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THE STAR TOTAL COLORING OF $P_m \times P_n$

Chao Zuo[1 §], Muchun Li[2], Ting Zhang[3], Changsheng An[4]
[1,2,3,4]School of Mathematics, Physics and Software Engineering
Lanzhou Jiaotong University
Lanzhou, 730070, P.R. CHINA
[1]e-mail: andyzuch@163.com

Abstract: The coloring problem of graphs is induced by computer science, which has widely application in networks. The coloring problem of graphs is to configure the colorings of each coloring ways of the graphs. All graphs considered in this paper are finite simple graphs. Let $G = G(V(G), E(G))$ be a graph, where $V(G)$ and $E(G)$ denote the vertex set and edge set of G. A proper total k-coloring of a graph G is a star total k-coloring if the colorings of vertices and edges of any path of length 3 in G are all different. The least number of k-spanning over all star total k-colorings of G, denoted by $\chi_{st}(G)$. It is called the star total chromatic number of G. In this paper, we discuss some the star total coloring of graph, and obtain the star total chromatic number of $P_m \times P_n$.

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1. Introduction

The coloring problem of graphs is widely applied in practice [3], [8], [2], [4], [7]. Some conditional coloring problems are introduced. Some network problem can be converted to the total coloring.

Definition 1. (see [1]) The product of simple graphs G and H is the simple graph $G \times H$ with vertex set $V(G) \times V(H)$, in which (u, v) is adjacent to (u', v') if and only if either $u = u'$ and $vv' \in E(H)$ or $v = v'$ and $uu' \in E(G)$.

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§Correspondence author

Definition 2. (see [6]) A proper total coloring of a graph G is called a star total k -coloring if no path of three vertices in G is bi colored. The least number of k spanning over all star total k -colorings of G, denoted by $\chi_{st}(G)$, is named with the *star total chromatic number* of G.

Conjecture. *For any simple graph G with maximum degree $\Delta(G)$, then* $\chi_{st}(G) \leq 3\triangle(G)$.

The paper does not define the terminologies and signs, please see [1] and [5].

2. Main Results

Lemma. (see [3], [8], [2]) *For any simple graph G with maximum degree $\Delta(G)$, if exist a $\chi_{st}(G)$, then $\chi_{st}(G) \geq 2\triangle(G) + 1$.*

Theorem. *Suppose P_m, P_n is a path with order m, n $(m \geq 1, n \geq 2)$, then:*

$$\chi_{st}(_{Pm\times Pn}) = \begin{cases} 3, & n = 2,\ m = 1; \\ 5, & n \geq 3,\ m = 1; \\ 6, & n = m = 2; \\ 7, & n > m = 2; \\ 9, & n \geq m \geq 3. \end{cases}$$

Proof. Let $V(P_m \times P_n) = \{v_{ij} | i = 1, 2, \cdots m;\ j = 1, 2, \cdots n\}$,

$$E(P_m \times P_n) = \{v_{ij}v_{(i+1)j} | i = 1, 2, \cdots m;\ j = 1, 2, \cdots n\} \cup \{v_{ij}v_{i(j+1)} | i = 1, 2, \cdots m;\ j = 1, 2, \cdots n\}.$$

When n=2, m=1. It is easy to know that $\chi_{st}(P_m \times P_n) = 3$; $v_{11}, v_{11}v_{12}, v_{12}$ are colored with colors 1, 2, 3.

When $n \geq 3$, m=1. According to the lemma, we know that $\chi_{st}(G) \geq 2\triangle(G) + 1$ =5.

So, we only need to give a 5-STC of G. It is easy to know that χ_{st} ($P_m \times P_n$)=5; $v_{11}, v_{11}v_{12}, v_{12}, v_{12}v_{13}, v_{13}...v_{1j}, v_{1j}v_{1(j+1)}...$ are colored with colors 1, 2, 3, 4, 5 alternately.

When m=n=2. It is easy to know that χ_{st} ($P_m \times P_n$)=6; $v_{11}, v_{11}v_{12}, v_{12}$ are colored with colors 4, 6, 3; $v_{21}, v_{21}v_{22}, v_{22}$ are colored with colors 1, 6, 2; $v_{11}, v_{11}v_{21}, v_{21}$ are colored with colors 4, 5, 1; $v_{12}, v_{12}v_{22}, v_{22}$ are colored with colors 3, 5, 2.

When n $>$ m=2. According to the lemma, we know that $\chi_{st}(G) \geq 2\triangle(G)+1$ =7.

So, we only need to give a 7-STC of G. It is easy to know that χ_{st} ($P_m \times P_n$)=7; $v_{11}, v_{12}, v_{13}...v_{1j}...$ are colored with colors 4, 3, 1, 2, alternately; $v_{11}v_{12}, v_{12}v_{13}, v_{13}v_{14}...v_{1j}v_{1(j+1)}...$ are colored with colors 6, 7, alternately; $v_{21}, v_{22}, v_{23}...v_{2j}...$ are colored with colors 1, 2, 4, 3 alternately; $v_{21}v_{22}, v_{22}v_{23}, v_{23}v_{24}...v_{2j}v_{2(j+1)}...$ are colored with colors 6, 7 alternately; $v_{11}v_{21}, v_{12}v_{22}, v_{13}v_{23}...v_{1j}v_{2j}...$ are colored with color 5.

When n $\geq$ m $\geq$ 3. According to the lemma, we know that $\chi_{st}(G) \geq 2\triangle(G)+1$ =9.

So, we only need to give a 9-STC of G. It is easy to know that χ_{st} ($P_m \times P_n$)=9; $v_{(5k+1)1}$, $v_{(5k+1)2}$, $v_{(5k+1)3}...v_{(5k+1)j}...$ are colored with colors 3, 2, 4, 5, 1, alternately (k=0, 1, 2...); $v_{(5k+2)1}, v_{(5k+2)2}$, $v_{(5k+2)3}...v_{(5k+2)j}...$ are colored with colors 5, 1, 3, 2, 4 alternately (k=0, 1, 2...); $v_{(5k+3)1}$, $v_{(5k+3)2}$, $v_{(5k+3)3}...v_{(5k+3)j}...$ are colored with colors 2, 4, 5, 1, 3 alternately (k=0, 1, 2...); $v_{(5k+4)1}$, $v_{(5k+4)2}$, $v_{(5k+4)3}...v_{(5k+4)j}...$ are colored with colors 1, 3, 2, 4, 5 alternately (k=0, 1, 2...); $v_{(5k)1}, v_{(5k)2}$, $v_{(5k)3}...v_{(5k)j}...$ are colored with colors 4, 5, 1, 3, 2 alternately (k=1, 2, 3...); $v_{ij}v_{i(j+1)}$ are colored with colors 9, 7 alternately; $v_{ij}v_{(i+1)j}$ are colored with colors 6, 8 alternately.

So, the theorem is true. □

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