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AN OPTIMAL CONTROL PROBLEM IN A MODEL OF THE INTERNATIONAL MARKET FOR EMISSIONS PERMITS

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Abstract: Within the framework of investigation of important for Russia aspects of the Kyoto Protocol, a simplified dynamical model of the international market for greenhouse gases emissions permits is considered. An optimal control problem is formulated, and a procedure for constructing optimal strategies of Russia's behavior is suggested. A possibility of obtaining algorithm's input data from integrated assessment models is discussed. A specific numerical analysis is performed.

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1. Introduction

The global climate change is one of the most important problems in the modern world. The driving forces of this process are not completely studied yet, and its ecological, social, and economic consequences are rather disputable and complicated from the analytical viewpoint. However, the experts are in agreement that the dramatic climate change observed in the last century is explained to

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some extent by the increase of the atmospheric concentration of greenhouse gases (GHG), first of all, CO_2, due to man's impact that is characterized by the essential increase of fossil fuel consumption. One of the efforts of the international community to control environmental impacts is the Kyoto Protocol developed by the United Nations Framework Convention on Climate Change and passed in December 1997. As is known, after the waivers of ratification of the Kyoto Protocol by the main countries producing GHG emissions, USA and China, the future of the Protocol directly depended on the position of Russia. However, even after the ratification of the Kyoto Protocol, the debate in Russia about future costs and benefits of being a Party to the Protocol has continued with hardly mitigated intensity since many statements of the Protocol and mechanisms of its application have an ambiguous value in the context of developing economy of Russia. In the discussion, arguments of proponents and opponents of Russia's participation in the Kyoto Protocol are often based on results of application of different mathematical models, mainly, of two types: (i) integrated models for evaluating regional and global effects of GHG reduction policies; (ii) optimization models.

In the present paper, a model-oriented approach to constructing optimal strategies of Russia's behavior on the international market for emissions permits is applied. This market is one of the Kyoto flexible mechanisms. A simple model is characterized by Russia's monopoly on the trade with the Annex B countries and by an opportunity of banking permits and optimizing their sale over time. Note that, due to the collapse of the industrial sectors in 1990's, Russia actually does not need to reduce emissions for selling permits, since the amount of Russian so called "hot air" is large enough. Therefore, Russia's monopoly is considered as the first approximation to a more complicated multipole market. The model uses a demand function describing the market price for emissions permits in the Annex B countries, a cost function for emissions abatement in Russia, and a temporal dynamics of the "hot air". To obtain specific dependencies, different integrated assessment models are applied.

2. The Simple Model of Dynamics of the Stock of Permits

To describe the process of emissions permits banking with an opportunity at every time moment to sell some amount on the market and/or to increase the stock by emissions abatement, we use a dynamical controlled system. Let $x(t)$ be the stock of permits that are banked at time t; $h(t)$ be the "hot air" available for sale (this function is actually regulated by the Protocol and assumed to be known); $q(t)$ be the emissions abatement; $u(t)$ be the amount of permits supplied for sale. The last two functions are control parameters. It is natural to equate the rate of the stock of permits, $\dot{x}(t)$, with the difference of two values, $h(t) + q(t)$ and $u(t)$. This results into the following differential equation:

$$\dot{x}(t) = h(t) + q(t) - u(t), \quad t \in [t_0, T]. \tag{1}$$

We assume that the initial time is a moment t_0 when the stock of permits equals zero, i.e.,

$$x(t_0) = 0. \tag{2}$$

It is evident that the "hot air" $h(t)$, the emissions abatement $q(t)$, and the amount of permits $u(t)$ supplied for sale can not exceed some definite values; this results into constraints of the form

$$a_1(t) \le q(t) \le b_1(t), \quad a_2(t) \le u(t) \le b_2(t), \quad a_3(t) \le h(t) \le b_3(t), \tag{3}$$

$$a_1(t) \ge 0, \quad a_2(t) \ge 0, \quad a_3(t) \ge 0,$$

$$x(t) \ge 0. \tag{4}$$

Note that constraints (3) provide fulfillment of the inequality $x(t) \le K$, where K is a constant, which can be explicitly written. Therefore, condition (4) is naturally replaced by the condition

$$0 \le x(t) \le K. \tag{5}$$

We assume that all the scalar functions from the right-hand part of (1) belong to the space $L_2([t_0, T]; \mathbb{R})$, functions $a_i(\cdot)$, $b_i(\cdot)$, $i = 1, 2, 3$, are continuous. In what follows, we consider any function $h(\cdot)$ as a known one. Functions $q(\cdot)$ and $u(\cdot)$ satisfying relations (3) and providing fulfillment of inequality (5) are called admissible controls. The set of all admissible controls is denoted by the symbol U_*. A solution of equation (1) is a Caratheodory solution and belongs to the space of absolutely continuous functions $\mathcal{A}([t_0, T]; \mathbb{R})$. A solution corresponding to a pair of admissible controls $(q(\cdot), u(\cdot)) \in U_*$ is denoted by $x(\cdot; q(\cdot), u(\cdot))$.

3. Statement of Optimal Control Problems

Let us formulate a problem of optimal control for system (1)–(3), (5).

Problem P1. It is required to find functions $q_*(\cdot)$ and $u_*(\cdot)$ solving the extremal problem

$$\max_{u,q} F(u, q), \tag{6}$$

$$F(u,q) = \int_{t_0}^{T} \beta(\tau)[P(u(\tau))u(\tau) - C(q(\tau))q(\tau)]\,d\tau + \beta(T)\pi(T)x(T), \tag{7}$$

$$x(T) = \int_{t_0}^{T} (h(\tau) + q(\tau) - u(\tau))\,d\tau, \tag{8}$$

and providing constraints (3), (5). Here $\beta(t)$ is the discount rate; $P(u(t))$ is the price of permits, which, as a rule, is inversely proportional to the amount of permits on the market; $C(q(t))$ is the cost function for marginal abatement, which, as a rule, is directly proportional to the level of abatement (it is determined by a so called regional MAC curve); $\pi(t)$ is the expected price of permits. Thus, the integral term characterizes the total income from operations on the market minus abatement costs, whereas the terminal one represents the cost of all emissions permits banked till the moment T; both with discounting. Optimization problems of a similar type have been investigated by many authors (see, for example, [6], [4], [2]).

Note that in the model described above the idea of possible banking of permits that can be profitable, firstly, due to the growth of demand and, consequently, of the market price for emissions permits in the future and, secondly, for decreasing own abatement costs (in a remote perspective) is realized.

We assume that the functions $P(\cdot)$ and $C(\cdot)$ are such that the functional $F(u,q)$ is strongly convex with respect to u and q. Since equality (8) is valid, functional (7) can be rewritten in the form:

$$F(u,q) = \int_{t_0}^{T} [(\beta(\tau)P(u(\tau)) - \beta(T)\pi(T))u(\tau) - (\beta(\tau)C(q(\tau)) - \beta(T)\pi(T))q(\tau) + +\beta(T)\pi(T)h(\tau)]\,d\tau. \tag{9}$$

Let us formulate an auxiliary problem of optimal control.

Problem P2. It is required to find functions $q_\alpha(\cdot)$ and $u_\alpha(\cdot)$ solving the extremal problem

$$\max_{u,q} F_\alpha(u,q), \tag{10}$$

$$F_\alpha(u,q) = \int_{t_0}^{T} [(\beta(\tau)P(u(\tau)) - \beta(T)\pi(T))u(\tau) - (\beta(\tau)C(q(\tau)) - \beta(T)\pi(T))q(\tau) + +\beta(T)\pi(T)h(\tau) - \alpha x^2(\tau)]\,d\tau, \tag{11}$$

$$x(\tau) = \int_{t_0}^{\tau} (h(\xi) + q(\xi) - u(\xi))\, d\xi,$$

and providing constraints (3), (5). Here $\alpha > 0$ is a small parameter.

Note that, in virtue of strong convexity of functionals (9) and (11) with respect to u and q, and convexity, boundedness, and closedness (in $L_2([t_0, T]; \mathbb{R}) \times L_2([t_0, T]; \mathbb{R})$) of the set U_*, Problems P1 and P2 have unique solutions, $(q_*(\cdot), u_*(\cdot)) \in U_*$ and $(q_\alpha(\cdot), u_\alpha(\cdot)) \in U_*$, respectively.

Theorem 1. *Let for any $\alpha > 0$ functions $q_\alpha(\cdot)$ and $u_\alpha(\cdot)$ solve Problem P2. Then, for the functional sequence $(q_\alpha(\cdot), u_\alpha(\cdot))$, the following convergence is valid as $\alpha \to 0$:*

$$(q_\alpha(\cdot), u_\alpha(\cdot)) \to (q_*(\cdot), u_*(\cdot)) \text{ weakly in } L_2([t_0, T]; \mathbb{R}) \times L_2([t_0, T]; \mathbb{R}), \tag{12}$$

where $(q_*(\cdot), u_*(\cdot))$ is the unique solution of Problem P1.

Proof. We have

$$(q_*(\cdot), u_*(\cdot)) = \arg\max_{q,u} \{F(q(\cdot), u(\cdot)) : (q(\cdot), u(\cdot)) \in U_*\}, \tag{13}$$

$$(q_\alpha(\cdot), u_\alpha(\cdot)) = \arg\max_{q,u} \{F_\alpha(q(\cdot), u(\cdot)) : (q(\cdot), u(\cdot)) \in U_*\}. \tag{14}$$

Let

$$I(q(\cdot), u(\cdot)) = -F(q(\cdot), u(\cdot)),$$

$y(t) = y(t; q(\cdot), u(\cdot))$ be a solution of the equation

$$\dot{y}(t) = x^2(t; q(\cdot), u(\cdot)), \quad y(t_0) = 0.$$

Then $y_\alpha(t) = y(t; q_\alpha(\cdot), u_\alpha(\cdot))$. In virtue of (11), (14), the following inequality is valid for any $(u, q) \in U_*$:

$$I(q(\cdot), u(\cdot)) + \alpha y(T; q(\cdot), u(\cdot)) \geq I(q_\alpha(\cdot), u_\alpha(\cdot)) + \alpha y_\alpha(T) \geq I(q_\alpha(\cdot), u_\alpha(\cdot)).$$

Consequently,

$$I(q(\cdot), u(\cdot)) \geq I(q_\alpha(\cdot), u_\alpha(\cdot)) - \alpha y(T; q(\cdot), u(\cdot)) \quad \forall (u, q) \in U_*. \tag{15}$$

To prove the theorem, it is sufficient to show that if $\alpha_k \to 0$ and

$$(q_{\alpha_k}(\cdot), u_{\alpha_k}(\cdot)) \to (\bar{q}(\cdot), \bar{u}(\cdot)) \quad \text{weakly in} \quad L_2([t_0, T]; \mathbb{R}) \times L_2([t_0, T]; \mathbb{R})$$

as $k \to \infty$, then the equalities

$$\bar{q}(\cdot) = q_*(\cdot), \quad \bar{u}(\cdot) = u_*(\cdot) \tag{16}$$

are fulfilled. Note that

$$\sup\{\|y(T; q(\cdot), u(\cdot))\| : (q(\cdot), u(\cdot)) \in U_*\} \le C < \infty.$$

Therefore,

$$\alpha y(T; q(\cdot), u(\cdot)) \to 0 \quad \text{as} \quad \alpha \to 0 \tag{17}$$

uniformly with respect to all $(q(\cdot), u(\cdot)) \in U_*$.

As is known [11], the functional $I(q(\cdot), u(\cdot))$ is weakly lower semicontinuous. Therefore,

$$\varliminf_{k\to\infty} I(q_{\alpha_k}(\cdot), u_{\alpha_k}(\cdot)) \ge I(\bar{q}(\cdot), \bar{u}(\cdot)). \tag{18}$$

From inequality (15), in virtue of (17) and (18), it follows that

$$I(q(\cdot), u(\cdot)) \ge I(\bar{q}(\cdot), \bar{u}(\cdot)) \quad \forall (u, q) \in U_*. \tag{19}$$

However, the solution of Problem P1 (problem (13)) is unique. This and (19) imply (16). The theorem is proved. □

Taking into account convergence (12), we solve auxiliary Problem P2 instead of Problem P1.

4. Algorithm for Solving Problem P2

Problem P2, being a problem of optimal control under phase constraints, needs special solving methods. The algorithm used in the present work is described in [7]. It consists in reduction of solving the problem with phase constraints to solving a sequence of classical optimal control problems, for example, by means of Pontryagin's maximum principle [10].

According to [7], an iterative procedure is designed for solving Problem P2. At each step k of this procedure, we solve the problem of finding functions $z^*(\cdot)$, $w_1^*(\cdot)$, and $w_2^*(\cdot)$ such that

$$(z^*(\cdot), w_1^*(\cdot), w_2^*(\cdot)) = \arg\max_{z, w_1, w_2} \left\{F_\alpha^1(z, w_1, w_2) : (z(\cdot), w_1(\cdot), w_2(\cdot)) \in Q\right\}. \tag{20}$$

Here the symbol Q stands for the set of Lebesgue measurable functions $z(\cdot)$, $w_1(\cdot)$, $w_2(\cdot)$ (acting as controls) satisfying the conditions

$$0 \le z(t) \le K,$$

$$a_1(t) + h(t) \le w_1(t) \le b_1(t) + h(t), \quad a_2(t) \le w_2(t) \le b_2(t), \tag{21}$$

and transferring a phase trajectory of the equation

$$\dot{\eta}(t) = g_C^k(t) z(t) - g_D^k(t) w_1(t) + g_D^k(t) w_2(t), \quad t \in [t_0, T], \tag{22}$$

from the initial state

$$\eta(t_0) = 0 \tag{23}$$

to the terminal state

$$\eta(T) = 0. \tag{24}$$

The performance criterion F_α^1 takes the form

$$F_\alpha^1(z, w_1, w_2) = \int_{t_0}^{T} [(\beta(\tau) P(w_2(\tau)) - \beta(T)\pi(T)) w_2(\tau) - \beta(\tau) C(w_1(\tau) \\ - h(\tau))(w_1(\tau) - h(\tau)) + \beta(T)\pi(T) w_1(\tau) - \alpha z^2(\tau)]\, d\tau. \tag{25}$$

The control $z(\cdot)$ corresponds to the given system's phase variable $x(\cdot)$, the control $w_1(\cdot)$ corresponds to the value $q(\cdot) + h(\cdot)$, and the control $w_2(\cdot)$ corresponds to the function $u(\cdot)$. The coefficients at the controls in (22) are found by the formulas:

$$g_C^k(t) = r_\alpha^k(t), \quad g_D^k(t) = \int_t^T r_\alpha^k(\tau)\, d\tau, \tag{26}$$

where

$$r_\alpha^k(t) = x_\alpha^k(t) - \int_{t_0}^{t} (h(\tau) + q_\alpha^k(\tau) - u_\alpha^k(\tau))\, d\tau. \tag{27}$$

The functions $(x_\alpha^k(\cdot), q_\alpha^k(\cdot), u_\alpha^k(\cdot))$ are calculated at the step $k-1$ (at the step 0, the values $x_\alpha^0(\cdot)$, $q_\alpha^0(\cdot)$, and $u_\alpha^0(\cdot)$ are chosen as functions providing the maximum of functional (11) under constraints (3)–(5); in particular, $x_\alpha^0(t) = 0$). Let $(z_\alpha^k(\cdot), w_{1\alpha}^k(\cdot), w_{2\alpha}^k(\cdot))$ be a solution of problem (20)–(25). The passage to the step $k+1$ is realized according to the following scheme. First, we calculate the function

$$\rho_\alpha^k(t) = z_\alpha^k(t) - \int\limits_{t_0}^{t} (w_{1\alpha}^k(\tau) - w_{2\alpha}^k(\tau))\, d\tau \tag{28}$$

and the coefficient (so called step size)

$$\tau_\alpha^k = \arg \min_{0 \le \tau \le 1} \|(1-\tau) r_\alpha^k(\cdot) + \tau \rho_\alpha^k(\cdot)\|_{L_2}^2. \tag{29}$$

Here the symbol $\|x(\cdot)\|_{L_2}$ means the norm of $x(\cdot)$ in the space $L_2([t_0,T];\mathbb{R})$, i.e., $\|x(\cdot)\|_{L_2} = \Big(\int\limits_{t_0}^{T} |x(\tau)|^2\, d\tau\Big)^{\frac{1}{2}}$.

Then we obtain the $(k+1)$th approximation to the solution of the given problem:

$$\begin{aligned} x_\alpha^{k+1}(t) &= x_\alpha^k(t) + \tau_\alpha^k (z_\alpha^k(t) - x_\alpha^k(t)),\\ q_\alpha^{k+1}(t) &= q_\alpha^k(t) + \tau_\alpha^k (w_{1\alpha}^k(t) - h(t) - q_\alpha^k(t)),\\ u_\alpha^{k+1}(t) &= u_\alpha^k(t) + \tau_\alpha^k (w_{2\alpha}^k(t) - u_\alpha^k(t)). \end{aligned} \tag{30}$$

The following theorem is true, see [7].

Theorem 2. *For each $\alpha > 0$, the sequence $(q_\alpha^k(\cdot), u_\alpha^k(\cdot))$ obtained according to (26)–(30) strongly converges in the metrics of the space $L_2([t_0,T];\mathbb{R}) \times L_2([t_0,T];\mathbb{R})$ to the element $(q_\alpha(\cdot), u_\alpha(\cdot))$ as $k \to \infty$:*

$$(q_\alpha^k(\cdot), u_\alpha^k(\cdot)) \to (q_\alpha(\cdot), u_\alpha(\cdot)).$$

5. On Solving Problem (20)–(25)

To solve problem (20)–(25), we apply Pontryagin's maximum principle [10]. Below the symbol α is omitted for brevity. Consider the case when the functions $C(v(t))$ and $P(v(t))$ (see functional (11)) are piece-wise linear with respect to their arguments at each time moment $t \in [t_0, T]$, i.e.,

$$\begin{aligned} &C(v(t)) = \alpha_1^i(t) v(t) + \alpha_2^i(t),\\ &\alpha_1^i(t) > 0, \quad v(t) \in [v_1^i(t), v_1^{i+1}(t)], \quad i = 0, \ldots, n_1 - 1,\\ &P(v(t)) = -\beta_1^i(t) v(t) + \beta_2^i(t),\\ &\beta_1^i(t) > 0, \quad v(t) \in [v_2^i(t), v_2^{i+1}(t)], \quad i = 0, \ldots, n_2 - 1, \end{aligned}$$

where $\{v_1^i(t)\}_{i=0}^{i=n_1}$ and $\{v_2^i(t)\}_{i=0}^{i=n_2}$ are partitions of the ranges of possible changes of $w_1(t) - h(t)$ and $w_2(t)$ at the moment t (i.e., $[a_1(t), b_1(t)]$ and $[a_2(t), b_2(t)]$) into n_1 and n_2 subintervals, respectively.

Following the maximum principle, we make up the Hamiltonian for problem (20)–(25):

$$\begin{aligned}H(\psi, z, w_1, w_2) = \psi(t)(q_C^k(t)z(t) - q_D^k(t)w_1(t) + q_D^k(t)w_2(t))\\ + (\beta(t)P(w_2(t)) - \beta(T)\pi(T))w_2(t)\\ - \beta(t)C(w_1(t) - h(t))(w_1(t) - h(t)) + \beta(T)\pi(T)w_1(t) - \alpha z^2(t).\end{aligned}$$

The given problem has a solution that provides the maximum of the Hamiltonian over parameters ($z(t)$, $w_1(t)$, and $w_2(t)$). We use the conditions $\frac{\partial H}{\partial z} = 0$, $\frac{\partial H}{\partial w_1} = 0$, and $\frac{\partial H}{\partial w_2} = 0$. Omitting unwieldy reasonings, we write out the solution depending on the additional variable $\psi(t)$:

$$z^k(t) = \begin{cases} \frac{\psi(t)q_C(t)}{2\alpha}, & \text{if} \quad 0 < \frac{\psi(t)q_C(t)}{2\alpha} < K, \\ 0, & \text{if} \quad \frac{\psi(t)q_C(t)}{2\alpha} \le 0, \\ K, & \text{if} \quad \frac{\psi(t)q_C(t)}{2\alpha} \ge K; \end{cases}$$

$$w_1^k(t) = \arg \max_{i\in[0,n_1-1]} J^i(w_1^{ki}(t)),$$

$$\begin{aligned}J^i(v) = -\alpha_1^i(t)\beta(t)v^2 + (2\beta(t)\alpha_1^i(t)h(t) - \beta(t)\alpha_2^i(t) + \beta(T)\pi(T)\\ - \psi(t)q_D^k(t))v - \alpha_1^i(t)\beta(t)h^2(t) + \beta(t)\alpha_2^i(t)h(t), \quad v \in [v_1^i(t), v_1^{i+1}(t)],\end{aligned}$$

$$w_1^{ki}(t) = \begin{cases} d_*^i(t), & \text{if} \quad v_1^i(t) < d_*^i(t) < v_1^{i+1}(t), \\ v_1^i(t), & \text{if} \quad d_*^i(t) \le v_1^i(t), \\ v_1^{i+1}(t), & \text{if} \quad d_*^i(t) \ge v_1^{i+1}(t), \end{cases}$$

$$d_*^i(t) = \frac{2\beta(t)\alpha_1^i(t)h(t) - \beta(t)\alpha_2^i(t) + \beta(T)\pi(T) - \psi(t)q_D^k(t)}{2\alpha_1^i(t)\beta(t)};$$

$$w_2^k(t) = \arg \max_{i\in[0,n_2-1]} G^i(w_2^{ki}(t)),$$

$$G^i(v) = -\beta_1^i(t)\beta(t)v^2 + (\beta(t)\beta_2^i(t) - \beta(T)\pi(T) + \psi(t)q_D^k(t))v, \qquad v \in [v_2^i(t), v_2^{i+1}(t)],$$

$$w_2^{ki}(t) = \begin{cases} d_{**}^i(t), & \text{if} \quad v_2^i(t) < d_{**}^i(t) < v_2^{i+1}(t), \\ v_2^i(t), & \text{if} \quad d_{**}^i(t) \le v_2^i(t), \\ v_2^{i+1}(t), & \text{if} \quad d_{**}^i(t) \ge v_2^{i+1}(t), \end{cases}$$

$$d_{**}^i(t) = \frac{\beta(t)\beta_2^i(t) - \beta(T)\pi(T) + \psi(t)q_D^k(t)}{2\beta_1^i(t)\beta(t)}.$$

Then, solving the system of canonical equations

$$\dot{\eta}(t) = g_C^k(t)z(t) - g_D^k(t)w_1(t) + g_D^k(t)w_2(t), \qquad \dot{\psi}(t) = 0,$$

we have

$$\psi(t) = c = \text{const.},$$

$$\eta(t) = \int_{t_0}^{t} (g_C^k(t)z(t) - g_D^k(t)w_1(t) + g_D^k(t)w_2(t))\, dt.$$

Substituting the obtained controls $(z^k(t), w_1^k(t), w_2^k(t))$ (depending on $\psi(t)$) into the last equality and using the boundary condition $\eta(T) = 0$, we find numerically the constant function $\psi(t)$. Then, we obtain explicit formulas for optimal controls in problem (20)–(25).

6. Results of Numerical Modeling

In the numerical experiments, system (1) was considered on the time interval $[2010, 2030]$, under the assumption that the Kyoto mechanisms are applicable on the whole interval (a so called "Kyoto Forever" scenario), i.e., under the assumption that the emissions levels (regulated by the Protocol) for the Annex B countries and the international market for emissions permits are preserved.

The unit of the stock of permits $x(t)$ was megaton of carbon equivalent (1 MtC); respectively, the controls $u(t)$ and $q(t)$ as well as the function $h(t)$ were measured in MtC per year. The dynamics of carbon dioxide CO_2 was studied. All prices were given in USD. As a forecast of the dependence of the market price for emissions permits on the amount of permits supplied for sale, under the conditions of Russia's monopoly (actually, as an estimate of the

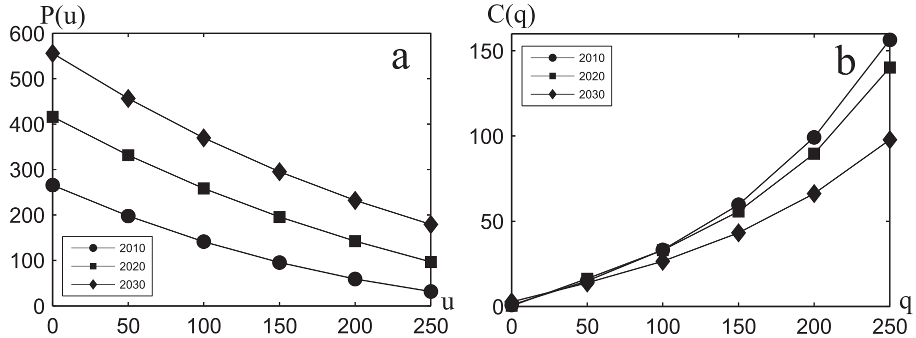

Figure 1: Input data: (a) law of demand; (b) MAC curve of Russia. Functions for 2010, 2020, and 2030 are presented.

demand for permits in the Annex B countries), the demand function $P(u(\cdot))$ from model GEMINI-E3 was chosen, see Figure 1a. This model is a general equilibrium model of the world economy, see [3]. The cost function $C(q(\cdot))$ for the marginal abatement depending on the level of abatement (a so called regional MAC curve) was taken from the same model, see Figure 1b. The linear interpolation was used between the pictured curves.

The constraints on the controls $u(t)$ and $q(t)$ were chosen as constants: $a_1(t) = a_2(t) = 0$, $b_1(t) = b_2(t) = 250$. We considered the process without discounting ($\beta = 1$), the parameter α (see (11)) was equal to 0.01, the expected price of permits at the terminal time $\pi(T)$ was equal to 0.

The main goal of the experiment was studying the dependence of the optimal dynamics of control parameters $u(t)$ and $q(t)$, the stock of permits $x(t)$, and the income obtained by Russia from operations on the market for permits on the amount of the "hot air", additionally (to the abatement) available for sale, i.e., on the function $h(t)$. Actually, the value of the function $h(t)$ is the difference between emissions at the moment t and the known emissions level of 1990 (the Kyoto level for Russia, 646 MtC). Then, using different scenarios of the economic development of Russia and applying different models forecasting the dynamics of CO_2 emissions, we obtained several scenarios of the dynamics of $h(t)$, see Table 1.

Remarks. 1. Variants (1)–(7) correspond to the following forecasts:

(1) - the reference scenario of the International Energy Outlook 2006, [12];

(2) - the forecast of the Energy Research Institute of RAS, [8];

(3) - the reference scenario of IV National Communication of RF, [13];

(4) - the innovation-active scenario of IV National Communication of RF, [13];

time	(1)	(2)	(3)	(4)	(5)	(6)	(7)
2010	155	169	155	132	85	186	300
2015	114	110	125	78	67	105	245
2020	69	52	93	19	57	41	199
2025	33	-13	60	-19	28	16	163
2030	-1	-85	25	-59	-3	6	136

Table 1: Estimates for the temporal dynamics of Russian "hot air", MtC per year

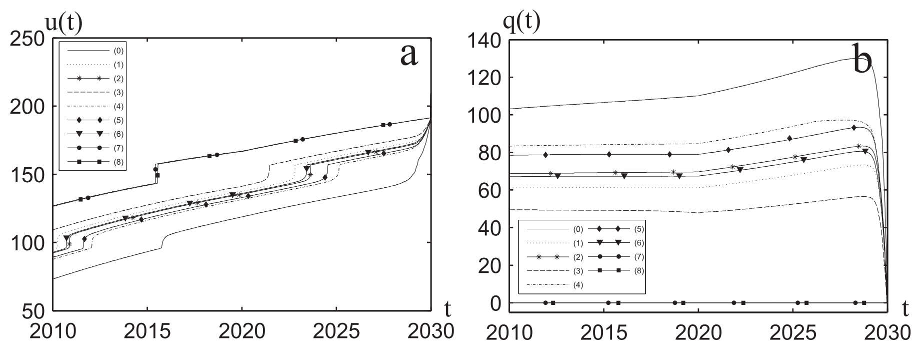

Figure 2: The temporal dynamics of (a) the amount of permits supplied for sale, $u(t)$; (b) the emissions abatement, $q(t)$. Variants (0)–(8).

(5) - the simulation results by MERGE, [9], [5];

(6) - the simulation results by EPPA, [1];

(7) - the simulation results by GEMINI-E3, [3].

2. Between the points listed in Table 1, the linear interpolation was used.

3. Negative values were replaced by zeros.

In addition to variants (1)–(7), two "extremal" cases with constant functions $h(t)$ were computed: variant (0), where $h(t) = 0$, and variant (8), where $h(t) = 163$. The temporal dynamics of the main output parameters was presented in Figures 2–3.

Note that, due to the method's error, there is a sense to limit the analysis by 2028. It is evident that the income from permits sale (see extremal problem (10)–(11)) should take a minimal (comparing with other variants) value in variant (0); this fact is confirmed by simulations. It turns out that variant (8) provides a maximally possible income over different functions $h(t)$ (in the case when rest parameters of the problem are fixed); the same result is obtained in variant (7). The maximality of the income in these variants is explained by the zero optimal value of $q(t)$ (see Figure 2b). Note that the least integer providing the maximum above was taken as the constant value of $h(t)$ in variant (8).

As is seen in Figure 2a, the amount of permits supplied by Russia for sale on the international market is varied in 2010 from 73 MtC up to 127 MtC, in 2020 from 119 MtC up to 167 MtC, in 2028 from 142 MtC up to 187 MtC (in variant (0) and in variants (7), (8), respectively). In all the variants, the amount of permits supplied for sale increases with time, and the growth rate is approximately the same (varies from 2.0% up to 3.4% per year). On the contrary, the emissions abatement is rather stable with time in all the variants (see Figure 2b; the most considerable growth is observed in variant (0)). The share of emissions reduction in the amount of permits supplied for sale is changed from evident 0% in variants (7), (8) up to 94% in 2010, 65% in 2020, and 59% in 2028 in variant (4) (with the exception of variant (0), when this index is not informative). In all the variants, it is inexpedient to use the disposable "hot air" at a time; the maximal banking of permits (the abatement is also taken into account) with the purpose of future income increase is observed in variant (6): 160 MtC in 2010 (with the exception of variants (7), (8), when this index is not informative). Note that the banking becomes possible due to the intertemporal optimization (on the whole time interval). As to the market price of permits (see Figure 3a), it rises from 116 USD/MtC in 2010 up to 229 USD/MtC in 2028 in variants (7), (8) (the minimal prices) and from 171 USD/MtC in 2010 up to 287 USD/MtC in 2028 in variant (0) (the maximal prices), in all the variants rather slowly (from 2.9% up to 3.9% per year) increasing with time.

For the comparative analysis of modeling results in variants (0)–(8), the histogram (Figure 3b), where the maximally possible income (for the whole time interval) is taken as 100%, is constructed. Analyzing the histogram, we conclude that the maximal income loss over forecasts (1)–(7) is 10.6% (in variant (4)), whereas the maximally possible loss is 20.5%. The average (over variants (1)–(7)) loss is rather small (6.2%). Hence, we can deduce that domestic resources of Russia (namely, an opportunity of relatively cheap (especially comparing with countries of European Union and Japan) emissions reduction in Russia due to incomplete realization of the energy effectiveness and energy saving potential) provide a considerable income from permits sale even in the case of unfavorable situation with the "hot air". It turns out that the dependence of the value of this income on the function $h(t)$ is not so essential, as one can suppose, when analyzing mathematical model (1)–(11).

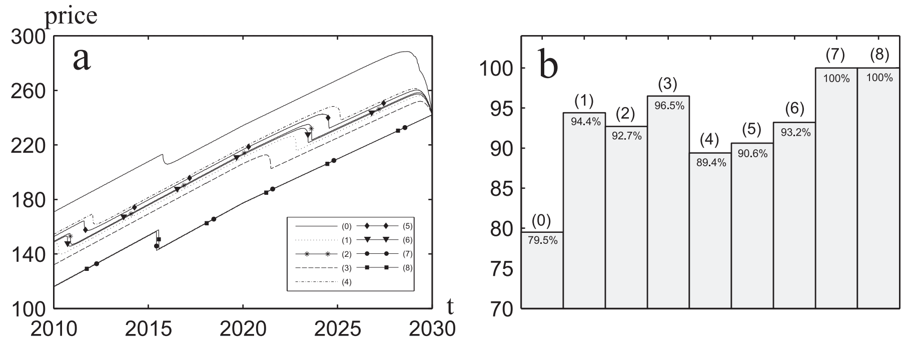

Figure 3: Modeling results: (a) the temporal dynamics of the price of permits; (b) the Russian income from permits sale (in % from the maximally possible income in the model). Variants (0)–(8).

7. Concluding Remarks

It should be noted that there is a high level of uncertainty in the specification of parameters of the model in question. In the paper, several scenarios forecasting the temporal dynamics of the "hot air" were studied. It is reasonable to consider the analysis of the dependence of optimal strategies of Russia's behavior on the international market for permits on variation of different model parameters (in particular, the functions presented in Figure 1) as one of the basic perspective directions in modeling.

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References

[1] M.H. Babiker, J.M. Reilly, M. Mayer, R.S. Eckaus, I. Sue Wing, R.C. Hyman, The MIT emissions prediction and policy analysis (EPPA) model: Revisions, sensitivities, and comparisons of results, *MIT Joint Program on*

the Science and Policy of Global Change, Report 71, Cambridge, MA, USA (2000).

[2] A. Bernard, A. Haurie, M. Vielle, L. Viguier, A two-level dynamic game of carbon emissions trading between Russia, China, and Annex B countries, *NCCR-WP4 Working Paper 11*, Swiss National Centre of Competence, University of Geneva (2002).

[3] A. Bernard, J. Reilly, M. Vielle, L. Viguier, Russia's role in the Kyoto Protocol, *Proceedings of the Annual Meeting of the International Energy Workshop Jointly Organized by EMF/IEA/IIASA*, Stanford University, USA (2002).

[4] P. Criqui, L. Viguier, Kyoto and technology at world level: costs of CO_2 reduction under flexibility mechanisms and technical progress, *International Journal of Global Energy Issues*, **14** (2000), 155-168.

[5] B. Digas, V. Rozenberg, Ya. Minullin, Computer modeling of economic consequences of Russia's participation in the Kyoto Protocol, In: *Proceedings of the VI International Conference on System Identification and Control Problems (SICPRO-2007)*, Moscow (2007), 1115-1128.

[6] B. Grimm, S. Pickl, A. Reed, Management and optimization of environmental data within emissions trading markets - VEREGISTER and TEMPI, In: *Emissions Trading and Business* (Ed-s: R. Antes, B. Hansjürgens, P. Letmathe), Physica-Verlag HD (2006), 165-176.

[7] A. Kryazhimskii, A. Ruszczyński, Constraint aggregation in infinite-dimensional spaces and applications, *IIASA Interim Report IR-97-051*, Laxenburg, Austria (1997).

[8] A. Makarov, V. Likhachev, Mitigation measures and policies from Russian perspective, In: *Proceedings of the Workshop on Post-2012 Climate policies*, IIASA, Laxenburg, Austria (2006).

[9] A. Manne, R. Mendelson, R. Richels, MERGE - a Model for Evaluating Regional and Global Effects of GHG reduction policies, *Energy Policy*, **23**, No. 1 (1995), 17-34.

[10] L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko, *Mathematical Theory of Optimal Processes*, Nauka, Moscow (1969).

[11] F.P. Vasil'ev, *Methods of Solution of Extremal Problems*, Nauka, Moscow (1981).

[12] *International Energy Outlook 2006*, Energy Information Administration, www.eia.doe.gov/oiaf/ieo/index.html.

[13] *The Fourth National Communication of the Russian Federation*, (Eds: Yu.A. Israel, A.I. Nakhutin), ANO "Meteoagentstvo Rosgidrometa", Moscow (2006).