dxdy$$
$$\geq k_\lambda(r)[\int_0^\infty x^{p(1-\frac{\lambda}{r})-1}f^p(x)dx]^{\frac{1}{p}}[\int_0^\infty y^{q(1-\frac{\lambda}{s})-1}g^q(y)dy]^{\frac{1}{q}}, \tag{3.10}$$

and

$$\int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy = \int_0^\infty y^{\frac{p\lambda}{s}-1}[\int_0^\infty \frac{f(x)}{(x+y)^\lambda}dx]^p dy$$
$$\geq [k_\lambda(r)]^p \int_0^\infty x^{p(1-\frac{\lambda}{r})-1} f^p(x)dx > 0. \quad (3.11)$$

If $\int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy = \infty$, considering that $0 < \int_0^\infty t^{p(1-\frac{\lambda}{r})-1} f^p(t)dt < \infty$, (3.11) takes the form of strict inequality. One has (3.9); if

$$0 < \int_0^\infty y^{q(1-\frac{\lambda}{s})-1} g^q(y)dy < \infty,$$

by (3.1), it following that (3.10) and (3.11) keep the strict forms. Thus inequality (3.9) holds.

On the other hand, if (3.9) is valid, by the same way of Theorem 3.2, one can show that (3.1) holds, which is equivalent to (3.9). By the equivalence of (3.1) and (3.9), one may conclude that the constant factor in (3.9) is the best possible. The theorem is proved. □

For $r = s = 2$, by (3.1), (3.5) and (3.9), one has

Corollary 3.4. *If $0 < p < 1$, $\frac{1}{p} + \frac{1}{q} = 1$, $\lambda > 0$, then one has the following equivalent inequalities*

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda} dxdy$$
$$> B(\frac{\lambda}{2}, \frac{\lambda}{2})[\int_0^\infty x^{p(1-\frac{\lambda}{2})-1} f^p(x)dx]^{\frac{1}{p}} [\int_0^\infty y^{q(1-\frac{\lambda}{2})-1} g^q(y)dy]^{\frac{1}{q}}, \quad (3.12)$$

$$\int_0^\infty x^{\frac{q\lambda}{2}-1}[\int_0^\infty \frac{g(y)}{(x+y)^\lambda}dy]^q dx < [B(\frac{\lambda}{2}, \frac{\lambda}{2})]^q \int_0^\infty y^{q(1-\frac{\lambda}{2})-1} g^q(y)dy, \quad (3.13)$$

$$\int_0^\infty y^{\frac{p\lambda}{2}-1}[\int_0^\infty \frac{f(x)}{(x+y)^\lambda}dx]^p dy$$
$$> [B(\frac{\lambda}{2}, \frac{\lambda}{2})]^p \int_0^\infty x^{p(1-\frac{\lambda}{2})-1} f^p(x)dx, \quad (3.14)$$

where the constant factors are all the best possible. In particular, for $\lambda = 1$, one has

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{x+y} dxdy$$
$$> \pi[\int_0^\infty x^{\frac{p}{2}-1} f^p(x)dx]^{\frac{1}{p}} [\int_0^\infty y^{\frac{q}{2}-1} g^q(y)dy]^{\frac{1}{q}}, \quad (3.15)$$

$$\int_0^\infty x^{\frac{q}{2}-1}[\int_0^\infty \frac{g(y)}{x+y}dy]^q dx < \pi^q \int_0^\infty y^{\frac{q}{2}-1} g^q(y)dy, \quad (3.16)$$

$$\int_0^\infty y^{\frac{p}{2}-1}[\int_0^\infty \frac{f(x)}{x+y}dx]^p dy > \pi^p \int_0^\infty x^{\frac{p}{2}-1} f^p(x)dx. \qquad (3.17)$$

Remark 3.5. (a) Inequalities (3.1), (3.5) and (3.9) are all equivalent.

(b) (3.12) is a reverse integral inequality of (1.1) with one parameter λ different from (1.3).

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References

[1] G.H. Hardy, J.E. Littiewood, G. Polya, *Inequalities*, Cambridge University Press, Cambridge (1952).

[2] J. Kuang, *Applied Inequalities*, Shangdong Science and Technology Press, Jinan (2004).

[3] D.S. Mintrinovic, J.E. Pecaric, A.M. Fink, *Inequalities Involving Functions and Their Integrals and Derivatives*, Kluwer Academic Publishers, Boston (1991).

[4] B. Yang, A reverse Hardy-Hilbert's integral inequality, *J. Jili. Univ.* (Scie. Edit.), **42**, No. 4 (2004), 489-493.

[5] B. Yang, On an extension of Hilbert's integral inequality with some parameters, *J. Math. Anal. Appl.*, **1** (2004), 1-8.

[6] B. Yang, A reverse of the Hardy-Hilbert's type inequality, *J. Sout. China Norm. Univ.* (Natu. Scie.), **30**, No. 6 (2005), 1012-1015.

[7] B. Yang, On best extensions of Hardy-Hilbert's inequality with two parameters, *J. Ineq. Pure Appl. Math.*, **6**, No. 3 (2005), 1-18.

[8] C. Zhao, L. Debnath, Some new inverse type Hilbert integral inequalities, *J. Math. Anal. Appl.*, **245** (2001), 248-265.