International Journal of Pure and Applied Mathematics
Volume 37 No. 1 2007, 29-42

PERIODICITY OF A ROSENZWEIG-MACARTHUR FOOD CHAIN WITH IMPULSIVE RATIO-HARVESTING PREY

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Abstract: In this paper, a Rosenzweig-MacArthur food chain with impulsive ratio-harvesting prey is investigated. By using Floquet theory and small amplitude perturbation skills, we discuss the boundary periodic solutions for predator-prey system under periodic pulsed conditions. The stability analysis of the boundary periodic solution yields an invasion threshold of the predator. Further, by use of the coincidence degree theorem and its related continuous theorem we prove the existence of the positive periodic solutions of the system when the value of the coefficient is large than the threshold.

AMS Subject Classification: 34C25, 92D25
Key Words: Rosenzweig-MacArthur food chain, impulsive effect, periodicity

Received: February 7, 2007

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1. Introduction

The Rosenzweig-MacArthur tritrophic food chains [12] is the following model

$$\begin{cases} \frac{dx_1(t)}{dt} = rx_1(t)(1-\frac{x_1(t)}{K}) - \frac{M_1x_1(t)x_2(t)}{A_1+x_1}, \\ \frac{dx_2(t)}{dt} = e_1\frac{M_1x_1(t)x_2(t)}{A_1+x_1} - c_2x_2(t) - \frac{M_2x_2(t)x_3(t)}{A_2+x_2}, \\ \frac{dx_3(t)}{dt} = e_2\frac{M_2x_1(t)x_2(t)}{A_2+x_2} - c_3x_3(t), \end{cases}$$

where x_1, x_2 and x_3 are prey, predator and superpredator biomass, r and K are prey growth rate and carrying capacity, respectively, and M_i, A_i and e_i $(i = 1, 2)$ are maximum predation rate, half saturation constant, efficiency of the predator and the superpredator. c_2, c_3 are the death rates of the predator and the superpredator, respectively. Such food chains have a very rich behavior, covering the whole spectrum of dynamic regimes, including chaos (see Hogeweg and Hesper [7]; Scheffer [13]; Hastings and Powell [6]; McCann and Yodzis [11]; Abrams and Roth [1]; S. Gakkhar and Naji Ma [5]; A. Klebanoff and A. Hastings [8]). As well known, countless organisms live in seasonally or diurnally forced environments. As Cushing [3] pointed out that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed (for example, those due to seasonal effects of weather, food supply, mating habits, hunting or harvesting seasons, etc.).

Periodic forcing and impulsive effect are two different kinds to simulate seasonal or other variation. Recently impulsive differential equations (IDE) have been introduced in almost every domain of applied sciences. Numerous examples are given in Bainov's and his collaborators' books (see [10,11]). Some IDE have been recently introduced in population dynamics by many authors involving: pulse vaccination [14], population ecology [10]. Our purpose of this paper is to discuss the existence of periodic solutions in the following Rosenzweig-MacArthur food chains with impulsive effects:

$$\begin{cases} \left.\begin{array}{l} \frac{dx_1(t)}{dt} = rx_1(t)(1-\frac{x_1(t)}{K}) - \frac{M_1x_1(t)x_2(t)}{A_1+x_1} \\ \frac{dx_2(t)}{dt} = e_1\frac{M_1x_1(t)x_2(t)}{A_1+x_1} - c_2x_2(t) - \frac{M_2x_2(t)x_3(t)}{A_2+x_2} \\ \frac{dx_3(t)}{dt} = e_2\frac{M_2x_1(t)x_2(t)}{A_2+x_2} - c_3x_3(t) \end{array}\right\} & t \neq t_k = kT, \\ \triangle x_1 = -px_1 & t = t_k = kT, \end{cases} \tag{1.1}$$

where $0 < p < 1$, T is the period of impulsive effects.

The organizations of the paper are as follows. In Section 2, we discuss the boundary periodic solutions for predator-prey system under periodic pulsed conditions. In Section 3, by using the coincidence degree theorem and its related continuous theorem we discuss the existence of the positive periodic solutions of the system (1.1). In the last section, we give out the conclusion of this paper.

2. Stability of Boundary Periodic Solution

We write the system (1.1) in a nondimensional form. Let

$$\tilde{x}_1(\tilde{t}) = \frac{x_1(t)}{K}, \quad \tilde{x}_2(\tilde{t}) = \frac{x_2(t)}{e_1 K}, \quad \tilde{x}_3(\tilde{t}) = \frac{x_2(t)}{e_1 e_2 K}, \quad \tilde{t} = rt, \quad m_1 = \frac{e_1 M_1}{r},$$
$$m_2 = \frac{e_2 M_2}{r}, \quad a_1 = \frac{A_1}{K}, \quad a_2 = \frac{A_2}{e_1 K}, d_2 = \frac{c_2}{r}, \quad d_3 = \frac{c_2}{r}, \quad w = rT.$$

Then (1.1) takes the form

$$\left\{ \begin{array}{l} \frac{dx_1(t)}{dt} = x_1(t)(1 - x_1(t)) - \frac{m_1 x_1(t) x_2(t)}{a_1 + x_1(t)} \\ \frac{dx_2(t)}{dt} = \frac{m_1 x_1(t) x_2(t)}{a_1 + x_1(t)} - d_2 x_2(t) - \frac{m_2 x_2(t) x_3(t)}{a_2 + x_2(t)} \\ \frac{dx_3(t)}{dt} = \frac{m_2 x_2(t) x_3(t)}{a_2 + x_2(t)} - d_3 x_3(t) \end{array} \right\}, \quad t \neq t_k = kw\,,$$
$$\triangle x_1 = -p x_1\,,\ t = t_k = kw\,, \quad (2.1)$$

where $0 < p < 1, i = 1, 2$, w is the period of impulsive effects.

Lemma 2.1. *Both the nonnegative cone $R^3_+ := \{(x_1, x_2, x_3) \in R^3 | x_i \geq 0, i = 1, 2, 3\}$ and the positive cone R^3_+ are positively invariant with respect to system (1.1).*

For the system (2.1), we know that the system (2.1) has the nonnegative boundary w-period solution

$$(x_{1e}(t), x_{2e}(t), x_{3e}(t)) = (0, 0, 0).$$

For this periodic solution, we have the following results.

Theorem 2.1. *Let $x(t) = (x_1(t), x_2(t), x_3(t))$ be any solution of system (2.1) with $x(0) > 0$.*

(1) If $\mu_1 := (1 - p)e^w < 1$, then the system (2.1) has a unique globally asymptotically stable positive $w-$ periodic solution $(x_{1e}(t), x_{2e}(t), x_{3e}(t))$, that is say, $\lim_{t\to\infty} x(t) = (0, 0, 0)$.

(2) If $\mu_1 := (1 - p)e^w > 1$, $(x_{1e}(t), x_{2e}(t), x_{3e}(t))$ is unstable.

Proof. (1) The local stability of periodic solution $(0, 0)$ may be determined by considering the behavior of small amplitude perturbations of the solution. Define

$$x_1(t) = u_1(t), \quad x_2(t) = u_2(t), \quad x_3(t) = u_3(t)\,,$$

there may be written

$$\begin{pmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_3(0) \end{pmatrix},$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & -d_2u_2 & 0 \\ 0 & 0 & -d_3u_3 \end{pmatrix} \Phi(t)$$

and $\Phi(\tau) = I$, the identity matrix. Hence the fundamental solution matrix is

$$\Phi(w) = \begin{pmatrix} e^w & 0 & 0 \\ 0 & e^{-d_2w} & 0 \\ 0 & 0 & e^{-d_3w} \end{pmatrix}.$$

The linearization of impulsive subsystem (2.1) becomes

$$\begin{pmatrix} u_1(nw^+) \\ u_2(nw^+)) \\ u_3(nw^+)) \end{pmatrix} = \begin{pmatrix} 1-p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1(nw) \\ u_2(nw) \\ u_3(nw) \end{pmatrix}.$$

We denote that

$$M = \begin{pmatrix} 1-p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(w).$$

The eigenvalues of the matrix M are the eigenvalues $\mu_1 = (1-p)e^w, \mu_2 = e^{-dw}$. If $\mu_1 = (1-p)\exp w < 1$, the solution $(0,0)$ is locally asymptotically stable. If $\mu_1 = (1-p)\exp w > 1$, then the periodic solution $(0,0)$ is unstable.

Now we prove that if $\mu_1 = (1-p)\exp w < 1$, the solution $(0,0)$ is globally asymptotically stable. We notice that

$$\begin{aligned} x_1(t) &= x_1(0)(1-p)^n \exp\left(\int_0^t 1 - x_1(l, x_1(0)) - \tfrac{m_1x_2(l)}{a_1+x_1(l)}\right)dl \\ &\le x_1(0)(1-p)^n \exp[(n+1)w], \quad t \in (nw, (n+1)w]. \end{aligned}$$

Hence $\lim_{t\to\infty} x_1(t) = 0$ and $\lim_{t\to\infty} x_2(t) = 0$. We complete the proof. □

We consider the impulsive equation

$$\begin{cases} \frac{dx_1}{dt} = x_1 - x_1^2(t), & t \ne t_k = kw, \\ \triangle x_1 = -px_1, & t = t_k = kw. \end{cases} \tag{2.3}$$

By simply calculating, we know the system (2.3) has uniquely positive w-

periodic solution

$$\begin{aligned} &x_{1s}(t) = \frac{x_{1s}(0)\exp[t-(k-1)w]}{1-x_{1s}(0)+x_{1s}(0)\exp[t-(k-1)w]}, \quad t \in ((k-1)w, kw], \\ &x_{1s}(0) = 1 - \frac{pe^w}{e^w - 1}, \end{aligned} \tag{2.4}$$

and we have that $\int_0^w x_{1s}(t)dt = w + \ln(1-p) > 0$. We calculate the multiplier μ_1 of the w-period solution $x_{1s}(t)$ and obtain that

$$\begin{aligned} \mu_{1s}: \ &= (1-p)\exp(\textstyle\int_0^w (1 - 2x_{1s}(t))dt) \\ &= (1-p)\exp(w - 2\ln(1-p) - 2w) \\ &= \exp(-w - \ln(1-p)) = \exp(-\textstyle\int_0^w x_{1s}(t)dt) < 1. \end{aligned} \tag{2.5}$$

Hence the w-period solution $x_{1s}(t)$ of the system (2.2) is globally asymptotically stable.

For the system (2.1), we know that the system system (2.1) has the non-negative boundary w-period solution

$$(x_{1s}(t), x_{2e}(t), x_{3e}(t)) = (x_{1s}(t), 0, 0).$$

For this periodic solution, we have the following results. For convenience, we denote

$$m_1^* := \frac{d_2}{\int_0^w \frac{x_{1s}(t)}{a_1 + x_{1s}(t)}dt} \tag{2.6}$$

is critical value for the stability of the boundary w-period solution $(x_{1s}(t), 0, 0)$, that is if $m_1 < m_1^*$, it is globally asymptotically stable; otherwise, it is unstable.

Theorem 2.2. *Let $x(t) = (x_1(t), x_2(t), x_3(t))$ be any solution of system (2.2) with $x(0) > 0$. Then we have that:*

(1) If $\mu_1 := (1-p)e^w > 1$ and $m_1 < m_1^*$, then the system (2.1) has a boundary $w-$ periodic solution $(x_{1s}, 0, 0)$, which is globally asymptotically stable, that is say, $\lim_{t\to\infty} x(t) = (x_{1s}, 0, 0)$.

(2) If $\mu_1 := (1-p)e^w > 1$, then there exists a constance $\lambda_0 > 0$, such that for each $m_1 \in (m_1^*, m_1^* + \lambda_0)$, there exists a stable w-periodic solution $(\tilde{x}_1, \tilde{x}_2, 0)$ of (2.1).

Proof. By Theorem 2.1, (1) is obvious. We shall prove (2).

We know that the $w-$period solution $(x_{1s}(t), 0)$ is a boundary periodic solution of the system (2.2). The local stability of periodic solution $(x_{1s}(t), 0)$ may be determined by considering the behavior of small amplitude perturbations of the solution. Define

$$x_1(t) = u_1(t) + x_{1s}(t), \quad x_2(t) = u_2(t), \quad x_3(t) = u_3(t)\,.$$

There may be written

$$\begin{pmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_3(0) \end{pmatrix},$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} (1-2x_{1s})u_1 & -m_1 u_2 & 0 \\ 0 & (m_1 x_{1s} - d_2)u_2 & 0 \\ 0 & 0 & -d_3 u_3 \end{pmatrix} \Phi(t)$$

and $\Phi(w) = I$, the identity matrix. Hence the fundamental solution matrix is

$$\Phi(\tau) = \begin{pmatrix} \exp(\int_0^w (1-2x_{1s}(t))dt) & * & 0 \\ 0 & \exp(\int_0^w (m_1 x_{1s}(t) - d_2)dt) & 0 \\ 0 & 0 & \exp(-d_3 w) \end{pmatrix}.$$

It is no need to give the exact form of $(*)$ as it is not required in the analysis that follows. The linearization of impulsive subsystem (2.2) become

$$\begin{pmatrix} u_1(nw^+) \\ u_2(nw^+) \\ u_3(nw^+) \end{pmatrix} = \begin{pmatrix} 1-p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1(nw) \\ u_2(nw) \\ u_3(nw) \end{pmatrix}.$$

We denote that

$$M = \begin{pmatrix} 1-p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(w),$$

hence we have

$$M = \begin{pmatrix} (1-p)\exp(\int_0^w (1-2x_{1s}(t))dt) & ** & 0 \\ 0 & \exp(\int_0^w (\frac{m_1 x_{1s}(t)}{a_1 + x_{1s}(t)} - d_2)dt) & 0 \\ 0 & 0 & \exp(-d_3 w) \end{pmatrix}.$$

It is no need to give the exact form of $(**)$ as it is not required in the analysis that follows. The eigenvalues of the matrix M are the eigenvalues $\mu_1^* = (1-p)\exp(\int_0^w (1-2x_{1s}(t))dt) < 1$, $\mu_2^* = \exp(\int_0^w (\frac{m_1 x_{1s}(t)}{a_1 + x_{1s}(t)} - d_2)dt)$ and $\mu_3^* = \exp(-d_3 w) < 1$. Hence, if $\mu_2^* < 1$, the solution $(x_{1s}(t), 0)$ is locally asymptotically stable. If $\mu_2^* > 1$ (or $m_1 > m_1^*$), then the periodic solution $(x_{1s}(t), 0, 0)$ is unstable.

In the system (2.1), if $x_3 = 0$, then the system (2.1) induces to the following impulsive system

$$\begin{cases} \left.\begin{array}{l} \frac{dx_1(t)}{dt} = x_1(t)(1-x_1(t)) - \frac{m_1x_1(t)x_2(t)}{a_1+x_1(t)}), \\ \frac{dx_2(t)}{dt} = \frac{m_1x_1(t)x_2(t)}{a_1+x_1(t)} - d_2x_2(t), \end{array}\right\} & t \neq t_k = kw, \\ \triangle x_1 = -px_1 & t = t_k = kw. \end{cases} \tag{2.8}$$

For this system, by using standard bifurcating theorem, we can prove the following results: if $\mu_1 := (1-p)e^w > 1$, then there exists a constance $\lambda_1 > 0$, such that for each $m_1 \in (m_1^*, m_1^* + \lambda_1)$, there exists a stable w-periodic solution $(\tilde{x}_1(m_1,t), \tilde{x}_2(m_1,t))$ of (2.8). We know if $m_1 \to m^*_{1+0}$, then $\|\tilde{x}_2(m_1,t)\| \to 0$. Hence there exists a constance $0 < \lambda_0 \leq \lambda_1$, such that for each $m_1 \in (m_1^*, m_1^* + \lambda_0)$, $\int_0^w (\frac{m_2\tilde{x}_2(m_1,t)}{a_2+\tilde{x}_2(m_1,t))} - d_3)dt < 0$ holds. Notice the w-periodic boundary solution $(\tilde{x}_1(m_1,t), \tilde{x}_2(m_1,t), 0)$ is a solution of system (2.1). By calculating the multipliers of $(\tilde{x}_1(m_1,t), \tilde{x}_2(m_1,t), 0)$, we know it is locally asymptotically stable. We complete the proof. □

3. Existence of Positive Periodic Solution

In this section, we will confine ourselves to prove that the system (2.1) can have positive w-periodic solutions under some conditions. The proof of the theorem is based on the Gains and Mawhin's continuation theorem [4]. For the reader's convenience, we introduce this theorem as follows.

Set X and Y are two real Banach spaces. Consider the operator equation

$$Lx = Nx$$

where $L : DomL \subset X \to Y$ is a linear bounded operator, $N : Y \to Y$ is a continuous operator. The mapping L will be called a Fredholm mapping of index zero if $dimKerL = codimImL < +\infty$ and ImL is closed in Y. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that

$$ImP = KerL, \quad ImL = KerQ = Im(I-Q)\,.$$

It follows that $L|_{domL \subset KerP} : (I-P)X \to ImL$ is invertible. We denote the inverse of that map by K_p. If Ω is an open subset of X, the mapping N will be called L-compact on Ω if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N : \overline{\Omega} \to X$ is compact. Since ImQ is isomorphic to $KerL$ there exist isomorphisms $J : ImQ \to KerL$.

Lemma 3.1. *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Assume:*

(a) For each $\lambda \in (0,1)$, $x \in \partial\Omega \cap DomL, Lx \neq \lambda Nx$.

(b) For each $x \in \partial\Omega \cap KerL, QNx \neq 0$.

(c) $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$.

Then $Lx = Nx$ *has at least one solution in* $\overline{\Omega} \cap DomL$.

In this section, for convenance, we denote the impulsive time points as $t_k = (k - \frac{1}{2})w$, $k = 1, 2, \ldots$.

Lemma 3.2. $x^*(t)$ *is an w-periodic solution of (2.1) with strictly positive components if and only if* $\ln x^*(t)$ *is an w-periodic solution of*

$$\begin{cases} \left.\begin{array}{l} \frac{dx_1}{dt} = 1 - e^{x_1(t)} - \frac{m_1 e^{x_2(t)}}{a_1 + e^{x_1(t)}}, \\ \frac{dx_2}{dt} = \frac{m_1 e^{x_2(t)}}{a_1 + e^{x_1(t)}} - d_2 - \frac{m_2 e^{x_3(t)}}{a_2 + e^{x_2(t)}}, \\ \frac{dx_3}{dt} = \frac{m_2 e^{x_2(t)}}{a_2 + e^{x_2(t)}} - d_3, \end{array}\right\} & t \neq t_k = (k - \frac{1}{2})w, \\ \left.\begin{array}{l} \triangle x_1 = x_1(t_k^+) - x_1(t_k) = \ln(1-p), \\ \triangle x_2 = x_2(t_k^+) - x_2(t_k) = 0, \\ \triangle x_3 = x_3(t_k^+) - x_3(t_k) = 0, \end{array}\right\} & t = t_k = (k - \frac{1}{2})w, \end{cases} \tag{3.1}$$

where $\ln\{x^*(t)\} = (\ln\{x_1^*(t)\}, \ln\{x_2^*(t)\}, \ln\{x_3^*(t)\})$.

With the help of Lemma 3.1, we can explore the existence of positive periodic solutions of (2.1) in a more direct way. In order to apply Lemma 3.1, we shall first embed our existence problem into the frame of the continuation theorem.

Define

$$C[0,w;t_1] = \left\{ x : [0,w] \to R^3 \;\middle|\; \begin{array}{l} x(t) \text{ is continuous with respect to} \\ t \neq t_1,\ x(t+0) \text{ and } x(t+0) \\ \text{exist at } t_1,\ x(t_1) = x(t_1 - 0). \end{array} \right\}$$

We introduce

$$X = \{C[0,w;t_1] | x(0) = x(w)\}, \quad \| x \|_c = \left\{ \sup_{t\in[0,w]} \| x \|, x \in X \right\},$$

where $\| \cdot \|$ is any norm in R^3, and

$$Z = X \times R^3, \quad \| z \|_Z = \| x \|_c + \| y \|, \quad z = (x,y) \in Z, \text{ with } x \in X,\ y \in R^3,$$

where $\| \cdot \|$ is any given norm of R^3. Then it is standard to show that both X and Z are Banach spaces when they are endowed with the norms $\| \cdot \|_c$ and $\| \cdot \|_Z$, respectively.

Let $DomL = X$, $L : DomL \to Z$, $Lx = (x', \triangle x(t_1))$. We define $N : X \to Z$

as following

$$Nx = \left\{ \left(\begin{array}{c} f_1 := 1 - e^{x_1(t)} - \frac{m_1 e^{x_2(t)}}{a_1 + e^{x_1(t)}} \\ f_2 := \frac{m_1 e^{x_2(t)}}{a_1 + e^{x_1(t)}} - d_2 - \frac{m_2 e^{x_3(t)}}{a_2 + e^{x_2(t)}} \\ f_3 := \frac{m_2 e^{x_2(t)}}{a_2 + e^{x_2(t)}} - d_3 \end{array} \right), \left(\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \right) \right\}, \quad x \in X.$$

We denote $b := (b_1, 0, 0)^T, b_1 = \ln(1-p)$. It is trivial to see that L is a bounded linear operator and $Ker(L) = R^2$,

$$\begin{array}{l} Ker(L) = \{x \in X | x = h \in R^2, t \in [0, \frac{w}{2}]; x = g \in R^2, t \in (\frac{w}{2}, w]\}, \\ ImL = \left\{z = (f, b) \in Z | \int_0^w f(s)ds + b = 0\right\}, \end{array}$$

and ImL is closed in Z, therefore, L is a Fredholm mapping of index zero. Define two projects P, Q as

$$\begin{array}{l} Px = \frac{1}{w} \int_0^w x(s)ds, \qquad x \in X; \\ Qz = Q(f, b) = \left(\frac{1}{w}[\int_0^w x(s)ds + b], 0\right). \end{array}$$

It is not difficult to show that P and Q are continuous projectors such that

$$ImP = KerL \ \ and \ \ ImL = KerQ = Im(I - Q),$$

and hence, the generalized inverse (to L) K_p exists. In the following, we first devote ourselves to deriving the explicit expression of $K_p : ImL \to KerP \cap DomL$. Take $z = (f, b) \in ImL$, then there exists an $x \in DomL \subset X$ such that

$$\begin{array}{ll} \dot{x}(t) = f(t), & t \neq t_k = (k - \frac{1}{2})w, \\ \triangle x(t) = b, & t = t_k = (k - \frac{1}{2})w. \end{array}$$

Then direct integration produces

$$x(t) = \int_0^t f(s)ds + \sum_{t > t_k} b + x(0). \tag{3.2}$$

Note that $x(t) \in KerP$, i.e., $\int_0^w x(s)ds = 0$, which, together with (3.2), implies

$$\int_0^w \int_0^t f(s)dsdt + wx(0) = 0\,,$$

then

$$x(t) = \int_0^t f(s)ds + \sum_{t > t_k} b - \frac{1}{w} \int_0^w \int_0^t f(s)dsdt\,.$$

That is,

$$K_p z = \int_0^t f(s)ds + \sum_{t > t_k} b - \frac{1}{w} \int_0^w \int_0^t f(s)dsdt,$$

then

$$QNx = \left\{\frac{1}{w}[\int_0^w f(s)ds + b], 0\right\}, \quad x \in X.$$

$$\begin{array}{ll} K_p(I - Q)Nx & = \int_0^t f(s)ds - \frac{t}{w} + \sum_{t > t_k} b + (\frac{1}{2} - \frac{t}{w}) \int_0^w f(s)ds \\ & \quad + \frac{b}{2} - \frac{1}{w} \int_0^w \int_0^l f(s)dsdl, \qquad x \in X. \end{array}$$

It is easy to check that QN and $K_p(I-Q)N$ are continuous by the Lebesgue convergence theorem and moreover, by the Arzera-Ascoli Theorem, $QN(\bar{\Omega})$ and $\overline{K_p(I-Q)N(\bar{\Omega})}$ compact for any open bounded set $\Omega \subset X$. Hence, N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

Now we at the point to find an open, bounded subset Ω which has been given in the above paragraph to satisfy the three conditions of Gaines and Mawhin's continuation theorem, which is Lemma 3.1.

We consider the operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, i.e.,

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= \lambda(1-\exp(x_1(t)) - \frac{m_1 e^{x_2(t)}}{a_1+e^{x_1(t)}}, \\ \frac{dx_2}{dt} &= \lambda(\frac{m_1 e^{x_1(t)}}{a_1+e^{x_1(t)}} - d_2 - \frac{m_2 e^{x_3(t)}}{a_2+e^{x_2(t)}}, \\ \frac{dx_2}{dt} &= \lambda(\frac{m_2 e^{x_2(t)}}{a_2+e^{x_2(t)}} - d_3), \end{aligned}\right\} & t \neq (k-\frac{1}{2})w. \\ \triangle x_1 = \lambda \ln(1-p) & t = (k-\frac{1}{2})w. \end{cases} \tag{3.3}$$

Assume that $x = (x_1(t), x_2(t), x_3(t)) \in X$ is a solution of (3.3) for a certain $\lambda \in (0,1)$. Integrating (3.3) over the interval $[0,w]$, we obtain

$$\begin{cases} \lambda \int_0^w (1-\exp(x_1(t)) - \frac{m_1 e^{x_2(t)}}{a_1+e^{x_1(t)}} dt + \lambda \ln(1-p) = 0, \\ \lambda \int_0^w (\frac{m_1 e^{x_1(t)}}{a_1+e^{x_1(t)}} - d_2 - \frac{m_2 e^{x_3(t)}}{a_2+e^{x_2(t)}}) dt = 0, \\ \lambda \int_0^w (\frac{m_2 e^{x_2(t)}}{a_2+e^{x_2(t)}} - d_3) dt = 0. \end{cases}$$

That is

$$\begin{cases} \int_0^w (1-e^{x_1(t)} - \frac{m_1 e^{x_2(t)}}{a_1+e^{x_1(t)}} dt + \ln(1-p) = 0, \\ \int_0^w (\frac{m_1 e^{x_1(t)}}{a_1+e^{x_2(t)}} - d_2 - \frac{m_2 e^{x_3(t)}}{a_2+e^{x_2(t)}}) dt = 0, \\ \int_0^w (\frac{m_2 e^{x_2(t)}}{a_2+e^{x_2(t)}} - d_3) dt = 0. \end{cases} \tag{3.4}$$

By comparing theorem, we notice that in the system (2.1), $x_1(t) \leq 1$ for t to be large enough, hence we obtain that for the system (3.3), $x_1(t) \leq 0$ for t to be large enough. From (3.4), one can derive

$$\begin{aligned} &\int_0^w |\dot{x}_1(t)| dt \leq \int_0^w (1 + e^{x_1(t)} + \frac{m_1 e^{x_2(t)}}{a_1+e^{x_1(t)}} dt + \ln(1-p) = 2(w + \ln(1-p)), \\ &\int_0^w |\dot{x}_2(t)| dt \leq \int_0^w \frac{2m_1 e^{x_1(t)}}{a_1+e^{x_1(t)}} dt \leq \frac{2m_1}{1+a_1}, \\ &\int_0^w |\dot{x}_3(t)| dt \leq \int_0^w (\frac{m_2 e^{x_2(t)}}{a_2+e^{x_2(t)}} + d_3) dt = 2wd_3. \end{aligned} \tag{3.5}$$

Since $x(t) \in X$, there exist $\xi_i \in [0,w]$ and $\zeta_i \in [0,w]$ such that

$$x_i(\xi_i) = \min_{t\in[0,w]} x_i(t), \quad x_i(\zeta_i) = \max_{t\in[0,w]} x_i(t).$$

From (3.4), we have

$$\frac{wm_2 \exp(x_2(\xi_2))}{a_2+\exp(x_2(\xi_2))} \leq \int_0^w \frac{m_2 \exp(x_2(t))}{a_2+\exp(x_2(t))} dt = wd_3 \leq \frac{wm_2 \exp(x_2(\zeta_2))}{a_2+\exp(x_2(\zeta_2))}.$$

Assume that $m_2 - a_2 d_3 > 0$, so we have

$$x_2(\xi_2) \le \ln(\tfrac{d_3}{m_2 - a_2 d_3}) \le x_2(\zeta_2). \tag{3.6}$$

We denote that

$$P_2 := \ln(\tfrac{d_3}{m_2 - a_2 d_3}) - \tfrac{2m_1}{1+a_1}; \quad H_2 := \ln(\tfrac{d_3}{m_2 - a_2 d_3}) + \tfrac{2m_1}{1+a_1}. \tag{3.7}$$

We have that

$$P_2 \le x_2(\zeta_2) - \int_0^w |\dot{x}_2(t)|dt \le x_2(t) \le x_2(\xi_2) + \int_0^w |\dot{x}_2(t)|dt \le H_2.$$

By (3.4), we have

$$\begin{cases} \int_0^w (1 - \exp(x_1(t)) - \frac{m_1 e^{P_2}}{a_1 + \exp(x_1(t))})dt + \ln(1-p) \ge 0, \\ \int_0^w (1 - \exp(x_1(t)) - \frac{m_1 e^{H_2}}{a_1 + \exp(x_1(t))})dt + \ln(1-p) \le 0. \end{cases} \tag{3.8}$$

That is

$$\begin{cases} 1 - \frac{m_1 e^{P_2}}{\max(a_1, 1)} + \frac{1}{w}\ln(1-p) \ge \exp(x_1(\xi_1)), \\ 1 - \frac{m_1 e^{H_2}}{a_1 + 1} + \frac{1}{w}\ln(1-p) \le \exp(x_1(\zeta_1)). \end{cases} \tag{3.9}$$

Suppose that $1 - \frac{m_1 e^{H_2}}{\max(a_1,1)} + \frac{1}{w}\ln(1-p) > 0$. Denote that

$$\begin{aligned} P_1 &:= \ln(1 - \tfrac{m_1 e^{P_2}}{a_1+1} + \tfrac{1}{w}\ln(1-p)) - 2(w + \ln(1-p)); \\ H_1 &:= \ln(1 - \tfrac{m_1 e^{H_2}}{\max(a_1,1)} + \tfrac{1}{w}\ln(1-p)) + 2(w + \ln(1-p)). \end{aligned} \tag{3.10}$$

We have that

$$P_1 \le x_1(\zeta_1) - \int_0^w |\dot{x}_1(t)|dt \le x_1(t) \le x_1(\xi_1) + \int_0^w |\dot{x}_1(t)|dt \le H_1.$$

By (3.4), we have

$$\begin{cases} \frac{m_1 e^{P_1}}{a_1 + e^{P_1}} - d_2 - \frac{m_2 \exp(x_3(\xi_3))}{a_2 + \exp(H_2)} dt \ge 0, \\ \frac{m_1 e^{H_1}}{a_1 + e^{H_1}} - d_2 - \frac{m_2 \exp(x_3(\zeta_3))}{a_2 + \exp(P_2)} dt \le 0. \end{cases} \tag{3.11}$$

Assume that $\ln(\frac{m_1 e^{P_1}}{a_1 + e^{P_1}} - d_2) - \ln(\frac{m_2}{a_2 + \exp(H_2)}) > 0$, then we have that

$$\begin{cases} \ln(\frac{m_1 e^{P_1}}{a_1 + e^{P_1}} - d_2) - \ln(\frac{m_2}{a_2 + \exp(H_2)}) \ge \exp(x_3(\xi_3)), \\ \ln(\frac{m_1 e^{H_1}}{a_1 + e^{H_1}} - d_2) - \ln(\frac{m_2}{a_2 + \exp(P_2)}) \le \exp(x_3(\zeta_3)). \end{cases} \tag{3.12}$$

Denote that

$$\begin{aligned} P_3 &:= \ln(\tfrac{m_1 e^{H_1}}{a_1 + e^{H_1}} - d_2) - \ln(\tfrac{m_2}{a_2 + \exp(P_2)}) - 2wd_3; \\ H_3 &:= \ln(\tfrac{m_1 e^{P_1}}{a_1 + e^{P_1}} - d_2) - \ln(\tfrac{m_2}{a_2 + \exp(H_2)}) + 2wd_3. \end{aligned} \tag{3.14}$$

Suppose that $P_3 < H_3$, then we have that

$$P_3 \le x_3(\zeta_3) - \int_0^w |\dot{x}_3(t)|dt \le x_3(t) \le x_3(\xi_3) + \int_0^w |\dot{x}_3(t)|dt \le H_3. \tag{3.15}$$

From (3.8), it follows that

$$\sup_{t\in[0,w]} |x_i(t)| \leq \sup\{|H_i|, |P_i|\} := M_i, \quad i = 1, 2, 3. \tag{3.16}$$

Clearly, M_i are independent of λ. Set $M = M_1 + M_2 + M_3 + M_0$, where M_0 is taken sufficiently large such that $\| \ln\{x^*\} \| = \| (\ln\{x_1^*\}, \ln\{x_2^*\}, \ln\{x_3^*\})^T \| = |\ln\{x_1^*\}| + |\ln\{x_2^*\}| + |\ln\{x_3^*\}| < M_0$, then $\| x \|_c < M$.

Let $\Omega = \{x \in X : \| x \|_c < M$, it is clear that Ω verifies the requirement (1) in Lemma 3.1. When $x \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, x is a constant vector in R^3 with $\| x \|_c = M$, then

$$QNx = \left\{ \begin{pmatrix} 1 - e^{x_1} - \frac{m_1 e^{x_2}}{a_1 + e^{x_2}} + \frac{1}{w}\ln(1-p) \\ \frac{m_1 e^{x_1}}{a_1 + e^{x_1}} - d_2 - \frac{m_2 e^{x_3}}{a_2 + e^{x_2}} \\ \frac{m_2 e^{x_2}}{1 + a_2 e^{x_2}} - d_3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\} \neq 0. \tag{3.17}$$

Obviously, we have that $QNx \neq 0$ for each $x \in \Omega \cap KerL$. Take

$$J : ImQ \to x, (f, 0) \to f,$$

then when $x \in \partial\Omega \cap KerL$, one obtains

$$JQNx = \begin{pmatrix} 1 - e^{x_1} - \frac{m_1 e^{x_2}}{a_1 + e^{x_2}} + \frac{1}{w}\ln(1-p) \\ \frac{m_1 e^{x_1}}{a_1 + e^{x_1}} - d_2 - \frac{m_2 e^{x_3}}{a_2 + e^{x_2}} \\ \frac{m_2 e^{x_2}}{1 + a_2 e^{x_2}} - d_3 \end{pmatrix}.$$

Now we suppose that

$$deg\{JQNx, \Omega \cap KerL, 0\} \neq 0. \tag{3.18}$$

By now we have proved that Ω verifies all the requirements in Lemma 2.1. Hence, (3.1) has at least one periodic solution $x^*(t) = (x_1^*(t), x_2^*(t), x_3^*(t))^T$ in $DomL \cap \bar{\Omega}$. Set $y^*(t) = \exp\{x^*(t)\}$, then $y^*(t) = (y_1^*(t), y_2^*(t), y_3^*(t))^T$ is an w-periodic solution of (2.1) with strictly positive components.

We regroup the above results as the following theorem 3.1.

Theorem 3.1. *For the system (2.1), assume that:*

(i) $w + \ln(1-p) > 0$ *and* $m_2 - a_2 d_3 > 0$;

(ii) $1 - \frac{m_1 e^{H_2}}{\max(a_1, 1)} + \frac{1}{w}\ln(1-p) > 0$, $\ln(\frac{m_1 e^{P_1}}{a_1 + e^{P_1}} - d_2) - \ln(\frac{m_2}{a_2 + \exp(H_2)}) > 0$ *and* $P_3 < H_3$;

(iii) $deg\{JQNx, \Omega \cap KerL, 0\} \neq 0$.

Then (2.1) has at least one w-periodic solution with strictly positive components.

4. Conclusion

In this paper, we introduce and study a Rosenzweig-MacArthur food chain with impulsive ratio-harvesting prey. By using Floquet Theorem and small amplitude perturbation skills, we have proved that if $\mu_1 < 1$, the periodic solution $(0,0,0)$ is globally asymptotically stable; if $\mu_1 > 1$, there exists m_1^* to play as the invasion threshold of the predator, that is to say, if $m_1 < m_1^*$, then the $(x_{1s}(t),0,0)$ is globally asymptotically stable and if $m_1^* < m_1 < m_1^* + \varepsilon$ and $\varepsilon > 0$ to be small enough, the system (2.1) has stable boundary w-periodic solution $(\tilde{x}_1(t),\tilde{x}_2(t),0)$. Using the coincidence degree theorem and its related continuous theorem we give out some sufficient conditions to ensure the existence of the positive periodic solutions of the system (2.1).

Acknowledgments

The research of Guoping Pang and Lansun Chen is supported by National Natural Science Foundation of P.R. China (10471117, 10526015) and Scientific Research Foundation of Guangxi Education Office, P.R. China (2006243).

The research of Fengyan Wang is supported by the Youth Science Foundation of Fujian Province (2006F3091) and the Science Foundation of Jimei University, China.

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