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HIERARCHICAL VORTICAL STRUCTURES IN A HOMOGENEOUS ISOTROPIC TURBULENT FLOW

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Abstract: Hierarchical vortices are extracted from a forced homogeneous isotropic turbulent field, and an automatic tracking of each vortex is attempted. The skeleton of each vortex is also defined by connecting the geometric centers of its structure. The Fourier low-pass filter is applied to the velocity field to extract structures of different scales. The automatic vortex-tracking scheme shows the capability of handling vortices even when they split into pieces, and the shapes of extracted skeleton agree well with those of the vortices.

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1. Introduction

Turbulence is a complicated and disordered fluid motion. Vortex motions of various scales appear one after another in a turbulent flow. They strongly affect each other before they disappear. The statistical aspects of these vortex motions have been discussed by Kolmogorov more than half a century ago. Thanks to the remarkable progress of computer technology, nowadays, a turbulent flow where the Taylor Reynolds number Re_λ is in the order of 10^3 can be handled by

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the direct numerical simulation (DNS), see [2]. However, the discussions on the topology and vortex dynamics of vortical structures are still mainly limited to the statistical approaches, instead of directly treating the individual structures. It is because a large-scale vortex exists as a group of fine-scaled filament vortices, and treating groups of vortices is not easy. The relations between structures of various scales are very complicated which make the understanding of turbulence more troublesome.

Our motivation is to extract vortical structures of different sizes and to highlight the characteristics of the large-scale vortical structures and their interaction with the fine-scale structures. In this study, the Fourier low-pass filter is applied to the velocity field to extract vortical structures of various scales from a homogeneous isotropic turbulent field. Then, the growth of extracted vortices is investigated by automatically tracking their motions and the central axis of each vortex (skeleton) is extracted.

2. Numerical Method

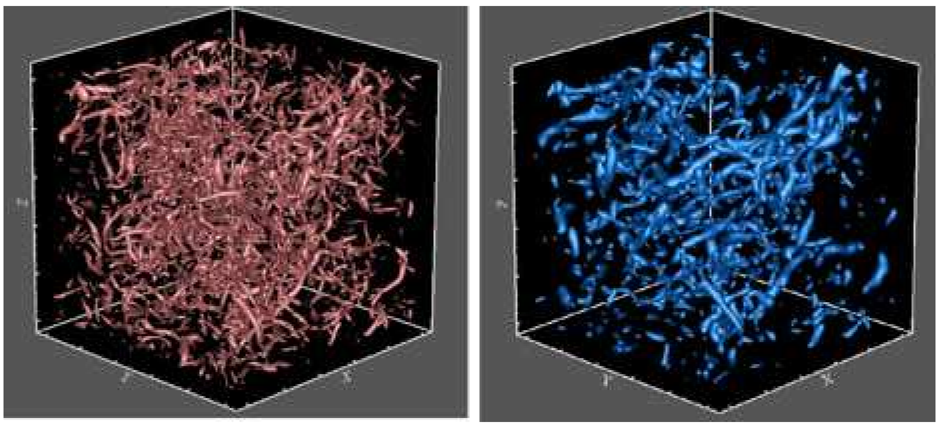
The target flow field is calculated by the Lattice Boltzmann Method (LBM) with an additional forcing term (see [4]) to obtain a statistically stationary three-dimensional, homogeneous and isotropic turbulence in a periodic box, $2\pi \times 2\pi \times 2\pi$, whose resolution is $256\times256\times256$. The energy is injected to the low wavenumbers, $0 < |\boldsymbol{k}| \leq 3$, where $\boldsymbol{k}$ represents the three dimensional wave number. The forcing takes place every time-step to excite a statistical stationary velocity field and the forcing scheme clearly satisfies the continuity equation. Initial condition of the velocity field is $\boldsymbol{u}(\boldsymbol{x}, 0) = (0,\ 0,\ 0)$ for all positions $\boldsymbol{x}$. Thus, the entire flow field is excited by the forcing scheme.

Characteristic quantities of stationary turbulence at $t/T_e = 420$ are shown in Table 1. Here, T_e is the large-eddy turnover-time defined by $T_e = l/u'_{\mathrm{rms}}$, l is the integral scale and u'_{rms} is the rms value of the velocity fluctuation. The Reynolds number Re_λ based on the Taylor microscale λ is approximately 180 with $k_{\mathrm{max}}\eta \simeq 1.2$, where k_{max} is the upper limit wavenumber, η the Kolmogorov length scale given by $\eta = (\nu^3/\varepsilon)^{1/4}$, and ν and ε are the kinematic viscosity and the energy dissipation rate.

The vortical structures are visualized using the Q method; Q is the second invariant of the characteristic equation of velocity gradient tensor. The threshold value of Q is determined so that the extracted structures will occupy 3% of the total volume of the computational domain. Q is non-dimensionalized by the averaged enstrophy.

N^3	ν	Re_λ	ε	l	λ	η
256	0.001	180	0.12	1.01	0.25	0.0096

Table 1: Characteristic quantities of turbulence



(a) non-filtered field (b) filtered field ($k_c = 16$)

Figure 1: Isosurfaces of Q with threshold 6.0(a) and 5.7(b).

3. Results and Discussion

Figure 1 shows the vortical structures before and after filtering. The Fourier decomposition is applied against the velocity field using a sharp cutoff low-pass filter with the cutoff wavenumber $k_c = 16$. This approach is basically the same as Tanahashi et al [3]. The flow field is filled with structures of almost the same scale, which look like fine strings entangled to each other. Large-scaled vortex structures cannot be observed in the non-filtered field (frame (a)), while the scale of vortical structure becomes larger and the number of vortices becomes smaller in the filtered field (frame (b)).

Figures 2 (a-1)-(a-3) show three isolated structures picked up from the large-scale flow field (k_c =16) using the Fourier filter. The corresponding structures in the fine-scale field ($k_c = 64$) that share the same grid points with the selected large-scale structures are also shown (b-1)-(b-3). In the fine-scale flow field, some small and thin filament vortices sharing the same location as the large-scale vortex can be observed. These fine vortices mainly appear at the places where the large vortex is folded or kinked. This figure supports the idea that the fine-scaled vortical structures contribute to the low wavenumber energy

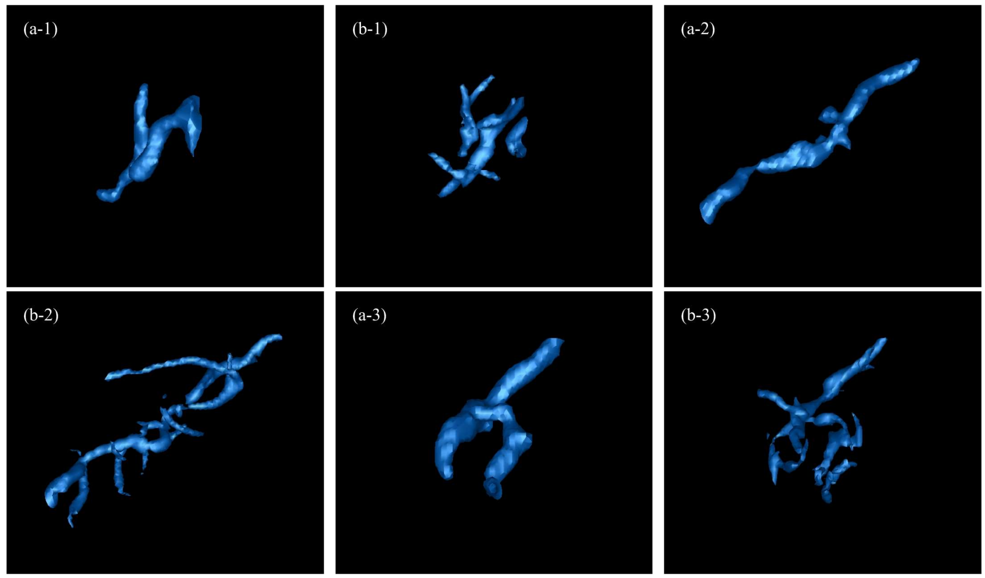


Figure 2: Examples of the isolated large vortices (a-1)-(a-3) at $k_c = 16$ and the corresponding structures in the fine-scale field (b-1)-(b-3) at $k_c = 64$

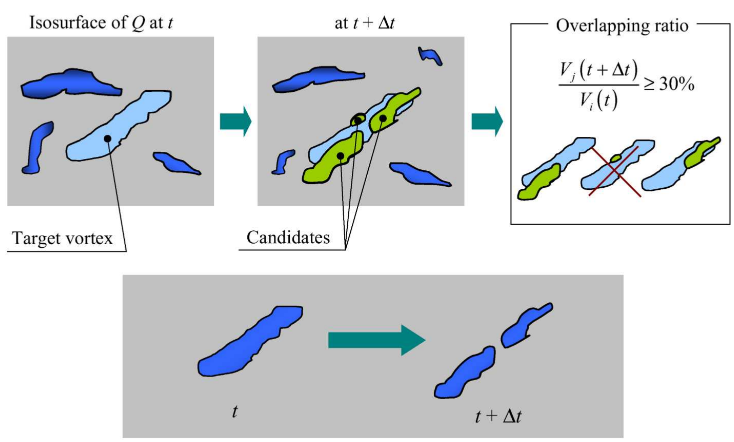


Figure 3: Automatic tracking procedure

components of velocity spectra by forming groups or clusters.

An automatic tracking of each vortex in the turbulent flow field is attempted. Figure 3 shows the automatic tracking procedure:

1. Find the maximum Q in the flow field.

2. Collect all neighboring points whose Q values are higher than the threshold and regard the group as one vortex.

3. Remove the Q values from the grid points within this vortex.

4. Repeat the procedures (1) to (3) until the maximum Q is lower than a given value.

5. Apply the procedure to the data set of the next time step.

6. The vortices that share the same grid points between two time steps are picked up.

7. If their overlapping ratios are larger than 30%, the couple are regarded as an identical vortex existing in different time.

Figure 4 shows the automatic tracking of one vortex in the filtered flow field at $k_c = 26$. In this study, if the number of grid points inside one vortex is less than a certain threshold, 52 for $k_c = 26$, i.e. if the volume is too small, the group of grid points is regarded as noise and removed from the flow field before tracking takes place. Target vortex consists of some tube-like structures, which are partly connected (frame (a)). It can be observed that some parts of the vortex are stretched and folded (frame (b) and (c)), and finally disappear (frame (d)).

The skeletons of the vortices shown in Figure 1(b) are shown in Figure 5. The centerline of a vortex is defined as the connected lines, each connecting the geometric points of its structure. As shown in the figure, the shape of each skeleton is quite similar to the vortical structure itself, even when the vortex is sharply curved. It is also found that the vorticity vectors inside a vortical structure are almost aligned with these extracted centerlines. The averaged angles between the two vectors are approximately 14 degrees in large scale ($k_c = 16$) and 19 degrees in fine scale ($k_c = 64$).

4. Conclusions

An attempt to extract multi-scaled vortical structures from a forced homogeneous isotropic turbulent field and an automatic tracking of each vortex were carried out. The skeleton of each extracted vortex was also obtained. The automatic vortex-tracking scheme showed the capability of tracking vortices even when they split into pieces, and the shapes of extracted skeletons agreed well with those of the vortices.

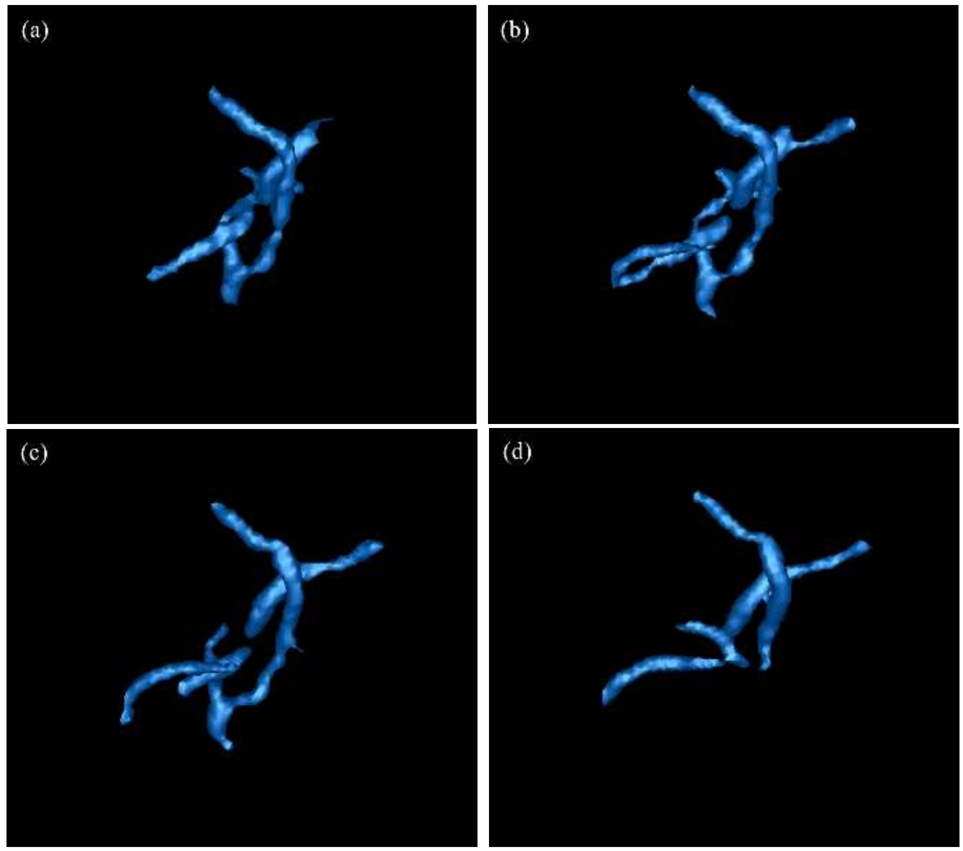


Figure 4: Automatic tracking of one vortex at $t = 0$ (a), 5 (b), 10 (c) and 15 (d) ($k_c = 26$)

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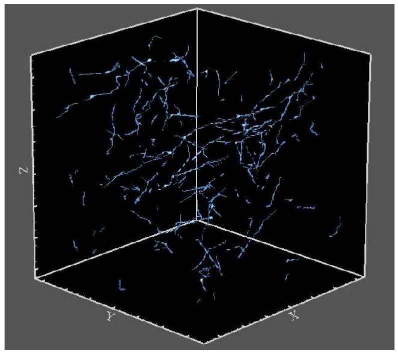

Figure 5: Skeletons of extracted vortices at $k_c = 16$

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