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# REDUCTION OF MULTI-VALUED INFORMATION SYSTEMS

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**Abstract:** Information granulation is very essential to human problem solving, and hence has a significant impact on the design and implementation of intelligent systems. Most granulation methods did not go deep in using topological structure. In this work, we aim to use general topological structures as tools for decision making in multi-valued information systems "MVIS". General relations are used to get granules that form subbase for topologies. These topologies are applied for obtaining discernibility matrix and reducts. The suggested topological structures open up the way for applying rich amount of topological facts and methods in the process of granular computing.

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## 1. Introduction

Today, information systems are widely used in many different sectors. Data, i.e. raw information, is gathered at an incredible rate. NASA's Earth Observing System gathers one terabyte of data every day. Rough Set Theory (RST) was proposed by Pawlak [10], [11] as an extension of the classical set theory.

An information granule is a concept that can be represented by classical set, fuzzy set or random set to reflect the granularity and hierarchy of information [2]. The evolutionary granular computing model can evolve to adapt to the database environment. The information granule is the basic building block to build the concepts and knowledge on higher level [1]. The knowledge discovery from real-life databases is a multi-phase process consisting of numerous steps, including attribute selection, discretization of real valued attributes, and rule induction.

The basic ideas of granular computing, i.e., problem solving with different granularities, have been explored in many fields, such as artificial intelligence, interval analysis, quantization, rough set theory, Dempster-Shafer theory of belief functions, divide and conquer, cluster analysis, machine learning, databases, and many others [18]. There is a renewed and fast growing interest in granular computing [13], [14], [6], [8], [7], [12], [15], [16], [17].

The approach we used depends on using more than one topological methods for granulations via general relations. These methods are applied and used separately to get the approximations and reducts. A comparison between different method is discussed and some examples are given.

## 2. Basic Concepts

Let $U$ be a finite set of elements called the universe and A non-empty finite set of attributes $a \in A$, such that $a : U \longrightarrow V_a$. The set $V_a$ is called the range of the attribute a. For an element $x \in U$ and an attribute $a \in A$, the pair $(x, a(x)) \in UxV_a$ indicates that $x$ has the attribute value $a(x)$. The pair $(U, A)$ is called an information system [3] and is often referred to as a single-valued information system "SVIS" [9]. In a SVIS, attributes $a \in A$ map elements $x \in U$ to a single attribute value $v = a(x)$ in the range $V_a$.

A MVIS is a generalization of the idea of a SVIS. In a MVIS, attribute functions are allowed to map elements to sets of attribute values [9]. More formally, we allow multi-valued attributes a such that $a : U \longrightarrow 2^{V_a}$. A subset $a(x) \subseteq V_a$ may also be referred to as an attribute value.

In a MVIS (U, A), each attribute $a \in A$ implies a relation $R_a \subseteq UxV_a$ by setting $xR_av \Longleftrightarrow v \in a(x)$.

A SVIS being a particular case of a MVIS.

A topological space [4] is a pair $(U, \tau)$ consisting of a set U and family of subset of U satisfying the following conditions:

(T1) $\phi \in \tau$ and $U \in \tau$.

(T2) $\tau$ is closed under arbitrary union.

(T3) $\tau$ is closed under finite intersection.

The pair $(U, \tau)$ is called a space, the elements of U are called points of the space, the subsets of U belonging to $\tau$ are called open set in the space, and the complement of the subsets of U belonging to $\tau$ are called closed set in the space; the family $\tau$ of open subsets of U is also called a topology for U.

It often happens that the open sets of space can be very complicated and yet they can all be described using a selection of fairly simple special ones. When this happens, the set of simple open sets is called a base or subbase (depending on how the description is to done). In addition, it is fortunate that many topological concepts can be characterized in terms of these simpler base or subbase elements.

Formally, a family $\beta \subseteq \tau$ is called a base for (U,$\tau$) iff every non-empty open subset of U can be represented as a union of subfamily of $\beta$.

Clearly, a topological space can have many bases. A family $S \subseteq \tau$ is called a subbase iff the family of all finite intersections is a base for $(U, \tau)$.

## 3. Reduction of Multi-Valued Information System

In the following definitions, we will get the reduction of MVIS according to the type of relation and the type of ordered pairs "after set, for set or twice".

**Definition 1.** Let U be a nonempty set of objects and R be a class of general binary relations on U, R=$\{r_1, r_2, r_3, ., r_n\}$, then (U, R) is called a generalized approximation space.

**Definition 2.** Let U be a nonempty set of objects and R be a class of general binary relations on U, each $r \in R$ yields a class $S_r = \{x_r : x \in U\}$ which called a subknowledge base.

In the following, we use general topological concepts for reduction of relations on MVIS.

**Definition 3.** Let R be a class of general binary relations, then a subbase for $\tau$ for all R is: $S_R = \bigcup_{r \in R} S_r$.

**Definition 4.** Let R be a class of general binary relations, then a base for $\tau$ for all R is: $\beta_R = \bigcap_{S_x \in S_R} S_x, \forall x \in U$.

**Definition 5.** Let $B \subseteq A$, $a \in B$, where A be a class of attributes, see [3]. $a$ is said to be superfluous attribute in B if: $\beta_B = \beta_{(B-a)}$

The set M is called a minimal reduct of B iff:

(i) $\beta_M = \beta_{(B)}$.

(ii) $\beta_M \neq \beta_{(B-a)}, \forall a \in M$.

The following example illustrates the notions given above.

**Example 1.** Let U={1,2,3,4,5}, A={r, p, q} as in Table 1, where:

| U/A | r | p | q |
|---|---|---|---|
| 1 | {E} | {H} | {S} |
| 2 | {E, G} | {H} | {S} |
| 3 | {G} | {H, B} | {R} |
| 4 | {G, A} | {B} | {R, F} |
| 5 | {A} | {T} | {F} |

Table 1.

r=Languages = {English, German, Arabic}={E, G, A}
p=Sports = {Tennis, Handball, Basketball}={T, H, B}
q=Skills = {Swimming, Running, Fishing}={S, R, F}

**Note.** Our choice for relation R depends on our view to the choice of objects, where we can choose a level of experience and any of objects having higher levels.

Let xRy iff $R(x) \subseteq R(y)$, then:
xry={(1,1),(1,2),(2,2),(3,3),(3,2),(3,4),(4,4),(5,5),(5,4)},
xpy={(1,1),(1,2),(1,3),(2,2),(2,1),(2,3),(3,3),(4,4),(4,3),(5,5)},
xqy={(1,1),(1,2),(2,2),(2,1),(3,3),(3,4),(4,4),(5,5),(5,4)}.

*First Step:* by using "after set", we get:
$S_r = \{\{1,2\},\{2\},\{2,3,4\},\{4\},\{4,5\}\} = \beta_r$,
$S_p = \{\{1,2,3\},\{3\},\{3,4\},\{5\}\} = \beta_p$,
$S_q = \{\{1,2\},\{3,4\},\{4\},\{4,5\}\} = \beta_q$.

Then the subbase for all attributes is:

$$S_A = \{\{1,2\},\{2,3,4\},\{4,5\},\{1,2,3\},\{3,4\},\{5\}, \{2,3,4,5\},\{1,2,3,4\},\{3,4,5\}\}.$$

The base is: $\beta_A = \{\{1,2\},\{2\},\{3\},\{4\},\{5\}\}$. Next consider:

$S_{(A-r)} = \{\{1,2,3\},\{3,4\},\{5\},\{1,2\},\{4,5\}\}$ ,

$\beta_{(A-r)} = \{\{1,2\},\{3\},\{4\},\{5\}\}$;

$S_{(A-p)} = \{\{1,2\},\{2,3,4\},\{4,5\},\{3,4\}\}$,
$\beta_{(A-p)} = \{\{1,2\},\{2\},\{3,4\},\{4\},\{4,5\}\}$;

$S_{(A-q)} = \{\{1,2\},\{2,3,4\},\{4,5\},\{1,2,3\},\{3,4\},\{5\}\}$,
$\beta_{(A-q)} = \{\{1,2\},\{2\},\{3\},\{4\},\{5\}\} = \beta_A$.

So we find that q is only superfluous attribute in A, and we have RED(A)={r, p}, CORE(A)={r, p}.

*Second Step.* by using "for set", we get:
${}_rS = \{\{1\},\{1,2,3\},\{3\},\{3,4,5\},\{5\}\}$,
${}_pS = \{\{1,2\},\{1,2,3,4\},\{4\},\{5\}\}$,
${}_qS = \{\{1,2\},\{3\},\{3,4,5\},\{5\}\}$.

Then the subbase for all attributes is:
${}_AS = \{\{1\},\{1,2,3\},\{3\},\{3,4,5\},\{5\},\{1,2\},\{1,2,3,4\},\{4\},\{5\}\}$.

The base is: ${}_A\beta = \{\{1\},\{1,2\},\{3\},\{4\},\{5\}\}$. Next consider:

${}_{(A-r)}S = \{\{1,2\},\{1,2,3,4\},\{4\},\{5\},\{3\},\{3,4,5\}\}$,
${}_{(A-r)}\beta = \{\{1,2\},\{3\},\{4\},\{5\}\}$;

${}_{(A-p)}S = \{\{1\},\{1,2,3\},\{3\},\{3,4,5\},\{5\},\{1,2\}\}$,
${}_{(A-p)}\beta = \{\{1\},\{1,2\},\{3\},\{5\},\{3,4,5\}\}$;

${}_{(A-q)}S = \{\{1\},\{1,2,3\},\{3\},\{3,4,5\},\{5\},\{1,2\},\{1,2,3,4\},\{4\}\}$,
${}_{(A-q)}\beta = \{\{1\},\{1,2\},\{3\},\{4\},\{5\}\} =_A \beta$.

So we find that q is only superfluous attribute in A, and we have RED(A)={r, p}, CORE(A)={r, p}.

**Note.** In spite of the relation which we use in this example is not symmetric, we find that: "for set" gives the same result when we use "after set" granules.

*Third Step.* By using "for set" and "after set" at the same time "union of them", we get:
$S_A$={{1},{1,2,3},{3},{4},{1,2},{2,3,4},{4,5},{1,2,3},{3,4},{5},{2,3,4,5}, {1,2,3,4},{3,4,5}}.

**Note.** There are many choice, one of them is the union. The base is: $\beta_A = \{\{1\},\{2\},\{3\},\{4\},\{5\}\}$. Next consider:

$S_{(A-r)} = \{\{1,2,3\},\{3,4\},\{5\},\{1,2\},\{4,5\},\{1,2,3,4\},\{4\},\{3\},\{3,4,5\}\}$,
$\beta_{(A-r)} = \{\{1,2\},\{3\},\{4\},\{5\}\}$;

$S_{(A-p)} = \{\{1,2\},\{2,3,4\},\{4,5\},\{3,4\},\{1\},\{1,2,3\},\{3\},\{3,4,5\},\{5\}\}$,
$\beta_{(A-p)} = \{\{1\},\{2\},\{3\},\{4\},\{5\}\} = \beta_A$;

$S_{(A-q)}$={{1,2},{2,3,4},{4,5},{1,2,3},{3,4},{5},{1},{3},{3,4,5},{1,2,3,4}, {4}},
$\beta_{(A-q)} = \{\{1\},\{2\},\{3\},\{4\},\{5\}\} = \beta_A$.

We find that $p$ and $q$ are superfluous relations in $A$ and,
$S_{(A-\{p,q\})} = S_r$={{1,2},{2},{2,3,4},{4},{4,5},{1},{1,2,3},{3},{3,4,5},{5}}
$\beta_{(A-\{p,q\})} = \beta_r = \{\{1\},\{2\},\{3\},\{4\},\{5\}\} = \beta_A$.

Then: RED(A)={r}, CORE(A)={r}.

**Definition 6.** Let R be a class of general binary relations, which make subbases on an universe U. Then discernibility matrix can be defined as:

$$M_R(x,y) = \{r \in R : r(x) \nsubseteq r(y)\}, \quad x,y \in U\,,$$

or $M_R(x,y) = \{r \in R : x\not R\ y\}, \ x,y \in U$.

**Example 2.** Continuation of Table 1, we get the discernibility matrix as shown in Table 2, where xRy iff A(x)⊆ A(y).

| UxU | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | - | - | r, q | r, p, q | r, p, q |
| 2 | r | - | r, q | r, p, q | r, p, q |
| 3 | r, p, q | p, q | - | p | r, p, q |
| 4 | r, p, q | r, p, q | r, q | - | r, p, q |
| 5 | r, p, q | r, p, q | r, p, q | p | - |

Table 2.

**Note.** The above discernibility matrix is not symmetric differs from Pawlak [8] which is symmetric.

**Definition 7.** Let R be a class of general binary relations, which make subbases on an universe U. Then discernibility function can be defined as:
For each $x,y \in U$, then $F_R(r_1,r_2,..,r_n) = \wedge\{\vee r : r \in M_R(x,y)\}$,

We are using the previous definition for reduction of general relations as in the following example.

**Example 3.** Continued from Example 2 and Table 2, we find that:
A(r, p, q)={r+q}.{r+p+q}.{r}.{p+q}.{p}= r.p.

Then the reduction of Table 1 is: RED(A)={r, p} and CORE(A)={r, p}.

**Note.** This result is the same as the result of using "after set" or "for set" alone.

**Definition 8.** Let R be a class of general binary relations, which make subbases on an universe U. Then discernibility function can be defined as:
For each $x \in U$, then $F_R(x) = \bigwedge_{y \in U} \{\vee r : r \in M_R(x, y), x \neq y\}$.

We are using the previous definition for reduction of general relations as in the following example.

**Example 4.** From Table 2 after eliminating the attribute q, we get the following table:

| UxU | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | - | - | r | r, p | r, p |
| 2 | r | - | r | r, p | r, p |
| 3 | r, p | p | - | p | r, p |
| 4 | r, p | r, p | r | - | r, p |
| 5 | r, p | r, p | r, p | p | - |

Table 3.

This gives us the decision making for each object using discernibility function as follows:

$$A(1) = r,\ A(2) = r,\ A(3) = p,\ A(4) = r,$$

$$A(5) = \{r + p\}.\{r + p\}.\{r + p\}.\{p\} = p\,,$$

$$r_{\{E\}} \longrightarrow 1\,, \quad r_{\{E,G\}} \longrightarrow 2\,, \quad p_{\{H,B\}} \longrightarrow 3\,,$$

$$r_{\{G,A\}} \longrightarrow 4\,, \quad p_{\{T\}} \longrightarrow 5\,.$$

## 4. Conclusion

MVIS is a system which has its value not single value. In MVIS, we can use a lot of general relations as required according to the type of information. We and others use "after set" as a subbase for topology but in this paper we use "after set", "for set" and "after set and for set at the same time". By using a new definition of discernibility matrix which used for MVIS, we can get reduction without finding after set or for set relations. Also we use it for decision making in MVIS.

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