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ON THE STAR TOTAL CHROMATIC NUMBER OF MYCIELSKI GRAPHS OF PATH AND CYCLE

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Abstract: A proper total k-coloring of a graph G is a star total k-coloring if the colorings of vertices and edges of any path of length 3 in G are all different. The least number of k spanning over all star total k-colorings of G, denoted by $\chi_{st}(G)$, is called the star total chromatic number of G. In this paper, we obtained the star total chromatic numbers of Mycielski of path and cycle graphs.

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1. Introduction

Graph theory is a sort of models which can be applied in various science fields such as computer science, physics, biology, chemistry, strategy, etc. Graph coloring is one of the chief topics in graph research. The four-color conjecture is firstly brought up in vertex coloring, which develops the research work in graph

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theory [1]-[10]. Later on, based on many theoretical and practical problems, numbers of mathematical experts began to study vertex coloring, edge coloring, total coloring, list coloring, etc. In this paper, we obtained the star total chromatic numbers of Mycielski of path and cycle graphs.

Definition 1. (see [12]) A proper total k-coloring of a graph G is a star total k-coloring if the colorings of vertices and edges of any path of length 3 in G are all different. The least number of k spanning over all star total k-colorings of G, denoted by $\chi_{st}(G)$, is called the star total chromatic number of G.

Definition 2. For a simple graph $G, V(G) = \{v_1, v_2, \cdots, v_n\}$, we refer to $M(G)$ as Mycielski if $V(M(G)) = \{v_1, v_2, \cdots, v_n; u_1, u_2, \cdots, u_n; w\}$, $E(M(G)) = E(G) \cup \{u_iv_j | v_iv_j \in E(G)\} \cup \{u_iw | i = 1, 2, \cdots, n\}$.

Conjecture. *For a simple graph G, if its girth $g(G) \geq 8$, then $\chi_{st}(G)) \leq 3\triangle(G)$.*

The other terminology can be found in [13], [14].

2. Main Results

Lemma. *For a simple graph, $\chi_{st}(G) \geq 2\triangle + 1$.*

Theorem 1. *Let P_n be a path with order n, then*

$$\chi_{st}(M(C_n)) = \begin{cases} 5 & n = 2\,, \\ 9 & n = 3\,, \\ 2n+1 & n \geq 4\,. \end{cases}$$

Proof. According to Lemma, it is easy to know that $\chi_{st}(M(P_n)) \geq 2n + 1$, there are three cases to be considered.

Case 1. When $n = 2$, we know that $\chi_{st}(M(P_2)) \geq 2\triangle + 1 = 5$, so we only need to give a 5-STC of $M(P_2)$. Let f be as follows: $f(w) = 2$; $f(u_1) = 5$; $f(u_2) = 4$; $f(v_1) = 1$; $f(v_2) = 3$; $f(wu_1) = 1$; $f(wu_2) = 3$; $f(v_1u_2) = 5$; $f(u_1v_2) = 4$; $f(v_1v_2) = 2$.

It is obvious that f is a 5-STC of $M(P_2)$.

Case 2. When $n = 3, d(M(P_3)) = 2$, $\chi_{st}(M(P_3)) \geq 2\triangle + 1 = 9$, so we only need to give a 9-STC of $M(P_3)$, Let f be as follows: $f(w) = 7$; $f(u_i) = i + 3$, $i = 1, 2, 3$; $f(v_i) = i$, $i = 1, 2, 3$; $f(wu_i) = i$, $i = 1, 2, 3$; $f(v_iv_{i+1}) = 2n + i$, $i = 1, 2$; $f(v_iu_{i+1}) = i + 3, i = 1, 2$; $f(u_1v_2) = 9$; $f(u_2v_3) = 1$.

It is obvious that f is a 9-STC of $M(P_3)$.

Case 3. When $n \geq 4$, we know that $\chi_{st}(M(P_n)) \geq 2n + 1$, so we only need to give a $2n+1$-STC of P_n. Let f be as follows: $f(w) = 1; f(wu_i) = i+1$, $i = 1, 2, \cdots, n$; $f(u_i) = i+n+1$, $i = 1, 2, \cdots, n$; $f(v_i) = i+1$, $i = 1, 2, \cdots, n$; $f(v_iv_{i+1}) = n+i-1$, $i = 1, 2$; $f(v_iv_{i+1}) = i-2$, $i = 3, 4, \cdots, n-1$; $f(v_iu_{i+1}) = n+i+1$, $i = 1, 2, \cdots, n-1$; $f(u_iv_{i+1}) = 2n-1+i$, $i = 1, 2$; $f(u_iv_{i+1}) = n+i-1$, $i = 3, 4, \cdots, n-1$. It is obvious that f is a $2n+1$-STC of $M(P_n)$. □

Theorem 2. *Let C_n be a cycle with order n, then*

$$\chi_{st}(M(C_n)) = \begin{cases} 10\,, & n = 3, 4\,, \\ 2n+1\,, & n \geq 5\,. \end{cases}$$

Proof. Case 1. When $n = 3, d(M(C_3)) = 2$, for any $u, v \in V(M(C_3)$ have $f(u) \neq f(v)$, there are seven vertices and twelve edges of $M(C_3)$, so we need seven colors to color the vertices of $M(C_3)$, but these seven colors can be used only once in the edge coloring of $M(C_3)$, there are still five edges should be colored with at least three colors, so $\chi_{st}(M(C_3)) = 10$. Now, we only need to give a 10-STC of $M(C_3)$. Let f be as follows: $f(w) = 1$; $f(wu_i) = i+1$, $i = 1, 2, 3$; $f(u_i) = i+4$, $i = 1, 2, 3$; $f(v_i) = i+1$, $i = 1, 2, 3$; $f(v_1v_2) = 1$; $f(v_2v_3) = 8$; $f(v_3v_1) = 9$; $f(v_iu_{i+1}) = i+4$, $i = 1, 2$; $f(v_3u_1) = 7$; $f(u_iv_{i+1}) = i+8$, $i = 1, 2$; $f(u_3v_1) = 10$. It is obvious that f is a 10-STC of $M(C_3)$.

Case 2. When $n = 4, d(M(C_4)) = 2$, for any $u, v \in V(M(C_4)$ have $f(u) \neq f(v)$, there are nine vertices and sixteen edges of $M(C_4)$, so we need nine colors to color the vertices of $M(C_4)$, but four of these nine colors can be used only once in the edge coloring of $M(C_4)$ and one color can be used twice in the edge coloring of $M(C_4)$, there are still ten edges should be colored with at least five colors, so $\chi_{st}(M(C_4)) = 10$. Now, we only need to give a 10-STC of $M(C_4)$. Let f be as follows: $f(w) = 1$; $f(wu_i) = i+1$, $i = 1, 2, 3, 4$; $f(u_i) = i+5$, $i = 1, 2, 3, 4$; $f(v_i) = i+1$, $i = 1, 2, 3, 4$; $f(v_1v_2) = 1; f(v_2v_3) = 10; f(v_3v_4) = 1; f(v_4v_1) = 8$; $f(v_iu_{i+1}) = 9$, $i = 1, 2; f(v_3u_4) = 8, f(v_4u_1) = 9$; $f(u_iv_{i+1}) = 7$, $i = 1, 3; f(u_2v_3) = 8; f(u_4v_1) = 8$. It is obvious that f is a 10-STC of $M(C_4)$.

Case 3. When $n \geq 5$, According to lemma,it is easy to know that $\chi_{st}(M(C_n)) \geq 2n+1$, So we only need to give a $2n+1$-STC of C_n. Let f be as follow:

$$f(w) = 1;\ f(wu_i) = i+1, \quad i = 1, 2, \cdots, n;$$

$$f(u_i) = i+n+1,\ i = 1, 2, \cdots, n; \quad f(v_i) = i+1,\ i = 1, 2, \cdots, n;$$

$$f(v_iv_{i+1}) = n+i-1,\ i = 1, 2; \quad f(v_iv_{i+1}) = i-2, \quad i = 3, 4, \cdots, n-1;$$

$$f(v_iv_1) = 1;\ f(v_iu_{i+1}) = n+i+1,\ i = 1, 2, \cdots, n-1;$$

$$f(v_nu_1) = 2n+1;\ f(u_iv_{i+1}) = 2n-1+i,\ i = 1, 2;$$

$$f(u_iv_{i+1}) = n+i-1,\ i = 3, 4, \cdots, n-1;\ f(u_nv_1) = 2n-1.$$

It is obvious that f is a $2n+1$-STC of C_n.

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