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AN EPQ INVENTORY MODEL WITH DETERIORATED AND IMPERFECT PRODUCTS UNDER FUZZY SENSE

G.C. Mahata[1], Pankaj Dutta[2] §

[1]Department of Mathematics
Sitananda College
Nandigram, Purba Medinipur, West Bengal, 721631, INDIA
e-mail: gourmahata@yahoo.com

[2]Department of Mathematics
Indian Institute of Technology
Kharagpur, 721302, INDIA
e-mail: pankaj@maths.iitkgp.ernet.in

Abstract: In this paper, we investigate an economic production quantity (EPQ) inventory model with fuzzy demand rate and fuzzy deterioration rate. The effect of loss of production quantity due to old/faulty machines, manufacturing defect, etc. are taken into consideration. To derive the estimate of the fuzzy annual total cost, we use the concept of signed distance method. Numerical example is provided to illustrate the computational procedure. The effect of changes in different parameters on the decision variables is discussed.

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Key Words: inventory, fuzzy demand rate, fuzzy deterioration rate, signed distance

1. Introduction

Deteriorating inventory models have been the subject of study for a number of researchers. Reference to deterioration is made by Whitin [11] who considered deterioration of goods at the end of a period of storage. Ghare and Schrader [4] first pointed out the effect of decay in inventory analysis. An EOQ model

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§Correspondence author

for items with variable rate of deterioration has been developed by Convert and Philip [2] by introducing two parameter weibull distribution. Philip [8] developed a three parameter weibull distribution for the deterioration time. Several researchers developed economic production lot-size models with different assumptions on the pattern of deterioration rate. Till now, the models have considered the deterioration rate as constant or dependent on time.

In the development of economic production quantity, usually researchers consider the demand rate, deterioration rate as constant in nature. In the real environments, it is observed that these quantities will have little changes from the exact values. Thus in practical situations, these variables should be treated as fuzzy variables. Recently fuzzy concept is introduced in the production/inventory problems. Zadeh [14] showed the intention of accommodating uncertainty in the non-stochastic sense rather than the presence of random variables. Roy and Maity [9] presented a fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Chang et al [1] presented a fuzzy model for inventory with backorder, where the backorder quantity was fuzzified as the triangular fuzzy number. Lee and Yao [5] and Lin and Yao [6] discussed the production inventory problems, where Lee and Yao [5] fuzzified the demand quantity and production quantity per day, and Lin and Yao [6] fuzzified the production quantity per cycle, treating them as the triangular fuzzy numbers. Yao et al [12] proposed the EOQ model in the fuzzy sense, where both order quantity and total demand were fuzzified as the triangular fuzzy numbers. Mahata et al [7] developed a joint economic lot size model for purchaser and vendor in fuzzy sense. Dutta et al [3] presented a single-period inventory model with two-ordering opportunity under fuzzy demand.

In this paper, we investigate an EPQ model with fuzzy demand rate and fuzzy deterioration rate. The loss of production quantity due to faulty/old machine, manufacturing defect etc. is taken into account. Here, we propose a fuzzy model with demand rate and deterioration rate are considered as fuzzy numbers. We use Yao and Wu's [13] ranking method for fuzzy numbers to find the estimate of the total cost in the fuzzy sense.

2. Mathematical Formulation

The mathematical model is developed on the basis of the following assumptions and notation:

1. The production rate k and demand rate d are assumed to be constants,

where $k > d$ always.

2. Replenishment rate is infinite and lead time is zero.

3. Shortages are not allowed.

4. A constant fraction θ, assumed to be small, of the on-hand inventory gets deteriorated per unit time and ϕ is the deterioration fraction of production rate per unit time.

5. C_1: (the holding cost per unit); p: (the selling price of the production per unit); r: (the purchase cost of raw material per unit); and b: (the set up cost per order, are known and constant).

6. T is the whole planning period; t_1 is the production time per cycle and t_2 is the length of each cycle.

7. $q_1(t)$ is the inventory level at time t, where $0 \le t \le t_1$, $q_2(t)$ is the inventory level at time t, where $t_1 \le t \le t_2$ and q is the actual production quantity received per cycle.

8. $k(1-\phi)$ is the actual production rate per unit time.

On the basis of the above assumptions the inventory level depleted in the following way. Initially the stock level is zero. Production begins at $t = 0$ and continues up to $t = t_1$. The observed production rate becomes less than the original production rate due to faulty machines and deterioration. The inventory accumulated during the production period t_1 after meeting up demand during the period $(0, t_1)$ and loss due to deterioration reaches to zero at time $t = t_2$ due to demand and deterioration during the period (t_1, t_2). The cycle then repeats itself for the entire period T. The differential equations governing the system in the interval $(0, t_2)$ are

$$\frac{dq_1}{dt} + \theta q_1 = k(1-\phi) - d, \quad 0 \le t \le t_1\,, \tag{1}$$

$$\text{and} \quad \frac{dq_2}{dt} + \theta q_2 = -\, d, \quad t_1 \le t \le t_2\,, \tag{2}$$

with the initial conditions

$$q_1(0) = 0, \quad q_1(t_1) = q_2(t_1) \ \text{ and } \ q_2(t_2) = 0\,. \tag{3}$$

The solutions of (1) and (2) with the conditions (3) are

$$q_1(t) = \frac{k(1-\phi) - d}{\theta}(1 - e^{-\theta t}) \quad , \quad 0 \le t \le t_1\,, \tag{4}$$

$$\text{and} \quad q_2(t_2) = \frac{d}{\theta}\left\{e^{\theta(t_2 - t)} - 1\right\} \quad , \quad t_1 \le t \le t_2\,. \tag{5}$$

Using second condition of (3), from (4) and (5) we obtain

$$t_2 = \frac{q}{d}(1+\frac{\theta t_1}{2}) - \frac{\theta q^2}{2d^2}\,.$$

The actual production quantity during the period t_1 is $q = k(1-\phi)t_1$ and the number of cycles$=\frac{T}{t_2}$.

The inventory holding cost per cycle, denoted by HC, is given by

$$\begin{aligned} HC &= C_1(\int_0^{t_1} q_1(t)dt + \int_{t_1}^{t_2} q_2(t)dt) \\ &= C_1[\{k(1-\phi)-d\}\Big\{\frac{t_1^2}{2} - \frac{\theta {t^3}_1}{6}\Big\} + d\Big\{\frac{(t_2-t_1)^2}{2} + \frac{\theta(t_2-t_1)^3}{6}\Big\}]\,. \quad (6) \end{aligned}$$

The cost for the loss of production quantity per cycle due to faulty machine

$$LP = rk\phi t_1\,. \quad (7)$$

In addition, the deterioration cost per cycle, denoted by DC is given by

$$DC = p\,\theta(\int_0^{t_1} q_1(t)dt + \int_{t_1}^{t_2} q_2(t)dt) = p(k(1-\phi)t_1 - dt_2)\,. \quad (8)$$

Hence, using (6), (8) and (7), the total inventory cost for the whole planning period T is

$$\begin{aligned} TC &= (HC+DC+LP+b)\frac{T}{t_2} \\ &= [(C_1+p\theta)\Big\{(k(1-\phi)-d)(\frac{t_1^2}{2} - \frac{\theta {t^3}_1}{6}) + d(\frac{(t_2-t_1)^2}{2} + \frac{\theta(t_2-t_1)^3}{6})\Big\} \\ &\qquad + rk\phi t_1 + b\,]\frac{T}{t_2} = A_1 + \theta A_2 + dA_3 - d\theta A_4 - \frac{\theta}{d}A_5, \quad (9) \end{aligned}$$

where

$$\begin{aligned} A_1 &= \frac{C_1 q T}{2}, \\ A_2 &= \Big\{pq + \frac{rq\phi}{1-\phi} + b\Big\}\frac{T}{2}, \\ A_3 &= \Big\{\frac{b}{q} + \frac{r\phi}{1-\phi} - \frac{C_1 q}{2k(1-\phi)}\Big\}\frac{T}{2}, \\ A_4 &= \Big\{\frac{rq\phi + (b+pq)(1-\phi)}{k(1-\phi)^2} - \frac{C_1 q^2}{6k^2(1-\phi)^2}\Big\}\frac{T}{2} \\ \text{and}\quad A_5 &= \frac{C_1 q^2 T}{12} \quad \text{and } \theta,\ d > 0\,. \end{aligned}$$

Particular Cases. (i) When $\phi \to 0, k \to \infty, \theta \to 0$, we obtain $Z(q) = \frac{C_1 qT}{2} + \frac{bdT}{q}$. This equation is same as the average total cost of a classical EOQ model with constant demand rate and no deterioration.

(ii) When $\phi \to 0, k \to \infty$, we get

$$Z(q) = \frac{C_1 q T}{2} + \frac{bdT}{q} + \frac{\theta T}{2}(b + pq - \frac{C_1 q^2}{6})$$

which is an EOQ model under deterioration and constant demand.

(iii) When $\phi \to 0$, we get

$$Z(q) = \frac{C_1 q T}{2}(1 - \frac{d}{k}) + \frac{bdT}{q} + \frac{\theta T}{2}(1 - \frac{d}{k})\Big\{b + pq - (1 + \frac{d}{k})\frac{C_1 q}{6}\Big\}$$

which is the EPQ model under deterioration.

(iv) When $\phi \to 0, \theta \to 0$, we get

$$Z_(q) = \frac{C_1 q T}{2}(1 - \frac{d}{k}) + \frac{bdT}{q},$$

which is the classical EPQ model.

In the development of EPQ, earlier authors have assumed that both the demand rate and deterioration rate are constant. But, in most of the cases the demand rate is uncertain in nature and in the real situation it is not always easy to determine the exact value of deterioration rate. We consider the demand rate d and deterioration rate θ to be triangular fuzzy numbers, $\widetilde{d} = (d-\Delta_1, d, d+\Delta_2)$ and $\tilde{\theta} = (\theta - \Delta_3, \theta, \theta + \Delta_4)$, where Δ_1, Δ_2, Δ_3 and Δ_4 are determined by the decision makers and satisfy the conditions $0 < \Delta_1 < d$, $0 < \Delta_2$ and $0 < \Delta_3 < \theta$, $0 < \Delta_4$. We express the fuzzy total cost as,

$$\widetilde{Z} = \widetilde{Z}(q) = A_1 + A_2\widetilde{\theta} + A_3\widetilde{d} - A_4\widetilde{d}\,\widetilde{\theta} - A_5\frac{\widetilde{\theta}}{\widetilde{d}}. \tag{10}$$

In order to get optimal order quantity and optimal total annual cost, we must defuzzify $\widetilde{Z}(q)$. One common method of defuzzification through the signed distance, as in [13], is adopted here.

For a fuzzy number $\widetilde{D}$, if $[D_L(\alpha), D_R(\alpha)]$ be the α-level set, then the signed distance from $\widetilde{D}$ to 0 is defined as:

$$d(\widetilde{D}, 0) = \frac{1}{2}\int_0^1 (D_L(\alpha) + D_R(\alpha))d\alpha\,. \tag{11}$$

Usually, to get the signed distance $d(\widetilde{Z}(q), 0)$, we should find the member function of the fuzzy number $\widetilde{Z}(q)$ through extension principle firstly. But in our model, the relationship between $\widetilde{Z}(q)$, $\widetilde{D}$ and $\widetilde{\theta}$ showed by equation (9) is every complex, and getting the member function of $\widetilde{Z}(q)$ is hardly possible. So, we get the α-cut of $\widetilde{Z}(q)$ directly through equation (10), and defuzzify $\widetilde{Z}(q)$ using (11).

Then we can obtain the signed distance of $\widetilde{Z}$ to 0 as,

$$d(\widetilde{Z},0) = A_1 + A_2 d(\widetilde{\theta},0) + A_3 d(\widetilde{d},0) - A_4 d(\widetilde{d}\,\widetilde{\theta},0) - A_5 d(\frac{\widetilde{\theta}}{\widetilde{d}},0)\,. \tag{12}$$

Thus using (11), the signed distance of fuzzy number $\widetilde{d}$ to 0 is

$$\begin{aligned} d(\widetilde{d},0) &= \frac{1}{4}[(d-\Delta_1)+2d+(d+\Delta_2)] = d+\frac{1}{4}(\Delta_2-\Delta_1) \\ \text{and}\ \ d(\widetilde{\theta},0) &= \theta+\frac{\Delta_4-\Delta_3}{4}. \end{aligned}$$

Now, we have to calculate the signed distance $d(\widetilde{d}\,\widetilde{\theta},0)$ and $d(\frac{\widetilde{\theta}}{\widetilde{d}},0)$. The left and right end points of the $\alpha - cut$ $(0 \le \alpha \le 1)$ of $\widetilde{d}$ and $\widetilde{\theta}$ are respectively

$$\begin{aligned} d_L(\alpha) &= (d-\Delta_1)+\Delta_1\alpha \quad \text{and} \quad d_U(\alpha) = (d+\Delta_2)-\Delta_2\alpha\,, \\ \theta_L(\alpha) &= (\theta-\Delta_3)+\Delta_3\alpha \quad \text{and} \quad \theta_U(\alpha) = (\theta+\Delta_4)-\Delta_4\alpha. \end{aligned}$$

The left and right end points of the $\alpha - cut$ $(0 \le \alpha \le 1)$ of $\widetilde{d}\,\widetilde{\theta}$ are

$$\begin{aligned} (d\ \theta)_L(\alpha) &= (d_L(\alpha))(\theta_L(\alpha)) = ((d-\Delta_1)+\Delta_1\alpha)((\theta-\Delta_3)+\Delta_3\alpha)\,, \\ \text{and}\quad (d\ \theta)_U(\alpha) &= (d_U(\alpha))(\theta_U(\alpha)) = ((d+\Delta_2)-\Delta_2\alpha)((\theta+\Delta_4)-\Delta_4\alpha)\,, \end{aligned}$$

respectively. Therefore, the signed distance of $\widetilde{d}\,\widetilde{\theta}$ to 0 is,

$$\begin{aligned} d(\widetilde{d}\,\widetilde{\theta},0) &= \frac{1}{2}\int_0^1 [(d\ \theta)_L(\alpha)+(d\ \theta)_U(\alpha)]d\alpha \\ &= [\frac{1}{2}\{(d-\Delta_1)(\theta-\Delta_3)+(d+\Delta_2)(\theta+\Delta_4)\}+\frac{1}{4}\{\Delta_1(\theta-\Delta_3) \\ &\quad +\Delta_3(d-\Delta_1)-\Delta_4(d+\Delta_2)-\Delta_2(\theta+\Delta_4)\}+\frac{1}{6}(\Delta_1\Delta_3-\Delta_2\Delta_4)]\,. \end{aligned}$$

Again, since $0 < d_L(\alpha) < d_U(\alpha)$, the left and the right end points of the $\alpha - cut$ $(0 \le \alpha \le 1)$ of $\frac{\widetilde{\theta}}{\widetilde{d}}$ are

$$\begin{aligned} (\frac{\theta}{d})_L(\alpha) &= \frac{\theta_L(\alpha)}{d_U(\alpha)} = \frac{(\theta-\Delta_3)+\Delta_3\alpha}{(d+\Delta_2)-\Delta_2\alpha} \quad \text{and} \\ (\frac{\theta}{d})_U(\alpha) &= \frac{\theta_U(\alpha)}{d_L(\alpha)} = \frac{(\theta+\Delta_4)-\Delta_4\alpha}{(d-\Delta_1)+\Delta_1\alpha}\,, \end{aligned}$$

respectively. Thus, the signed distance of $\frac{\widetilde{\theta}}{\widetilde{d}}$ to 0 is as follows:

$$\begin{aligned} d(\frac{\widetilde{\theta}}{\widetilde{d}},0) &= \frac{1}{2}\int_0^1 [(\frac{\theta}{d})_L(\alpha)+(\frac{\theta}{d})_U(\alpha)]d\alpha \\ &= \frac{1}{2}[\frac{(d\Delta_4+\theta\Delta_1)}{\Delta_1^2}\ln(\frac{d}{d-\Delta_1})-\frac{\Delta_4}{\Delta_1}+\frac{(d\Delta_3+\theta\Delta_2)}{\Delta_2^2}\ln(\frac{d+\Delta_2}{d})-\frac{\Delta_3}{\Delta_2}]\,, \end{aligned}$$

which is positive, since $(\frac{\theta}{d})_L(\alpha) > 0$ and $(\frac{\theta}{d})_U(\alpha) > 0$ are continuous function on $0 \leq \alpha \leq 1$; hence the above result of definite integral must be positive. Using the results of $d(\widetilde{d},0)$, $d(\widetilde{\theta},0)$, $d(\widetilde{d}\,\widetilde{\theta},0)$ and $d(\frac{\widetilde{\theta}}{\widetilde{d}},0)$ in (12), we obtain

$$Z(q) = d(\widetilde{Z},0) = A_1 + A_2(\theta + \frac{\Delta_4 - \Delta_3}{4}) + A_3(d + \frac{\Delta_2 - \Delta_1}{4})$$
$$- A_4 \times [\frac{1}{2}\{(d-\Delta_1)(\theta-\Delta_3) + (d+\Delta_2)(\theta+\Delta_4)\}$$
$$+ \frac{1}{4}\{\Delta_1(\theta-\Delta_3) + \Delta_3(d-\Delta_1) - \Delta_4(d+\Delta_2) - \Delta_2(\theta+\Delta_4)\} + \frac{1}{6}(\Delta_1\Delta_3 - \Delta_2\Delta_4)]$$
$$- A_5 \times \frac{1}{2}[\frac{(d\Delta_4 + \theta\Delta_1)}{\Delta_1^2}\ln(\frac{d}{d-\Delta_1}) - \frac{\Delta_4}{\Delta_1} + \frac{(d\Delta_3 + \theta\Delta_2)}{\Delta_2^2}\ln(\frac{d+\Delta_2}{d}) - \frac{\Delta_3}{\Delta_2}],$$

where $Z(q)$ regarded as the estimate of total cost in the fuzzy sense.

3. Numerical Example

In this section, a numerical example is considered to illustrate the models. The following values of the parameters are used in the example.

Let, $C_1 = 10$, $b = 500$, $T = 40$, $p = 3$, $r = 1$, $k = 10$, $d = 2$, $\phi = 0.01$, $\theta = 0.04$ in proper units. The economic production lot size q_* and the minimum total cost Z_* in the crisp case can be obtain from (9). Using the software *LINGO*, the solution of the crisp model is obtained as, $q_* = 16.91$ and $Z_* = 5233.87$.

For the proposed model, we considered the demand rate d and deterioration rate θ as triangular fuzzy numbers $\widetilde{d} = (d - \Delta_1,\ d,\ d + \Delta_2)$ and $\widetilde{\theta} = (\theta - \Delta_3,\ \theta,\ \theta + \Delta_4)$ respectively. We solve the economic production lot size q^* and the minimum total cost Z^* from (13) for the whole planning period in the fuzzy sense for various sets of $(\Delta_1,\ \Delta_2)$ and $(\Delta_3,\ \Delta_4)$ satisfying the conditions $0 < \Delta_1 < d$ and $0 < \Delta_2$ and $0 < \Delta_3 < \theta$ and $0 < \Delta_4$. The results are summarized in Table 1.

From Table 1 it is observed that, if $\Delta_1 < \Delta_2$ decreases, then the solutions in the fuzzy sense increases marginally in comparison to the solution obtained in crisp case but this trend changes in the opposite direction for the case when $\Delta_1 > \Delta_2$. For $\Delta_1 = \Delta_2$, the solutions in fuzzy sense and crisp sense almost remains unchanged. From Table 1, it is observed that for fixed values of (Δ_1, Δ_2), as the estimate of deterioration rate in the fuzzy sense $d(\widetilde{\theta},0)$ increases, the solutions in fuzzy sense increases marginally. Again, for fixed values of (Δ_3, Δ_4), it is observed that $\Delta_3 < \Delta_4$ decreases, then the solutions in fuzzy sense increases marginally in comparison to the solution obtained in crisp case but this

Δ_1	Δ_2	Δ_3	Δ_4	$d(\widetilde{d},\widetilde{0}_1)$	$d(\widetilde{\theta},\widetilde{0}_1)$	$d(\frac{\widetilde{\theta}}{\widetilde{d}},\widetilde{0}_1)$	$d(\widetilde{d\theta},\widetilde{0}_1)$	q^*	Z^*	$\frac{q^*-q_*}{q_*}\times 100$	$\frac{Z^*-Z_*}{Z_*}\times 100$
0.4	0.6	0.005	0.002	2.05	0.0390	0.0197	0.0806	17.18	5269.63	1.60	0.68
			0.005	2.05	0.0400	0.0201	0.0818	17.21	5272.41	1.77	0.74
			0.009	2.05	0.0410	0.0207	0.0834	17.25	5276.10	2.01	0.81
		0.008	0.006	2.05	0.0395	0.0199	0.0809	17.20	5269.48	1.71	0.68
			0.008	2.05	0.0400	0.0202	0.0817	17.22	5271.33	1.83	0.72
			0.011	2.05	0.0408	0.0207	0.0829	17.25	5274.09	2.01	0.77
		0.010	0.007	2.05	0.0393	0.0199	0.0805	17.20	5267.83	1.71	0.65
			0.010	2.05	0.0400	0.0203	0.0817	17.23	5270.60	1.89	0.70
			0.015	2.05	0.0413	0.0210	0.0837	17.28	5275.19	2.19	0.79
0.5	0.5	0.005	0.002	2.00	0.0393	0.0202	0.0787	16.93	5225.00	0.12	-0.17
			0.005	2.00	0.0400	0.0206	0.0800	16.96	5227.67	0.30	-0.12
			0.009	2.00	0.0410	0.0212	0.0817	17.00	5231.20	0.53	-0.05
		0.008	0.006	2.00	0.0395	0.0205	0.0792	16.95	5224.64	0.23	-0.18
			0.008	2.00	0.0400	0.0208	0.0800	16.97	5226.42	0.35	-0.14
			0.011	2.00	0.0408	0.0212	0.0813	17.00	5229.07	0.53	-0.09
		0.010	0.007	2.00	0.0393	0.0204	0.0788	16.94	5222.93	0.18	-0.21
			0.010	2.00	0.0400	0.0209	0.0800	16.97	5225.59	0.35	-0.16
			0.015	2.00	0.0413	0.0216	0.0821	17.03	5229.99	0.71	-0.07
0.6	0.4	0.005	0.002	1.95	0.0393	0.0208	0.0769	16.67	5178.97	-1.42	-1.05
			0.005	1.95	0.0400	0.0212	0.0782	16.70	5181.52	-1.24	-1.00
			0.009	1.95	0.0410	0.0219	0.0799	16.75	5184.89	-0.95	-0.94
		0.008	0.006	1.95	0.0395	0.0211	0.0774	16.69	5178.41	-1.30	-1.06
			0.008	1.95	0.0400	0.0214	0.0783	16.71	5180.11	-1.18	-1.03
			0.011	1.95	0.0408	0.0218	0.0796	16.75	5182.64	-0.95	-0.98
		0.010	0.007	1.95	0.0393	0.0210	0.0770	16.69	5176.63	-1.30	-1.09
			0.010	1.95	0.0400	0.0215	0.0783	16.72	5179.16	-1.23	-1.05
			0.015	1.95	0.0413	0.0223	0.0805	16.77	5183.36	0.83	-0.97

Table 1: Optimal solution for the model with fuzzy demand rate and fuzzy deterioration rate

trend changes in the opposite direction for the case when $\Delta_3 > \Delta_4$.

4. Conclusions

In the development of EPQ model, most of the earlier researchers have considered demand rate and deterioration rate as constant quantity. But in real life situations, the demand rate and deterioration rate are not exactly constant but slightly disturbed from their original crisp value. This motivates us to develop an EPQ inventory model with finite production rate, fuzzy demand rate and fuzzy deterioration rate. Loss of production incurred due to faulty/old machines, manufacturing defect etc. have also been taken into account by considering a fraction of production rate deteriorates per unit time. Using the concept of signed distance method, we have derived the estimate of fuzzy total cost. The fuzzy model has been explained with the help of a numerical example. The sensitivity of the decision variables, economic production quantity q and the total cost Z, for changes in the parameters Δ_1, Δ_2, Δ_3 and Δ_4 have also been investigated and important observations have been discussed.

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