International Journal of Pure and Applied Mathematics
Volume 40 No. 1 2007, 21-31

ON AN EMPIRICAL BAYES TEST FOR TRUNCATION PARAMETERS USING NA SAMPLES

Bao-Li Liu[1], Hui-Guang Kang[2], Yong Xu[3] §

[1]Department of Computer Science and Technology
Northwestern Polytechnical University
Xi'an, Shaanxi, 710072, P.R. CHINA

[2]Department of Mathematics
Anyang Teachers College
Anyang, Henan, 455000, P.R. CHINA

[3]Department of Applied Mathematics
Northwestern Polytechnical University
Xi'an, Shaanxi, 710072, P.R. CHINA
e-mails: hsux3@163.com, hsux3@263.net

Abstract: An empirical Bayes test (EBT) is considered in the two-side truncation distribution family using NA samples. The kernal estimation of probability density is adopted to construct the EBT, and under some certain conditions the proposed EBT is asymptotic optimal. Finally, an example satisfying conditions of theorem is given.

AMS Subject Classification: 62C12
Key Words: NA samples, empirical Bayes test (EBT), Bayes risk, kernel function, asymptotic optimality

1. Introduction

Empirical Bayes (EB) approach was introduced by Robbins (1956), and up to now a lot of investigators have examined empirical Bayes estimation (EBE) or empirical Bayes test (EBT) in the case of i.i.d. samples, see [1], [2], [9], [8], [12], [3], [13], [17], [10],. However, in reliability, penetration theory and multivariable

Received: July 25, 2007

§Correspondence author

analysis the random samples are not always but negatively associated (NA). Recently, Xu and Shi [18], [19], [20], [16], [15], explored the EBT or EBE for one-side truncation parameter in the case NA samples, and Wei [4] considered the EBT for exponential distribution family.

Consider the truncation parameter with p.d.f. of the following form

$$f(x|\theta) = u(x)A(\theta)I_{[\theta,m\theta]}(x)\,, \tag{1}$$

where $m > 1$ is a constant, and $u(x)$ is positive, integrable and bounded with lower bound strictly greater than 0 on $[\theta, m\theta]$, $A(\theta) = [\int_\theta^{m\theta} u(x)dx]^{-1}$, $\theta \in (0, +\infty)$. The truncation parameter θ of our interest has a prior distribution $G(\theta)$ with p.d.f. $g(\theta)$ over $\theta \in (0, +\infty)$. Then the marginal density of X is

$$f(x) = \int_0^\infty f(x\,|\theta\,)dG(\theta) = u(x)\int_{x/m}^x A(\theta)g(\theta)d\theta \triangleq u(x)v(x),$$

and let $f(x)$ be bounded and have$s$th bounded derivative with $s \geq 1$ a given natural number.

This paper is to construct the EBT with asymptotic optimality in the two-side truncation distribution family under NA samples. Firstly, we introduce the definition of NA sequence by Joag-Dev and Proschan [7].

Definition 1.1. Random variables $X_1X_2\cdots X_n (n \geq 2)$ are said to be NA if for every pair of disjoint subsets T_1 and T_2 of $\{1, 2, 3, \cdots, n\}$, $\mathrm{Cov}\,(f_1(X_i, i \in T_1), f_2(X_j, j \in T_2)) \leq 0$, where f_1 and f_2 are increasing or decreasing for every variable such that this covariance exists. Random variables sequence $\{X_i, i \in N\}$ are said to be NA if for every natural number $n \geq 2$, $X_1X_2\cdots X_n$ are negatively associated.

The hypothesis to be tested is

$$H_0 : \theta \leq \theta_0 \leftrightarrow H_1 : \theta > \theta_0. \tag{2}$$

The loss function is taken to be

$$L_j(\theta, d_j) = (1-j)a(\theta-\theta_0)I_{(\theta-\theta_0>0)} + ja(\theta_0-\theta)I_{(\theta_0-\theta\geq 0)}, \quad j = 0, 1\,, \tag{3}$$

where a is a positive constant, $D = \{d_0, d_1\}$is the action space, d_i denotes the acceptation of H_i, $i = 0, 1$, I_A denotes indicative function.

The loss function is a weighted linear loss function, and this paper is to construct the EBT under NA samples. The paper is organized as follows. We construct an EBT for the truncation parameters using NA samples and investigate the asymptotic optimality of the proposed EBT in Section 2. In Section 3 some lemmas are given to complete the proofs in Section 4. At last we give an example satisfying the conditions of the theorems.

2. Empirical Bayes Test

Let $r(x) = P(\text{accept} H_0 | X = x)$ be a randomized decision rule. Then the Bayes risk of $r(x)$ can be written as,

$$R(r, G) = \int_\Omega \int_\Delta \{L(\theta, d_0) r(x) + [1 - r(x)] L(\theta, d_1)\} f(x|\theta) dG(\theta) dx$$
$$= \int_\Omega Q(x) r(x) dx + C_G \, ,$$

where $\Omega = (0, +\infty), \Delta = [x/m, x]$,

$$C_G = \int_\Omega \int_\Delta L(\theta, d_1) f(x|\,\theta) dG(\theta) dx.$$

$$Q(x) = \int_{\Delta \bigcap (\theta_0, \infty)} (\theta - \theta_0) f(x\,|\theta) dG(\theta) - \int_{\Delta \bigcap (0, \theta_0)} (\theta_0 - \theta) f(x\,|\theta) dG(\theta)$$
$$= \int_{x/m}^{x} \theta f(x\,|\theta) dG(\theta) - \theta_0 f(x) = v_2(x) - \theta_0 f(x),$$

where $v_2(x) = u(x) \int_\Delta \theta A(\theta) g(\theta) d\theta$.

Therefore the Bayes decision rule is

$$r_G(x) = \begin{cases} 1\,, & Q(x) \le 0, \\ 0\,, & Q(x) > 0. \end{cases}$$

It is easy to show that $r_G(x)$ is a Bayes test with respect to $G(\theta)$. Since $G(\theta)$ is unknown and $r_G(x)$ cannot be applied, we introduce the EB approach. Suppose $(X_1, \theta_1), \cdots, (X_n, \theta_n)$, (X, θ) are identically distributed samples, where $X_1, \cdots, X_n$ (past samples) and X(present samples) have the same density function $f(x\,|\theta)$; θ_i and θ have the same prior distribution $G(\theta)(i = 1, 2, 3, \cdots)$. Let $X_1, X_2, \cdots, X_n (n \ge 2)$ be identically distributed NA sequence, $f(x)$ be density function and $f(x) \in C_{s,\alpha},\ x \in R^1$, where $C_{s,\alpha}$ denotes a family of probability density function with s times derivatives and its absolute value not exceeding α, $\alpha \in Z^+$,$Z^+$is the set of the positive integers, $s > 1$, $s \in N$. It is assumed in this paper that NA sequence has the property:

$$(C)\sum_{i,j=1}^{\infty}[-\mathrm{Cov}\,(X_i,X_j)]<\infty.$$

Let $Q_n(x)=Q_n(x_1,\cdots,x_n;x)$ be the estimator of $Q(x)$. We define the EBT of $r_G(x)$ by

$$r_n(x)=\begin{cases}1, & \text{if } Q_n(x)\le 0,\\ 0, & \text{otherwise.}\end{cases} \tag{4}$$

Then the over all Bayes risk of $r_n(x)$ is

$$R_n \triangleq R_n(r_n(x),G)=E_n\int Q(x)r_n(x)dx+C_G\,, \tag{5}$$

where E_n stands for the expectation with respect to the joint distribution of $(X_1,\cdots,X_n)$.

For any $G(\theta)\in F^1$, the EBT $r_n(x)$ is said to be asymptotically optimal (a.o) if $R_n\to R_G$ as $n\to\infty$. Moreover if for a $q>0$, $R_n-R_G=O(n^{-q})$, then the EBT $r_n(x)$ is said to be asymptotically optimal with convergence rate of $O(n^{-q})$, where F^1 is the prior distribution family of θ, $R_G=R(r_G,G)=\inf_r R(r,G)$ is the Bayes risk of $r_G(x)$.

We can define the estimator of $f(x)$ by

$$f_n(x)=(nh)^{-1}\sum_{i=1}^{n}K\Big(\frac{X_i-x}{h}\Big)\,, \tag{6}$$

where $h=h_n(>0)$, and $\lim_{n\to\infty}h_n=0$. $K(\cdot)$ is a kernel function, satisfying the following conditions:

(1) $K(y)=0$, if $y\notin(0,1)$.

(2) $|K(y)|\le M$ for all y, M is a positive number.

(3) $\int y^tK(y)dy=\begin{cases}1, & \text{as } t=0,\\ 0, & \text{as } t=1,2,\cdots,s-1.\end{cases}$

Assume that the following conditions hold (for all $x>0$),

(A1) $\sum_{j=1}^{\infty}xv(m^jx)<\infty$,

(A2) $m^jA(m^jx)g(m^jx)\to 0 \quad (j\to\infty)$, (A3) $\sum_{j=1}^{\infty} f^{(s)}(m^j x) < \infty,\ \sum_{j=1}^{\infty} f(m^j x) < \infty,\ \ x\sum_{j=1}^{\infty} f^{(s)}(m^j x) < \infty,$

(A4) $\sum_{j=1}^{\infty} \int_{x/m}^{x} x f^{(s)}(m^j\theta)d\theta < \infty,$

where $v(m^j x) = \int_{m^{j-1}x}^{m^j x} A(\theta)g(\theta)d\theta = f(m^j x)[u(m^j x)]^{-1}$.

Noting (A2) and

$$\frac{d}{dx}v(m^j x) = m^j A(m^j x)g(m^j x) - m^{j-1}A(m^{j-1}x)g(m^{j-1}x),$$

we get

$$A(x)g(x) = -\sum_{j=1}^{\infty} \frac{dv(m^j x)}{dx}.$$

Therefore

$$v_2(x) = u(x)\int_{x/m}^{x} \theta A(\theta)g(\theta)d\theta = u(x)\int_{x/m}^{x} -\sum_{j=1}^{\infty} \theta\frac{dv(m^j\theta)}{d\theta}d\theta.$$

We need the following lemma to simplify $v_2(x)$.

Lemma 2.1. (see [5]) *Let $\{f_n\}$ be a sequence of measurable functions on (Ω, F_1, μ), where (Ω, F_1, μ) is a measurable space, and μ is a σ-finite measure. If either $\sum_{n=1}^{\infty} \int_\Omega f_n^+ d\mu < \infty$ or $\sum_{n=1}^{\infty} \int_\Omega f_n^- d\mu < \infty$, then $\sum_{n=1}^{\infty} f_n$ is integral for μ (i.e. $\int_\Omega \sum_{n=1}^{\infty} f_n d\mu < \infty$), and $\int_\Omega \sum_{n=1}^{\infty} f_n d\mu = \sum_{n=1}^{\infty} \int_\Omega f_n d\mu$, where f_n^+ and f_n^- stand for the positive and negative parts of the f_n^+ respectively. Since*

$$\sum_{j=1}^{\infty}\int_{x/m}^{x} \theta[\frac{dv(m^j\theta)}{d\theta}]d\theta$$
$$= -xv(x)/m + (1-1/m)x\sum_{j=1}^{\infty} v(m^j x) - \sum_{j=1}^{\infty}\int_{x/m}^{x} v(m^j\theta)d\theta\,.$$

From the condition (A1):

$$\sum_{j=1}^{\infty}\int_{x/m}^{x} [\theta\frac{d}{d\theta}v(m^j\theta) + d\theta = (1-1/m)x\sum_{j=1}^{\infty} v(m^j x) < \infty$$

By Lemma 2.1, we get $\int_{x/m}^{x} \sum_{j=1}^{\infty} \theta\, dv(m^j\theta) = \sum_{j=1}^{\infty} \int_{x/m}^{x} \theta \frac{d}{d\theta} v(m^j\theta)\ d\theta$. Hence

$$v_2(x) = -u(x) \sum_{j=1}^{\infty} \int_{x/m}^{x} [\theta \frac{dv(m^j\theta)}{d\theta}] d\theta$$

$$\triangleq u(x)[xv(x)/m - (1-1/m)xT(x) + \psi_2(x)]\,,$$

where $T(x) = \sum_{j=1}^{\infty} f(m^j x)[u(m^j x)]^{-1}$, $\psi_2(x) = \sum_{j=1}^{\infty} \int_{x/m}^{x} f(m^j\theta)[u(m^j\theta)]^{-1} d\theta$.

We define the estimation of $v_2(x)$ as $v_{2n}(x)$, namely,

$$v_{2n}(x) = (x/m)f_n(x) - u(x)(1-1/m)xT_n(x) + u(x)\psi_{2n}(x)\,,$$

where $f_n(x)$ is defined by (6), and

$$T_n(x) = \sum\nolimits_{j=1}^{\infty} f_n(m^j x)[u(m^j x)]^{-1},$$

$$\psi_{2n}(x) = \sum_{j=1}^{\infty} \int_{x/m}^{x} f_n(m^j\theta)[u(m^j\theta)]^{-1} d\theta,$$

Therefore, the estimator of $Q(x)$ can be constructed as follows,

$$Q_n(x) = v_{2n}(x) - \theta_0 f_n(x).$$

For the EBT $r_n(x)$, we claim the following theorem.

Theorem. *Suppose the conditions (A1) $\sim$ (A4) hold and $f(x)$ is s times differentiable, $\sup_x f(x) < \infty$, $\sup_x \left|f^{(s)}(x)\right| < \infty$. Then as $h_n = n^{-1/(2s+1)}$, we have $\lim_{n\to\infty} R_n = R_G$.*

3. Some Lemmas

In this section it is assumed that M is a positive constant in different cases even in the same expression.

Now we introduced the following lemmas to complete the proof of theorem.

Lemma 3.1. *Let R_n and R_G be defined above, then*

$$0 \le R_n - R_G \le a \int |Q(x)|\, P\left(|Q_n(x) - Q(x)| \ge |Q(x)|\right) dx\,.$$

Proof. See Lemma 1 in Johns and Van Ryzin [14]. □

Lemma 3.2. *Let X and Y be NA variables with finite covariance for differentiable functions g_1 and g_2 ,*

$$|\mathrm{Cov}\,(g_1(X), g_2(X))| \leq \sup_X \left|g_1'(X)\right| \sup_Y \left|g_Y'(Y)\right| [-\mathrm{Cov}\,(X,Y)].$$

As g_1 and g_2 are not differentiable on finite or countable set E_0^1 and E_0^2, we have

$$|\mathrm{Cov}\,(g_1(X), g_2(X))| \leq \sup_{X\in R^1\setminus E_0^1} \left|g_1'(X)\right| \sup_{Y\in R^1\setminus E_0^2} \left|g_Y'(Y)\right| [-\mathrm{Cov}\,(X,Y)].$$

Proof. See Lemma 2.1 in [6]. □

Lemma 3.3. *Let $f_n(x)$ be defined by (1.3), $\sup_x |f(x)| < \infty$, $\sup_x \left|f^{(s)}(x)\right| < \infty$, then for every $0 < \delta < 1$,*

$$E_n\,|f_n(x) - f(x)| \to 0 \;\; (n \to \infty), \;\text{as}\; nh^4 \to \infty.$$
$$|E_n f_n(x) - f(x)| \leq M n^{-s/(2s+1)} f^{(s)}(x).$$
$$E_n\,|f_n(x) - f(x)|^{2\delta} \leq M n^{-\frac{\delta s}{s+1}} [\{f(x)\}^\delta + \{\left|f^{(s)}(x)\right|\}^{2\delta}] \qquad (2)$$

when $h = n^{-\frac{1}{2(s+1)}}$.

Proof. (1) $E_n\,|f_n(x) - f(x)|^2 \leq (\mathrm{Var}\,[f_n(x)]) + |E_n f_n(x) - f(x)|^2$. But

$$\mathrm{Var}\,[f_n(x)] = (nh)^{-2}\mathrm{Var}\,[\sum_{i=1}^n k(\frac{X_i - x}{h})]$$
$$= (nh)^{-2}\{\sum_{i=1}^n \mathrm{Var}\,[k(\frac{X_i - x}{h})] + 2\sum_{i<j} \mathrm{Cov}\,(k(\frac{X_i - x}{h}), k(\frac{X_j - x}{h}))\}$$
$$\leq (nh)^{-2}\{nE[k(\frac{X_i - x}{h})]^2 + 2\sum_{i<j} \mathrm{Cov}\,(k(\frac{X_i - x}{h}), k(\frac{X_j - x}{h}))\},$$

where $(nh)^{-2}nE[k(\frac{X_i-x}{h})]^2 = (nh)^{-1}\int_0^1 f(x+hu)k^2(u)du \leq M(nh)^{-1}$.

From Lemma 3.2 we get $(nh)^{-2}\sum_{i<j} \mathrm{Cov}\,(k(\frac{X_i-x}{h}), k(\frac{X_j-x}{h})) \leq M(nh^4)^{-1}$. Then $\mathrm{Var}\,[f_n(x)] \to 0,\; nh^4 \to \infty$.

Note that

$$|E_n f_n(x) - f(x)| = \frac{h^s}{s!}\int_0^1 f^{(s)}(x + hu\xi)k(u)u^s du$$
$$\leq Mh^s \text{and } h_n \to 0\,(n \to \infty)\,,$$

then

$$|E_n f_n(x) - f(x)| \to 0, (n \to \infty).$$

Therefore $E_n\,|f_n(x) - f(x)| \to 0 \quad (n \to \infty)$ as $nh^4 \to \infty$.

(2) Considering Lemma 3.1 in [11], condition (C) and C_r inequality, one arrives at

$$E_n\,|f_n(x) - f(x)|^{2\delta} \leq M\{(\mathrm{Var}\,[f_n(x)])^\delta + |E_n f_n(x) - f(x)|^{2\delta}\}\,.$$

Then

$$E_n\,|f_n(x) - f(x)|^{2\delta} \leq Mn^{-\frac{\delta s}{s+1}} \mathrm{as} h = n^{-\frac{1}{2(s+1)}}.$$

The lemma is proved. □

Lemma 2.4. *Let (A1)∼(A4) and the conditions of Lemma 3.2 hold and* $h_n = n^{-1/(2s+1)}$. *Then for any* $0 < \delta \leq 1$,

$$|E_n v_{2\,n}(x) - v_2(x)| \leq Mn^{-s/(2s+1)}\,.$$

Proof. Note that (A1-A4) and Lemma 3.2-3.3, one has

$$\begin{aligned}
|E_n v_{2\,n}(x) - v_2(x)| &\leq M|E_n x f_n(x) - x f(x)| \\
&\quad + M|E_n x T_n(x) - xT(x)| + M|E_n \psi_{2n}(x) - \psi_2(x)| \\
&\leq M\{|E_n x f_n(x) - x f(x)| + \sum_{j=1}^{\infty} \frac{\left|E_n x f_n(m^j x) - x f(m^j x)\right|}{u(m^j x)} \\
&\quad + \sum_{j=1}^{\infty}\int_{x/m}^{x} \frac{\left|E_n f_n(m^j\theta) - f(m^j\theta)\right|}{u(m^j\theta)} d\theta\} \\
&\leq M\{x f^{(s)}(x) + \sum_{j=1}^{\infty} x f^{(s)}(m^j x) + \sum_{j=1}^{\infty}\int_{x/m}^{x} f^{(s)}(m^j\theta)d\theta\}n^{-s/(2s+1)} \\
&\leq Mn^{-s/(2s+1)}.
\end{aligned}$$

The proof is completed. □

4. Proof of Theorem

By Lemma 3.1 and Markov inequality, we get

$$0 \le R_n - R_G \le \int |Q(x)| P\{|Q_n(x) - Q(x)| \ge |Q(x)|\} \, dx \triangleq \int B_n(x) dx.$$

$$\begin{aligned} B_n(x) &= |Q(x)| P\{|Q_n(x) - Q(x)| \ge |Q(x)|\} \\ &\le |Q(x)| E_n |Q_n(x) - Q(x)| / |Q(x)| \\ &\le M\{E_n \mid f_n(x) - f(x) \mid + E_n \, |v_{2n}(x) - v_2(x)|\} \, . \end{aligned}$$

By Lemmas 3.3-3.4, we get $E_n|Q_n(x) - Q(x)| \le Mn^{-s/(2s+1)}$.

Therefore $0 \le B_n(x) \le Mn^{-s/(2s+1)}$. Using the Dominated Convergence Theorem, we arrive at

$$\lim_{n\to\infty} \int B_n(x) dx = \int \lim_{n\to\infty} B_n(x) dx = 0.$$

So that $\lim\limits_{n\to\infty} R_n - R_G = 0$. End of the proof.

Remark. From Theorem, one can find that the proposed EB test is asymptotical optimal, and its convergence rate will be obtained in our future work.

5. An Example

Let $f(x|\theta) = \frac{1}{\theta} I_{[\theta, 2\theta]}(x)$, $g(\theta) = \theta \ e^{-\theta} I_{(0,\infty)}(\theta)$. Then $u(x) = 1$, $A(\theta) = \frac{1}{\theta}$,$\theta \in (0, +\infty)$. So

$f(x) = v(x) = e^{-x/2} - e^{-x}$, straightforward calculating can yield:

(A1) $\sum\limits_{j=1}^{\infty} x[e^{-2^{j-1}x} - e^{-2^j x}] < \infty$,

(A2) $m^j A(m^j x) g(m^j x) \to 0 \quad (j \to \infty)$,

(A3) $\sum\limits_{j=1}^{\infty} f^{(s)}(m^j x) < \infty$, $\sum\limits_{j=1}^{\infty} f(m^j x) < \infty$,$x \sum\limits_{j=1}^{\infty} f^{(s)}(m^j x) < \infty$,

(A4) $\sum\limits_{j=1}^{\infty} \int_{x/m}^{x} x f^{(s)}(m^j \theta) d\theta < \infty$,

we can check that (A1)$\sim$(A4) and all conditions of Theorem.

Acknowledgements

This work was partly supported by NPU Youth Foundation.

References

[1] S.Y. Huang, Empirical Bayes testing procedures in some nonexponential families using asymmetric Linex Loss function, *J. Statist. Plann. Inference*, **46** (1995), 293-309.

[2] S.Y. Huang, T.C. Liang, Empirical Bayes estimation of the truncation parameter with linex loss, *Statistica Sinica*, **7** (1997), 755-769.

[3] T.Z. Hu, et al, Convergence rates for empirical Bayes two-action problems in scale exponential family, *Mathematical Statistics and Applied Probability*, **7**, No. 1, 86-96.

[4] L.S. Wei, The empirical Bayes test problem for scale exponential family: In the case of NA samples, *Acta Mathematicae Applicatae Sinica*, **23**, No. 3 (2000), 403-412.

[5] H. Liang, The convergence rate of estimation for two-dimensional two-sided truncation distribution families, *Mathematical Statistics and Applied Probability*, **4** (1991), 433-447, In Chinese.

[6] C. Su et al, Moment inequalities and weak convergence for negatively associated sequences, *Sciences in China, Series A*, **26**, No. 12 (1996), 1091-1099.

[7] K. Joag-Dev, F. Proschan, Negative association of random variables with application, *Ann. Stasist.*, **11** (1983), 286-295.

[8] T.C. Liang, Monotone empirical Bayes tests for a discrete normal distribution, *Staist. Prob. Lett.*, **44** (1999), 241-149.

[9] T.C. Liang, On empirical Bayes tests in a positive exponential family, *J. Statist. Plann. Inference*, **83** (2000), 169-181.

[10] Y.M. Ma, N. Balakarishnan, Empirical Bayes estimation for truncation parameters, *J. Statist. Plann. Inference*, **84** (2000), 111-120.

[11] R.J. Karunamuni, Optimal rates of convergence of empirical Bayes tests for the continuous one-parameter exponential family, *The Annals of Statistics*, **24**, No. 1 (1996), 212-231.

[12] Y.M. Shi, Empirical Bayes estimation for parameter of two-sided truncated distribution under Linex loss function, *Appl. Math. J. Chinese Univ. Ser. A*, **15**, No. 4 (2000), 475-483.

[13] R.S. Singh, L.S. Wei, Empirical Bayes with rate and best possible rates of convergence in-family: Estimation case, *Ann. Inst. Statist. Math.*, **44** (1992), 435-439.

[14] V.R. Johns, Convergence rates in empirical Bayes two-action problems: Continuous case, *Ann. Math. Statist.*, **43** (1972), 934-947.

[15] Y.S. Xu, Y. Xu, Y.M. Shi, Convergence rates of empirical Bayes test for one-side truncation parameters with asymmetric loss functions, *International Journal of Pure and Applied Mathematics*, **27**, No. 1 (2006), 21-29.

[16] Y. Xu, Y.M. Shi, Empirical Bayes test for truncation parameter with na samples, *Mathematicae Applicatae*, **14**, No. 4 (2001), 98-102.

[17] Y. Xu, Y.M. Shi, Empirical Bayes test for truncation parameters using linex loss, *Bulletin of the Institute of Mathematics, Academia Sinica*, **3** (2004), 207-220.

[18] Y. Xu, X.L. Shi, Y.M. Shi, The EB estimation of parameter in truncated family with LINEX loss using NA Samples, *Chinese Journal of Mathematics*, **2** (2004), 124-130.

[19] Y. Xu, S.L. Wang, Y.M. Shi, Empirical Bayes estimation for truncation parameters using NA samples, *Chinese Quarterly Journal of Mathematics*, **17**, No. 4 (2002), 43-47.

[20] J.F. Zhao, D.Q. Chen, Y. Xu, Y.M. Shi, Empirical Bayes test for one-side truncation parameters with asymmetric loss functions using NA samples, *International Journal of Pure and Applied Mathematics*, **27**, No. 1 (2006), 11-20.