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MATHEMATICAL MODEL FOR THE WASTE MATERIAL FLOWS AND CONTENTS OF TOXIC COMPOUNDS MINIMIZATION IN ELECTROLYTIC COPPER PRODUCTION

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1. Introduction

In pyrometalurgy electrolytic copper production basic supply are copper concentrates, whose component composition varied at certain limits. The production flows in several continuous processes (drying, flash smelting, converting, anode refining and electrolytic refining), during each of its are obtained waste material flows: technology gases, contaminated water flows, solid wastes (slag, converter dusts, filter cake, etc.), contained hazardous components [2, 3].

The rational use of the energy resources, the basics and auxiliary materials, as minimizing the volume of the generated wastes and the content of hazardous components of them, obtained by different technology facilities, is a inseparable as well control of activities in any factory.

For a deciding a mathematical model of the base is in minimizing the wastes on the task framed on element material balance of the production process.

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2. Mathematical Model of Element Mass Balance

In order to achieve the main goal to facilitate and simplify the calculations needed, the production process is divided to separate structures, and for each separate structure its own material balance is presented. The material balance features the following general type:

$$M_{in} = M_{out}\,, \tag{1}$$

where:

M_{in} – sum of the input mass streams (2): the initial and the auxiliary materials; water (process water, heat/cool.); air (technological, ventilation, etc.), recirculated materials and the exploited energy resources.

M_{out} – sum from the output mass flows (4): the commercial products; and the solid, liquid and gaseous material flows.

$$M_{in} = M_x + M_{y1} + M_{y1} + M_z + M_E + M_W + M_A \pm \sum \Delta_{in}\,, \tag{2}$$

where:

M_x– mass of initial materials;

M_{y1}– mass of auxiliary materials for the primary technologic process;

M_{y2}– mass of auxiliary materials for the secondary technologic process;

M_z– mass of recirculated materials (dusts, slag concentrate);

M_E– mass of used energy sources (technological fuels, water steam, oxygen, etc);

M_W– mass of used water;

M_A– mass of used air;

$\sum \Delta_{in}$ – error during measuring/calculation of the input mass flows.

$$\sum \Delta_{in} = \pm\Delta_x \pm \Delta_{y1} \pm \Delta_{y2} \pm \Delta_z \pm \Delta_E \pm \Delta_W \pm \Delta_A; \tag{3}$$

Δ_x– error during measuring/calculation of the mass of the initial materials;

Δ_{y1}– error during measuring/calculation of the mass of the auxiliary materials for the primary technological process;

Δ_{y2}– error during measuring/calculation of the mass of the auxiliary materials for the secondary technological process;

Δ_z– error during measuring/calculation of mass of recirculated materials (dusts, slag concentrate);

Δ_E– error during measuring/calculation of the mass of the energy sources (technological fuels, water steam, oxygen, etc.);

Δ_W– error during measuring/calculation of the mass of the used water;

Δ_A– error during measuring/calculation of the mass of the used air

$$M_{out} = \mathrm{M_{p1}} + \mathrm{M_{p2}} + \mathrm{M_{rg}} + \mathrm{M_{wg}} + \mathrm{M_{ww}} + \mathrm{M_{ws}} + \mathrm{M_{pack2}} \pm \sum \Delta_{o\upsilon\tau}; \quad (4)$$

M_{p1} – mass of the obtained intermediate products;

M_{p2} – mass of the obtained final products;

M_{rg} – mass flow of the ventilation gasses;

M_{wg} – mass flow of the technological gasses;

M_{ww} – mass flow of the waste liquid material flows;

M_{ws}– mass of the solid wastes;

$\sum \Delta_{o\upsilon\tau}$ – error during measuring or calculation of the masses of outgoing material flows

$$\sum \Delta_{o\upsilon\tau} = \pm\Delta_{p1} \pm \Delta_{p2} \pm \Delta_{rg} \pm \Delta_{wg} \pm \Delta_{ww} \pm \Delta_{ws}; \quad (5)$$

Δ_{p1} – error during measuring/calculation of the mass of the obtained intermediate products;

Δ_{p2} – error during measuring/calculation of the mass of the obtained final products;

Δ_{rg} – error during measuring/calculation of the mass flow of the ventilation gasses;

Δ_{wg} – error during measuring/calculation of the mass flow of the technological gasses;

Δ_{ww} – error during measuring/calculation of the mass flow of the waste liquid material flows;

Δ_{ws} – error during measuring/calculation of the mass of the solid wastes.

In order to find the of exact solution of the equation describing the material balance, it is necessary to minimize the errors due to measuring or calculation ($\sum \Delta_{in}$ and $\sum \Delta_{o\upsilon\tau}$).

Two criteria have been introduced. The first one considers the waste quantities generated by the technology and it is called the *zero-waste criterion.* The second criterion reflects the presence of toxic components in the outgoing material flows.

2.1. Criterion for Waste Minimization During Utilization of a Sertain Technology

ε_1 – level of zero-waste generation of the technology through the useful products quantities (6a) and (6b)

$$\varepsilon_1 = \pm \frac{(M_{p2} \pm e_{prod})}{100\,(M_{in} + \pm \sum \Delta_{in})}, [t/y]\,, \tag{6a}$$

$$\varepsilon_1 = \frac{(M_{p2} \pm \Delta_{prod})}{M_{in} \pm \sum \Delta_{in}} 100[\%]; \tag{6b}$$

e_{prod}– average percentage error during determination of the final product (electrolytic copper);

ε_2 – level of zero-waste generation of the technology through quantity of waste material flows (7a) and (7b).

$$\varepsilon_2 = \pm \frac{(M_{p2} \pm e_{prod})}{100\,(M_{in} + \pm \sum \Delta_{in})}\,, \tag{7a}$$

$$\varepsilon_2 = \frac{\left(\mathrm{M_{rg}} \pm \Delta_{rg} + \mathrm{M_{wg}} \pm \Delta_{wg} + \ \mathrm{M_{ww}} \pm \Delta_{ww} + \ \mathrm{M_{ws}} \pm \Delta_{ws}\right)}{M_{in} \pm \sum \Delta_{in}} 100\,. \tag{7b}$$

For verification of the criterion's precision, the following equation (dependency) could be used:

$$\varepsilon_1 = 100 - \varepsilon_2[\%]\,. \tag{8}$$

2.2. Minimization of Hazardous Compounds Quantity in the Waste Materials Flows

The quantities of the specific toxic (hazardous) component in the waste gasses, wastewaters and/or solid waste flows are calculated

2.3. Determining of Quantity of the Solid Waste Flows – Q_{WS}

$$Q_{WS} = \frac{\sum_{i=1}^{N} q_{ws}}{N}\,. \tag{9}$$

The overall quantities of the solid waste flows are calculated as an average sum of the different solid waste flows (q_{ws}).

$$q_{ws,k} = \frac{\sum_{s=1}^{m} \left(\frac{\overline{A_{k,s}} + \overline{e_{k,s}}}{NDE_{k,s}} WSh_s \right)}{\sum_{s=1}^{m} WS_s}, \tag{10}$$

where:

$\overline{A}_{k,s}$– average value [g/t] for concentration of the hazardous component k in the flow s of outgoing solid wastes discharged in the environment, $s = 1, 2, ..., m$ flows of solid wastes;

$\overline{e}_{k,s}$– average value of the fixed error for the hazardous component k in the flow s of outgoing solid wastes [g/t];

WSh_s– mass of the flow $s$consisting of solid waste and aqueous slime flows containing the hazardous component k, [t/y];

WS_s– total mass of the flow $s$consisting of outgoing solid wastes released in the atmosphere [t/y];

$NDE_{k,s}$– permissible emissions norm for the hazardous component k in the flow s consisting of solid wastes, pre-calculated and expressed as [g/t].

2.4. Determining the Quantity of the Liquid Waste Materials Flows – Q_{WW}

$$Q_{WW} = \frac{\sum_{i=1}^{N} q_{ww}}{N}, \tag{11}$$

$$q_{ww,k} = \frac{\sum_{l=1}^{u} \left(\frac{\overline{A_{k,l}} + \overline{e_{k,l}}}{NDE_{k,l}} WWh_l \right)}{\sum_{l=1}^{u} WW_l}, \tag{12}$$

where:

$\bar{A}_{k,l}$ – average value, g/m^{3}for the concentration of the hazardous component k in the flow l consisting of liquid waste discharged in the environment, $l = 1, 2, ..., u$ liquid waste flows;

$\bar{e}_{k,l}$ – average value of the fixed error resulting from the calculations of $\bar{A}_{k,s}$, [g/m^3];

$NDE_{k,l}$ - Permissible Emissions Norm for the hazardous component k;

WWh_l- volume of the flow$l$consisting of liquid waste, containing the hazardous component k, discharged in the environment [m^3/y];

WW_l- total volume of the flow $l$consisting liquid waste discharged in the environment [m^3/y].

2.5. Determining of the Quantity of the Waste Gaseous Material Flows – Q_{WG}

$$Q_{WG} = \frac{\sum_{i=1}^{N} q_{wg}}{N}\,, \tag{13}$$

$$q_{wg,k} = \frac{\sum_{g=1}^{v}\left(\frac{\overline{A_{k,g}}+\overline{e_{k,g}}}{NDE_{k,g}} WGh_g\right)}{\sum_{g=1}^{v} WG_g}\,, \tag{14}$$

$\bar{A}_{k,g}$ – average value, g/1000Nm^3 for the concentration of the hazardous component k in the flow g consisting of gaseous waste discharged in the atmosphere;

$g = 1, ..., v$ flows of gaseous waste;

$\bar{e}_{k,g}$ – error from the measurement/calculation of the $\bar{A}_{k,g}$, g/1000Nm^3;

$NDE_{k,g}$ - Permissible Emissions Norm for the hazardous component k in the flow g expressed as g/1000Nm^3;

WGh_g- volume of the flow g, 1000 Nm^3/y from waste gaseous flows containing the toxic component k, discharged in the atmosphere, [Nm^3/y];

$WG-$ total volume of the flow g, 1000 Nm^3/y, of waste gaseous flows discharged in the atmosphere, [Nm^3/y].

2.6. Mass Flow Balances of the Toxic Components

2.6.1. Balances of the Mass Flows of the Toxic Components

In order to use the mass flows balances equations relevant to the hazardous components, first of all detailed assessment of the components toxicity in the raw and auxiliary materials have to be performed, as well as an assessment of the toxicity of the compounds produced by them after implementation of a specific production technology. The assessment results should be analyzed in the light of the legal requirements in force [4, 5].

$$\sum X_{in} \pm \Delta x_{in} = \sum X_{out} \pm \Delta x_{out}\,, \tag{15}$$

where:

$\sum X_{in}-$ sum of the masses of the component x in raw and auxiliary materials for the primary process, in raw and auxiliary materials for secondary processes, recirculated materials, used water, air and fuels.

$\sum X_{out}-$ sum of the masses of the component x in the obtained products, waste gasses, waste waters and solid wastes.

The components toxicity assessment in the raw and auxiliary materials, as well as toxicity of the compounds produced by these components after realization of a specific technology.

$$\varepsilon_3 = \frac{Q_{SW}\sum\limits_{s=1}^{m} WSh_s + Q_{WW}\sum\limits_{l=1}^{u} WWh_l + Q_{SG}\sum\limits_{g=1}^{v} WGh_g}{\sum\limits_{s=1}^{m} WS_s + \sum\limits_{l=1}^{u} WW_l + \sum\limits_{g=1}^{v} WG_g} . \tag{16}$$

In equations (17), (19) criteria for determining of toxicity of solids, liquids and gasses outgoing flows (waste gasses, waste waters and solid wastes).

$$\varepsilon_4 = \frac{\sum\limits_{s=1}^{m} WSh_s}{\sum\limits_{s=1}^{m} WS_s} \leq 1 , \tag{17}$$

$$\varepsilon_5 = \frac{\sum\limits_{l=1}^{u} WWh_l}{\sum\limits_{l=1}^{u} WW_l} \leq 1 , \tag{18}$$

$$\varepsilon_6 = \frac{\sum\limits_{g=1}^{v} WGh_g}{\sum\limits_{g=1}^{v} WG_g} \leq 1 . \tag{19}$$

3. Conclusion

1. The analysis of technological process for copper production from sulfide input materials shows that the generated waste solid, liquid and gaseous flows contain significant amounts of toxic compounds. The improper control and management of these flows could cause serious harmful impacts on the environmental components.

2. On the base of the components mass balance, a mathematical model was developed, providing the possibility to regulate the ingoing material flows parameters in such a mode, as to minimize the outgoing waste material flows and the toxic components concentrations in them.

3. The main criteria for quantitative minimization of the released hazardous components, as well as for solving concrete optimisation tasks have been determined.

4. The proposed mathematical model could be applied for upgrade and improvement of existing automation control systems relevant to various technological processes in the metallurgy and chemical industries.

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