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FUZZY CONGRUENCE ON BI-SEMIRINGS

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Abstract: In this paper, we define the concept of fuzzy congruence on the bi-semirings. The properties of bi-semirings S with fuzzy congruence relation are discussed. We prove that if S is a reversible bi-semirings with fuzzy congruence relation, then the kernel of fuzzy congruence relation is a fuzzy ideal of S. And some homomorphic properties of bi-semirings with fuzzy congruence relation are investigated at the end.

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1. Introduction

After the concept of semi-ring was first proposed by Dedekind in the original work of algebraic number theory in 1894, it was widely used by many well-known scholars. Such as Noether, Krull et al, they used it in researching the ideal of ring. In 1899, it was quoted by Hilbert in his discussion about the axiom of natural number and non-negative rational number. In recent years, the semi-ring theory achieves a big progress, the main viewpoints in investigating the semi-ring focus on ring and semi-group. Only after Redei and A. Costa summarized the main part of theory which related with semi-ring in 1967 and

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in 1974 respectively, then semi-ring became a main branch in algebra. In 1991, Golan discussed the semi-ring theory in system in his literature [1], then semi-ring genuinely became a subject.

Semi-ring is a algebra system which includes addition and multiplication and satisfies combinative law and distributive law. Its application is much wider than ring and its concept and characteristic are widely applied in many subjects, such as functional analysis, topology, graph theory, optimization theory, automatic theory, format language theory and quanta physics and so on.

In the semi-ring theory, the structure of semi-ring is always the main subject we investigated, but congruence on semi-ring is the main tool that we use to investigate the structure of semi-ring. Semi-ring and lattice-order semi-group were intensively investigated by many mathematicians in 1950. The methods they employed can be found in the ring theory and lattice-order group respectively. In general, the congruence and ideal on semi-group, group, ring and lattice play a very significant role in depicting these algebra structures. In [2], they proposed the concept of bi-semirings which is a much more general concept than semi-ring and distributive lattice-order semi-group, and discussed some basic properties of bi-semirings, they also investigated the depicting of the congruence on bi-semirings for the purpose to unify the methods which were employed in researching semi-ring and lattice-order semi-group.

Since the first literature about fuzzy mathematics [3] which was written by Zadeh was published in 1965, The strong life-force of fuzzy mathematics was fully represented in researching the basic application theory. In early 1970, the investigation of fuzzy algebra theory had been commenced [4]. Fuzzy equivalence relation and fuzzy congruence were vital tools in researching the fuzzy algebra theory. N. Kuroki first proposed the concepts of fuzzy congruence on semi-group and fuzzy ideal, F.A. Alghuhair investigated the fuzzy congruence pair on reversible semi-group, in [5], they defined the concept of fuzzy congruence, and investigated the homomorphism properties of ring with fuzzy congruence relation. In [6], the concept of fuzzy congruence on reversible semi-ring with addition was defined and the homomorphism properties of additional reversible semi-ring with fuzzy congruence was discussed. Naturally, the investigation for the definition and properties of fuzzy congruence relation on bi-semirings is a significative work.

In this paper we define the fuzzy congruence relation on bi-semirings based on the concept of bi-semirings, the properties of bi-semirings with fuzzy congruence relation is discussed. We prove that: if S is a reversible bi-semirings with fuzzy congruence, then the kernel of the fuzzy congruence is a fuzzy ideal of S, and we also give some homomorphism properties of bi-semirings with fuzzy congruence relations.

2. Basic Concepts and Properties

A non-empty set S is said to be a bi-semirings, if there three algebra operations "$+$", "$\bigoplus$", "$\bullet$" on S are defined, and they satisfy:

(1) $(s,+),(s,\bigoplus),(s,\bullet)$ are all semi-groups;

(2) for $\forall a,b,c \in S$ they satisfy the following properties:

i) $a(b+c)=ab+ac,(b+c)a=ba$;

ii) $a(b\bigoplus c)=ab\bigoplus ac,(b\bigoplus c)a=ba\bigoplus ca$;

iii) $a+(b\bigoplus c)=(a+b)\bigoplus(a+c),(b\bigoplus c)+a=(b+a)\bigoplus(c+a)$;

iv) $a\bigoplus(b+c)=(a\bigoplus b)+(a\bigoplus c),(b+c)\bigoplus a=(b\bigoplus c)+(c\bigoplus a)$.

Definition 2.1. Let S be a bi-semirings, a fuzzy relation α on S means a mapping from $S\times S$ to the unit interval [0,1], let α,β be two fuzzy relations on S, then the product $\alpha\circ\beta$ between α and β is defined as: $(\alpha\circ\beta)(a,b)=\sup_{x\in s}[\min\{\alpha(a,x),\beta(x,b)\}]$, where $\alpha\leq\beta$ if and only if $\alpha(x,y)\leq\beta(x,y),\forall(x,y)\in S$

A fuzzy set μ on X means a mapping from X to [0,1] we denote all fuzzy sets on X by $F(x)$, $\mu_\lambda=\{x\in X|\mu(\lambda)\geq\lambda\}$ and $\mu(\lambda)=\{x\in X|\mu(x)>\lambda\}$ is called λ level set and λ strong level set of μ respectively.

The fuzzy relation μ on S is called the fuzzy equivalence relation on S, if it satisfies:

(1) $\mu(a,a)=1$ (fuzzy reflexive);

(2) $\mu(a,b)=\mu(b,a)$ (fuzzy symmetric);

(3) $\mu\circ\mu\subseteq\mu$ (fuzzy transitive).

3. The Congruence Relation on Bi-Semirings

Definition 3.1. Let μ be the fuzzy equivalence relation on fuzzy bi-semirings, then μ is called the fuzzy congruence relation on S, for $\forall x\in S$, if it satisfies:

(1) $\mu(a+x,b+x)\geq\mu(a,b)$;

(2) $\mu(ax,bx)\geq\mu(a,b)$ and $\mu(xa,xb)\geq\mu(a,b)$;

(3) $\mu(a\bigoplus x,b\bigoplus x)\geq\mu(a,b)$ and $\mu(x\bigoplus a,x\bigoplus b)\geq\mu(a,b).\forall a,b\in S$.

We denote the characteristic function of the duality relation f on S by x_f, $con(s)$ and $con_f(s)$ denotes the whole congruence sets and the whole fuzzy congruence sets on S respectively, then we have the following results.

Theorem 3.1. *Let f be the duality relation on bi-semirings S, then f is the equivalence relation (congruence relation) on S, if and only if x_f is the fuzzy equivalence relation (congruence relation) on S.*

Proof. See Theorem 2.4 in [7]. □

Theorem 3.2. *μ is the fuzzy equivalence relation (congruence relation) on S, if and only if for $\forall\lambda \in [0,1], \mu_\lambda$ is the equivalence relation (congruence relation) on S.*

Proof. Necessity. Let μ is the fuzzy congruence relation on S, for $\forall\lambda \in [0,1]$ $\mu_\lambda = \{(a,b)|a,b \in S, \mu(a,b) \geq \lambda\}$. Obviously, μ_λ is reflexive and symmetric, if $(a,b),(b,c) \in \mu_\lambda$, then $\mu(a,b) \geq \lambda$, $\mu(b,c) \geq \lambda$, since μ is transitive, $\mu(a,c) \geq \mu \circ \mu(a,c) = \sup_{x\in S}[\min\{\mu(a,x),\mu(x,c)\}] \geq \min\{\mu(a,b),\mu(b,c)\} \geq \lambda$. Therefore $(a,c) \in \mu_\lambda$ for $\forall x \in S, (a,b) \in \mu_\lambda$, $\mu(a+x,b+x) \geq \mu(a,b) \geq \lambda$ $\mu(xa,xb) \geq \mu(a,b) \geq \lambda, \mu(ax,bx) \geq \mu(a,b) \geq \lambda$ $\mu(x\bigoplus a, x\bigoplus b) \geq \mu(a,b) \geq \lambda, \mu(a\bigoplus x, b\bigoplus x) \geq \mu(a,b) \geq \lambda$, thus $(a+x,b+x) \in \mu_\lambda, (ax,bx) \in \mu_\lambda, (xa,xb) \in \mu_\lambda, (a\bigoplus x, b\bigoplus x) \in \mu_\lambda, (x\bigoplus a, x\bigoplus b) \in \mu_\lambda$, so μ_λ is the congruence relation on S.

Sufficiency. If $\forall\lambda \in [0,1]$, μ_λ is the congruence relation on bi-semirings, then $\forall\lambda \in [0,1]$, $\forall a \in S, \mu(a,a) \geq \lambda$, we obtain $\mu(a,a) = 1$. From $\mu(a,b) = \lambda$, it is easy to know that $(a,b) \in \mu_\lambda$, and $\mu(b,a) \geq \lambda = \mu(a,b) = \lambda; \forall(a,c) \in S \times S, \forall x \in S$. Let $\min\{\mu(a,x),\mu(x,c)\} = \lambda$, then $(a,x) \in \mu_\lambda, (x,c) \in \mu_\lambda$ therefore we obtain $(a,c) \in \mu_\lambda$, i.e. $\forall x \in S$, $\mu(a,c) \geq \min\{\mu(a,x),\mu(x,c)\} = \lambda$, so $\mu(a,c) \geq \sup_{x\in S}[\min\{\mu(a,x),\mu(x,c)\}] = \mu\circ\mu(a,c)$, μ is the fuzzy equivalence relation on S, $\forall x \in S$, if $\mu(a,b) = \lambda$. Then $(a,b) \in \mu_\lambda$, so we get $(a+x,b+x) \in \mu_\lambda$, i.e. $\mu(a+x,b+x) \geq \lambda = \mu(a,b)$.

Similarly, we obtain the following conclusion: $\mu(ax,bx) \geq \lambda = \mu(a,b)$, $\mu(xa,xb) \geq \lambda = \mu(a,b)$ $\mu(a\bigoplus x, b\bigoplus x) \geq \lambda = \mu(a,b), \mu(x\bigoplus a, x\bigoplus b) \geq \lambda = \mu(a,b)$

Then μ is the fuzzy congruence relation on S. □

Definition 3.2. (Definition 3.1 in [8]) If the semi-group of the bi-semirings S, we denote by $(s,+),(s,\bigoplus),(s,\bullet)$ are all reversible semi-groups (i.e. each element has a unique reverse element), then $(s,+,\bigoplus,\bullet)$ is called a reversible bi-semirings.

Let μ be the fuzzy equivalence relation on bi-semirings S, for $\forall x \in S$, we define $\mu_a(x) = \mu(a,x)$, then we have the following theorems.

Theorem 3.3. *If bi-semirings $(s,+,\bigoplus,\bullet)$ is reversible, θ is the zero element on S, then μ_θ is a fuzzy ideal of S.*

Proof. Let μ be the fuzzy congruence on reversible bi-semirings S, then $\forall x, y \in S$, we have $\mu_\theta(x-y) = \mu(\theta, x-y) = \mu(y-y, x-y) \geq \mu(y,x) = \mu(x,y) \geq \mu \circ \mu(x,y) = \sup_{x\in S}[\min\{\mu(x,z), \mu(z,y)\}] \geq \min\{\mu(x,\theta), \mu(\theta, y)\} = \min\{\mu_\theta(x), \mu_\theta(y)\}$, and $\forall \gamma \in S, \mu_\theta(\gamma x) = \mu(\theta, \gamma x) = \mu_\theta(\gamma\theta, \gamma x) \geq \mu(\theta, x) = \mu_\theta(x)$.

Similarly $\mu_\theta(\gamma x) \geq \mu_\theta(\gamma)$, thus $\mu_\theta(\gamma x) \geq \min\{\mu_\theta(\gamma), \mu_\theta(x)\}$. In addition,

$$\begin{aligned}\mu_\theta(x \bigoplus y^{-1}) &= \mu(\theta, x \bigoplus y^{-1}) = \mu(y \bigoplus y^{-1}, x \bigoplus y^{-1}) \geq \mu(y,x) \\ &= \mu(x,y) \geq \mu \circ \mu(x,y) = \sup_{x\in s}[\min\{\mu(x,z), \mu(z,y)\}] \geq \min\{\mu(x,\theta), \mu(\theta,y)\} \\ &= \min\{\mu_\theta(x), \mu_\theta(y)\}\,.\end{aligned}$$

Similarly, $\mu_\theta(y \bigoplus x^{-1}) \geq \min\{\mu_\theta(x), \mu_\theta(y)\}$. Therefore, μ_θ is the fuzzy ideal on S. □

Theorem 3.4. *Let μ be the fuzzy congruence relation on reversible bi-semirings S and $a \in S$, then $\forall x \in S, \mu_a(x) = \mu_\theta(x-a)$, and $\mu_a(x) = \mu_\theta(x \bigoplus a^{-1})$*

Proof. We only have to prove $\mu(a,x) = \mu(\theta, x-a), \forall x \in S$. While

$$\begin{aligned}\mu(\theta, x-a) &= \mu(a-a, x-a) \geq \mu(a,x) \geq \mu \circ \mu(a,x) \\ &= \sup_{x\in S}[\min\{\mu(a,y), \mu(y,x)\}] \geq \min\{\mu(a,a), \mu(a,x)\} \\ &= \mu(a,x) = \mu(a, x-a+a) \geq \mu_\theta(x-a)\,.\end{aligned}$$

In the same reason, we have $\mu_a(x) = \mu_\theta(x \bigoplus a^{-1})$.

So we complete the proof of this theorem. □

Theorem 3.5. *Let μ be the fuzzy congruence relation on reversible bi-semirings S, then $\forall a, b \in S, \mu_a = \mu_b$ if and only if $\mu_\theta(a-b) = 1$ or $\mu_\theta(a \bigoplus b^{-1}) = 1$. Equivalently, $\mu_a = \mu_b$ if and only if $\mu(a,b) = 1$*

Proof. Suppose $\mu_a = \mu_b$, then $\forall a, b \in S$, $\mu(a,x) = \mu(b,x)$, so

$$\mu_\theta(a-b) = \mu(a-b, \theta) = \mu(a,b) = \mu_a(b) = \mu_b(b) = \mu(b,b) = 1\,.$$

On the contrary, if $\mu_\theta(a-b) = \mu(\theta, a-b) = 1$, then $\forall x \in S$:

$$\begin{aligned}\mu_a(x) &= \mu(a,x) \geq \mu(\theta, x-a) \geq \mu \circ \mu(\theta, x-a) \\ &= \sup_{y\in S}[\min\{\mu(\theta, y), \mu(y, x-a)\}] \geq \min\{\mu(\theta, b-a), \mu(b-a, x-a)\} \\ &= \mu(b-a, x-a) \geq \mu(b,x) = \mu_b(x), \text{ thus } \mu_b \supseteq \mu_a\,.\end{aligned}$$

Symmetrically, we also get $\mu_b \subseteq \mu_a$, so $\mu_a = \mu_b$.

This complete the proof of Theorem 3.5. □

Let μ be the fuzzy congruence on S, we define the operations between μ_a and μ_b as follows:

$$\mu_a \bigotimes \mu_b(x) = \begin{cases} \sup_{x=y+z}[\min\{\mu_a(y), \mu_b(z)\}], & \text{if } x = y+z\,, \\ 0, & \text{if } x \neq y+z\,, \end{cases}$$

$$\mu_a \bigodot \mu_b(x) = \begin{cases} \sup_{x=yz}[\min\{\mu_a(y), \mu_b(z)\}], & \text{if } x = yz\,, \\ 0, & \text{otherwise}\,, \end{cases}$$

$$\mu_a * \mu_b(x) = \begin{cases} \sup_{x=yz}[\min\{\mu_a(y), \mu_b(z)\}], & \text{if } x = y \cdot z\,, \\ 0, & \text{otherwise}\,. \end{cases}$$

Theorem 3.6. *Let μ be the fuzzy congruence relation on bi-semirings S, then $\mu_a \bigotimes \mu_b(x) = \mu_{a+b}$*

Proof. First we prove the operation $\bigotimes$ is well-defined.

Suppose $\mu_a = \mu_b, \mu_c = \mu_d$, then according to Theorem 3.5, we have, $\mu(a,b) = \mu(c,d) = 1$, therefore $\mu(a+c, b+d) = \mu_\theta[(a-b)+(c+d)] \geq \mu_\theta(a-b) \wedge \mu_\theta(c-d) = \mu(a,b) \wedge \mu(c,d) = 1$, so $\mu_a \bigotimes \mu_b = \mu_c \bigotimes \mu_d$, $\forall y, z \in S$, if $x = y+z$, then:

$$\begin{aligned} \mu_{a+b}(x) = \mu(a+b, x) &= \mu_\theta[x-(a+b)] = \mu_\theta[y+z-(a+b)] \\ &\geq \mu_\theta(y-a) \wedge \mu_\theta(z-b) = \mu(a,y) \wedge \mu(b,z) = \mu_a(y) \wedge \mu_b(z)\,. \end{aligned}$$

Therefore, $\mu_{a+b}(x) \geq \sup_{x=y+z}[\min\{\mu_a(y), \mu_b(z)\}]$, i.e. $\mu_{a+b} \geq \mu_a \bigotimes \mu_b$.

On the contrary, for $\forall x \in S$ if x can be expressed as $x = y+z, (y, z \in S)$, then:

$$\begin{aligned} &\mu_a \bigotimes \mu_b(x) = \sup_{x=y+z}[\min\{\mu(a,y), \mu(b,z)\}] \\ &\geq \sup_{\substack{x=y+z \\ \mu(a,y)\neq\mu(b,z)}}[\min\{\mu(a,y), \mu(b,z)\}] = \sup_{\substack{x=y+z \\ \mu(a,y)\neq\mu(b,z)}}[\min\{\mu_\theta(y-a), \mu_\theta(z-b)\}] \\ &\quad = \sup_{\substack{x=y+z \\ \mu(a,y)\neq\mu(b,z)}}[\min\{\mu_\theta(y+z)-(a+b))\}] = \mu(a+b, y+z) = \mu(a+b, x), \end{aligned}$$

i.e. $\mu_a \bigotimes \mu_b \geq \mu_{a+b}$. Thus $\mu_a \bigotimes \mu_b = \mu_{a+b}$.

Similarly, we can define the binary operations $\bigodot$ and $*$ on S/μ as: $\mu_a \bigodot \mu_b = \mu_{ab}$, $\mu_a * \mu_b = \mu_{a \cdot b}$.

From the definition we obtain that if μ is the fuzzy congruence relation on bi-semirings S, then $(S/\mu, \bigotimes, \bigodot, *)$ is bi-semirings. □

4. The Homomorphism of Bi-Semirings

Definition 4.1. Let S and $\overline{S}$ both are bi-semirings, mapping $f : S \longmapsto \overline{S}$, if f keeps operation, i.e. for $\forall x, y \in S$, $f(x+y) = f(x) + f(y), f(xy) = f(x)f(y), f(x \cdot y) = f(x) \cdot f(y)$, then f is a homomorphism from S to $\overline{S}$.

Let S and $\overline{S}$ be two bi-semirings and $f : S \longmapsto \overline{S}$ be a homomorphism, then

$$ker(f) = \{(a,b)|f(a), a, b \in S\}$$

is the congruence relation on S, thereby its characteristic function $\chi_{ker(f)}$ is the fuzzy congruence on S and $\chi_{ker(f)}(a,b) = \begin{cases} 1, & \text{if } f(a) = f(b)\,, \\ 0, & \text{if } f(a) \neq f(b)\,. \end{cases}$

Theorem 4.1. *Let μ be the fuzzy congruence relation on bi-semirings S, $(S/\mu, \bigotimes, \bigodot, *)$ is a bi-semirings, we define $\mu^\sharp : S \longmapsto S/\mu, \mu^\sharp(a) = \mu_a$, then $\mu^\sharp$ is a homomorphism.*

Proof. For $\forall a, b \in S$, we have:
$\mu^\sharp(a) \bigotimes \mu^\sharp(b) = \mu_a \bigotimes \mu_b = \mu_{a+b} = \mu^\sharp(a+b)$,
$\mu^\sharp(a) \bigodot \mu^\sharp(b) = \mu_a \bigodot \mu_b = \mu_{a \cdot b} = \mu^\sharp(a \cdot b)$,
$\mu^\sharp(a) * \mu^\sharp(b) = \mu_a * \mu_b = \mu_{ab} = \mu^\sharp(ab)$.
Then $\mu^\sharp$ is a homomorphism mapping from S to S/μ. □

Theorem 4.2. *Let S and $\overline{S}$ be two bi-semirings and $f : S \longmapsto \overline{S}$ be a homomorphism, then the kernel of fuzzy relation $\chi_{ker(f)}$ is a fuzzy congruence relation on S, and there exists a single homomorphism $g : S/\chi_{ker(f)} \longmapsto \overline{S}, g(\chi_{ker(f)})_a = f(a)$, such that: $f = g \circ (\chi_{ker(f)})^\sharp$.*

Proof. Let $a, b \in S$, then from the definition of $\mu^\sharp$ we have:
$\mu^\sharp(a+b) = \mu_{a+b} = \mu_a \bigotimes \mu_b = \mu^\sharp(a) \bigotimes \mu^\sharp(b)$,
$\mu^\sharp(ab) = \mu_{ab} = \mu_a * \mu_b = \mu^\sharp(a) * \mu^\sharp(b)$,
$\mu^\sharp(a \cdot b) = \mu_{a \cdot b} = \mu_a \bigodot \mu_b = \mu^\sharp(a) \bigodot \mu^\sharp(b)$.
Since $\forall a \in S, g : S/\chi_{ker(f)} \longmapsto \overline{S}, g(\chi_{ker(f)})_a = f(a)$, if $a, b \in S, (\chi_{ker(f)})_a = (\chi_{ker(f)})_b$, then $\chi_{ker(f)}(a,b) = 1, (a,b) \in \chi_{ker(f)}$. Therefore $g((\chi_{ker(f)})_a) = f(a) = f(b) = g((\chi_{ker(f)})_b)$ and

$$\begin{aligned} g((\chi_{ker(f)})_a \bigotimes (\chi_{ker(f)})_b) = g((\chi_{ker(f)})_{a+b}) = f(a+b) = f(a) + f(b) \\ = g((\chi_{ker(f)})_a) \bigotimes g((\chi_{ker(f)})_b)\,, \end{aligned}$$

$$g((\chi_{ker(f)})_a * (\chi_{ker(f)})_b) = g((\chi_{ker(f)})_{ab}) = f(ab) = f(a)f(b)$$

$$= g((\chi_{ker(f)})_a) * g((\chi_{ker(f)})_b)\,,$$

$$g((\chi_{ker(f)})_a \bigodot (\chi_{ker(f)})_b) = g((\chi_{ker(f)})_{a\cdot b}) = f(a \cdot b) = f(a) \cdot f(b)$$
$$= g((\chi_{ker(f)})_a) \bigodot g((\chi_{ker(f)})_b)\,.$$

So g is a homomorphism. Suppose $a \in S$, then

$$g \circ (\chi^{\sharp}_{ker(f)}(a)) = g(\chi^{\sharp}_{ker(f)}(a)) = g((\chi_{ker(f)})_a) = f(a)\,,$$

i.e. $g \circ (\chi_{ker(f)})^{\sharp} = f$.

5. Conclusion

In this paper, based on the concept of bi-semirings, we introduce the definition of fuzzy congruence relation on bi-semirings, some properties of bi-semirings S with fuzzy congruence relation are discussed and prove that the kernel of the fuzzy congruence relation is a fuzzy ideal of bi-semirings S. Restricted by time and author's knowledge level, some properties have not been discussed yet. Readers who interested in this aspect can investigate the properties of fuzzy congruence family and fuzzy congruence quotient set of bi-semirings S about μ, we believe that many benefit conclusions will be achieved.

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