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A COMPETITIVE STAGE-STRUCTURED MODEL WITH TIME DELAY

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Abstract: In this paper, we study the asymptotic behavior of a competitive stage-structured model with time delay. Sufficient conditions for a positive equilibrium to be globally asymptotically stable are obtained. Moreover, it is investigated the conditions that the positive equilibrium is unstable for any $\tau \geq 0$.

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1. Introduction

The competitive system is an important population model and has been studied by many authors [1], [13], [10], [2], [3]. However, these authors always assume that during the whole life histories, each individual admits the same density-dependent rate as well as the identical ability to bear and to compete with other species, which clearly is unrealistic. Because for many animals, whose babies

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are raised by their parents or are dependent on the nutrition from the eggs they stay in, the babies are much weaker than the mature, their competition with other individuals of the community can be ignored, so they do not have the ability to produce babies. Therefore, it is practical to introduce the stage structure into the competitive model.

On the other hand, it is well known that time delays in ecological system can have a considerable influence on the qualitative behavior of these systems, it is generally recognized that some kind of time delays are inevitable in population interactions and tend to be destabilizing in the sense that longer delays may destroy the stability of positive equilibrium [5], [9]. Recently persistence and stability of a population dynamical system involving time delays have been considered by many authors [9], [6], [14], [4], [11], [8], [7], [12]. H.I. Freedman and K. Gopalsamy in [6] have shown that if a time delay is incorporated into the resource limitation of the logistic equation, then it has destablizing effect on the stability of the system. In papers [4], [11], the authors have shown that for some systems the stability switches can happen many times and the system will eventually become unstable when time delays increase. Reference [12] has investigated that for certain values of the delay, there occurs an unstable equilibrium with periodic oscillations. However, time delays sometimes have no effect on uniform persistence and stability of the solutions of the systems.

In this paper, we intend to consider the stage structure of two species. For the simplicity of our model, we only consider the stage structure of immature and mature species X (their sizes of population are written as x_1, x_2, respectively), and do not consider the stage structure of species Y (its size of population is written as y), and two species satisfy the following assumptions:

$(A1)$ The birth rate of the immature population is proportional to the existing mature population with a proportional constant α (cf. the term αx_2 in $(1.1a)$; for the immature population, the death rate and transformation rate of mature are proportional to the existing immature population with proportionality constants r_1 and ω (cf. the terms $r_1 x_1$ and ωx_1 in $(1.1a)$).

$(A2)$ The death rate of mature population is proportional to the existing mature population with a proportionality constant r_2 (cf. the term $r_2 x_2$ in $(1.1b)$); the mature population is density restriction (cf. the term ηx_2^2 in $(1.1b)$).

$(A3)$ The immature species X and species Y do not attack each other, furthermore, when individuals of mature competitors X and Y are still young, the ability of species X attacking species Y each other is too weak to be ignored. $\bar{\tau}_1 \geq 0$ denotes the time of species Y to attain ability to attack mature species X, constant $\bar{\tau}_2 \geq 0$ denotes the time of mature species X being stronger to compete with species Y; The mature species X and species Y compete with each other in a bilinear fashion (cf. the terms $bx_2(t)y(t-\bar{\tau}_1)$ and $cx_2(t-\bar{\tau}_2)y(t)$ in (1.1b) and (1.1c)).

($A4$) The species Y satisfies the logistic competitive model.

According to $A1 - A4$, we can investigate the following stage-structured competitive model:

$$\dot{x}_1(t) = \alpha x_2(t) - r_1 x_1(t) - \omega x_1(t)\,, \tag{1.1a}$$

$$\dot{x}_2(t) = \omega x_1(t) - r_2 x_2(t) - \eta x_2^2(t) - b x_2(t) y(t-\bar{\tau}_1)\,, \tag{1.1b}$$

$$\dot{y}(t) = y(t)(\beta - a y(t) - c x_2(t-\bar{\tau}_2))\,, \tag{1.1c}$$

with initial conditions

$$\begin{aligned} &x_1(0) > 0,\ x_2(s) = \phi(s) \geq 0,\ y(s) = \psi(s) \geq 0,\ s \in [-\bar{\tau},\ 0]\,, \\ &\phi(0) > 0,\ \psi(0) > 0\,, \end{aligned} \tag{1.2}$$

where $\bar{\tau} = \max\{\bar{\tau}_1,\ \bar{\tau}_2\}$.

The organization of this paper is as follows. In the next section, we obtain positivity and boundedness of the solutions of the model (1.1). In Section 3, the nonnegative equilibria and their local stability properties are analyzed. In Section 4, we investigate the sufficient conditions that the positive equilibrium of system (1.1) is globally asymptotically stable. It is also obtained the conditions that the positive equilibrium is unstable for any $\tau \geq 0$.

2. Preliminaries and Lemmas

First, to prove the following theorems, we need some lemmas and definition.

Lemma 2.1. *Every solution $(x_1(t),\ x_2(t),\ y(t))$ of system (1.1) with initial conditions (1.2) exists in the interval $[0,\ +\infty)$ and remains positive for all $t \geq 0$.*

Proof. It is true because

$$\dot{x}_1|_{x_1=0} = \alpha x_2 > 0 \quad \text{for} \quad x_2 > 0\,,$$
$$\dot{x}_2|_{x_2=0} = \omega x_1 > 0 \quad \text{for} \quad x_1 > 0\,,$$
$$y(t) = \psi(0) exp[\int_0^t (\beta - a y(s) - c x_2(s-\bar{\tau}_2)) ds] > 0 \quad \text{for} \quad \psi(0) > 0. \qquad \square$$

Lemma 2.2. *Every solution of system (1.1) with initial conditions (1.2) is bounded for $t \geq 0$.*

Proof. Choose the function $\rho(t) = x_1(t) + x_2(t) + y(t)$ and calculate the derivative of $\rho(t)$ along the solution of (1.1), we have

$$\dot{\rho}(t) = -r_1 x_1 + (\alpha - r_2) x_2 - \eta x_2^2 + \beta y - a y^2 - b x_2 y(t-\bar{\tau}_1) - c y x_2(t-\bar{\tau}_2)\,.$$

For a positive constant ε ($\varepsilon < r_1$), we have

$$\dot{\rho} + \varepsilon\rho \leq (-r_1 + \varepsilon)x_1 + (\alpha + \varepsilon - r_2)x_2 - \eta x_2^2 + (\beta + \varepsilon)y - ay^2 .$$

Hence, there exists a positive number c such that $\dot{\rho} + \varepsilon\rho < c$. Further

$$\rho(t) < \frac{c}{\varepsilon} + (\rho(0) - \frac{c}{\varepsilon})e^{-\varepsilon t} .$$

Here we obtain the boundedness of the positive solutions of system (1.1). The proof is completed. □

Definition 2.3. The positive equilibrium of system (1.1) is said to be globally asymptotically stable if the positive equilibrium is asymptotically stable and attracts all positive solutions.

For the sake of simplicity, we put in dimensionless form the model (1.1) by rescaling the variables:

$$x_1 = \frac{\alpha}{c}\bar{x}_1, \quad x_2 = \frac{r_1 + \omega}{c}\bar{x}_2, \quad y = \frac{1}{r_1 + \omega}\bar{y} ,$$

and then using as dimensionless time, $u = (r_1 + \omega)t$. For convenience, in the following we replace $\bar{x}_i$ ($i = 1,\ 2$), $\bar{y}$ and u by x_i ($i = 1,\ 2$), y and t, respectively. Then system (1.1) leads to

$$\begin{cases} \dot{x}_1(t) = x_2(t) - x_1(t) , \\ \dot{x}_2(t) = dx_1(t) - ex_2(t) - fx_2^2(t) - x_2(t)y(t - \tau_1) , \\ \dot{y}(t) = y(t)(p - qy(t) - x_2(t - \tau_2)) , \end{cases} \tag{2.1}$$

where $d = \frac{\alpha\omega}{(r_1+\omega)^2}$, $e = \frac{r_2}{r_1+\omega}$, $f = \frac{\eta}{c}$, $p = \frac{\beta}{r_1+\omega}$, $q = \frac{a}{b}$, $\tau_1 = (r_1 + \omega)\bar{\tau}_1$, $\tau_2 = (r_1+\omega)\bar{\tau}_2$. Obviously, system (2.1) has the same qualitative property as system (1.1).

3. Equilibria and their Local Stability

Let $\tilde{E} = (\tilde{x}_1,\ \tilde{x}_2,\ \tilde{y})$ denote any nonnegative equilibrium point of (2.1), i.e., $\tilde{x}_1,\ \tilde{x}_2,\ \tilde{y} \geq 0$. Then $(\tilde{x}_1,\ \tilde{x}_2,\ \tilde{y})$ must satisfy the algebraic equation

$$x_2 - x_1 = 0, \quad dx_1 - ex_2 - fx_2^2 - x_2y = 0, \quad y(p - qy - x_2) = 0 .$$

Clearly, $E_0 = (0,\ 0,\ 0)$ and $E_1 = (0,\ 0,\ \frac{p}{q})$ are two equilibria. If $d > e$, then $E_2 = (\frac{d-e}{f},\ \frac{d-e}{f},\ 0)$ is a nonnegative equilibrium.

We assume,

$(H1)$ $$\frac{p}{q} < d - e < fp ,$$

$(H2)$ $$fp < d - e < \frac{p}{q} .$$

It is easy to obtain, if (H1) or (H2) holds, then system (2.1) has a unique positive equilibrium $E^* = (x_1^*,\ x_2^*,\ y^*)$, where

$$x_1^* = x_2^* = \frac{dq - eq - p}{fq - 1}, \quad y^* = \frac{fp - d + e}{fq - 1}\,.$$

To determine the local stability of the equilibria $E_0,\ E_1,\ E_2$ and E^*, we linearized (2.1) about $\tilde{E} = (\tilde{x}_1,\ \tilde{x}_2,\ \tilde{y})$ ($\tilde{E}$ may be $E_0,\ E_1,\ E_2$ or E^*):

$$\begin{aligned}
\dot{X}_1(t) &= -X_1(t) + X_2(t)\,,\\
\dot{X}_2(t) &= dX_1(t) - (e + 2f\tilde{x}_2 + \tilde{y})X_2(t) - \tilde{x}_2 Y(t-\tau_1)\,,\\
\dot{Y}(t) &= (p - 2q\tilde{y} - \tilde{x}_2)Y(t) - \tilde{y}X_2(t-\tau_2)\,.
\end{aligned}$$

The corresponding characteristic equation is

$$\begin{vmatrix} \lambda+1 & -1 & 0 \\ -d & \lambda + e + 2f\tilde{x}_2 + \tilde{y} & \tilde{x}_2 e^{-\lambda\tau_1} \\ 0 & \tilde{y}e^{-\lambda\tau_2} & \lambda - p + 2q\tilde{y} + \tilde{x}_2 \end{vmatrix} = 0. \tag{3.1}$$

For $(\tilde{x}_1,\ \tilde{x}_2,\ \tilde{y}) = E_0$, we have the following theorem easily.

Theorem 3.1. *The equilibrium $E_0 = (0,\ 0,\ 0)$ is unstable.*

For $(\bar{x}_1,\ \bar{x}_2,\ \bar{y}) = E_1$, (3.1) reduces to

$$(\lambda + p)[\lambda^2 + (e + \frac{p}{q} + 1)\lambda + e + \frac{p}{q} - d] = 0\,. \tag{3.2}$$

Hence:

(i) If $d - e > \frac{p}{q}$, then equation (3.2) has a positive root. Therefore E_1 is unstable.

(ii) If $d - e < \frac{p}{q}$, then all solutions of equation (3.2) have negative real parts. Therefore E_1 is asymptotically stable.

Thus, we have the following theorem.

Theorem 3.2. *If $d - e < \frac{p}{q}$, then the nonnegative equilibrium E_1 is asymptotically stable; if $d - e > \frac{p}{q}$, then E_1 is unstable.*

By the similar discussions as those in proof of Theorem 3.2, we can obtain the following results.

Theorem 3.3. *If $d - e > fp$, then the nonnegative equilibrium E_2 is asymptotically stable; if $0 < d - e < fp$, then E_2 is unstable.*

From Theorem 3.2 and Theorem 3.3, we obtain the following results.

Corollary 3.1. *If $d - e < \min\{fp,\ \frac{p}{q}\}$, then system (2.1) has no positive equilibrium, E_1 is asymptotically stable, E_2 is unstable.*

Corollary 3.2. *If $d - e > \max\{fp,\ \frac{p}{q}\}$, then system (2.1) has no positive equilibrium, E_2 is asymptotically stable, E_1 is unstable.*

To determine the local stability of the positive equilibrium E^*, we let $\tilde{E} = E^*$, and (3.1) reduces to

$$\lambda^3 + A\lambda^2 + B\lambda + C = D(\lambda + 1)e^{-\tau\lambda}, \tag{3.3}$$

where

$$\begin{aligned} &A = e + 2fx_2^* + y^* + qy^* + 1, \\ &B = eqy^* + 2fqx_2^*y^* + qy^{*2} + e + 2fx_2^* + qy^* - d, \\ &C = eqy^* + 2fqx_2^*y^* + qy^{*2} - dqy^*, \\ &D = x_2^*y^* \quad \tau = \tau_1 + \tau_2. \end{aligned} \tag{3.4}$$

Consequently, we can obtain

$$A^2 - 2B > 0, \quad B^2 - 2AC - D^2 > 0, \quad C - D = (fq - 1)x_2^*y^*. \tag{3.5}$$

Theorem 3.4. *Let (H1) hold, then the positive equilibrium E^* $(x_1^*,\ x_2^*,\ y^*)$ of system (2.1) is asymptotically stable for any $\tau \geq 0$.*

Proof. By putting $\tau = 0$ and $\tilde{E} = (x_1^*,\ x_2^*,\ y^*)$ in the characteristic equation (3.3), we obtain

$$\lambda^3 + A\lambda^2 + (B - D)\lambda + (C - D) = 0.$$

By the Routh-Hurwitz Theorem, all the characteristic roots have negative real parts if

$$A(B - D) > C - D. \tag{3.6}$$

From (3.4), we have $B > C$. Hence (3.6) holds. Consequently, when $\tau = 0$, the positive equilibrium $(x_1^*,\ x_2^*,\ y^*)$ is asymptotically stable.

Let $\lambda = i\sigma,\ \sigma \in R$, (3.3) reduces to

$$-i\sigma^3 - A\sigma^2 + B\sigma i + C = D(\sigma i + 1)[\cos(\sigma\tau) - i\sin(\sigma\tau)].$$

Hence

$$\begin{cases} -\sigma^3 + B\sigma = D[\sigma\cos(\sigma\tau) - \sin(\sigma\tau)], \\ -A\sigma^2 + C = D[\cos(\sigma\tau) + \sigma\sin(\sigma\tau)]. \end{cases}$$

It follows by taking the sum of squares that

$$(\sigma^3 - B\sigma)^2 + (A\sigma^2 - C)^2 = D^2(\sigma^2 + 1),$$

or

$$\sigma^6 + (A - 2B)\sigma^4 + (B^2 - D^2 - 2AC)\sigma^2 + C^2 - D^2 = 0. \tag{3.7}$$

This is a cubic equation about σ^2. From (3.5), we have $A^2 - 2B > 0$, $C^2 - D^2 > 0$ and $B^2 - D^2 - 2AC > 0$. It suffices to show that equation (3.3) has no pure imaginary root. This implies the positive equilibrium E^* $(x_1^*,\ x_2^*,\ y^*)$ is asymptotically stable when $\tau \geq 0$. The proof is completed. □

Hypothesis (H1) in system (2.1) is equivalent to:

$$(H3) \qquad \frac{\beta b}{a} < \frac{\alpha\omega}{r_1+\omega} - r_2 < \frac{\beta\eta}{c}$$

in system (1.1).

Obviously, if (H3) holds, then system (1.1) has a unique positive equilibrium $E = (X_1^*,\ X_2^*,\ Y^*)$, where

$$X_2^* = \frac{a\alpha\omega - ar_2(r_1+\omega) - \beta b(r_1+\omega)}{(r_1+\omega)(a\eta - bc)}, \quad X_1^* = \frac{\alpha}{r_1+\omega}X_2^*, \quad Y^* = \frac{\beta - cX_2^*}{a}\,.$$

From Theorem 3.4, we have:

Theorem 3.5. *The positive equilibrium E $(X_1^*,\ X_2^*,\ Y^*)$ of system (1.1) is asymptotically stable for any $\tau \geq 0$ provided that (H3) holds.*

4. Global Properties of the Positive Equilibrium

We now turn to study the global stability of the positive equilibrium $E(X_1^*, X_2^*, Y^*)$ of (1.1).

Theorem 4.1. *If (H3) holds, then the positive equilibrium $E(X_1^*, X_2^*, Y^*)$ of (1.1) is globally asymptotically stable provided that*

$$(H4) \qquad r_1+\omega > \alpha, \quad r_2 + \eta X_2^* > \omega + bX_2^*, \quad a > c\,.$$

Proof. System (1.1) can be rewritten as

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= \alpha(x_2(t) - X_2^*) - (r_1+\omega)(x_1(t) - X_1^*),\\
\frac{dx_2(t)}{dt} &= \omega(x_1(t) - X_1^*) - r_2(x_2(t) - X_2^*) - \eta(x_2 + X_2^*)(x_2 - X_2^*)\\
&\quad - by(t-\tau_1)(x_2 - X_2^*) - bX_2^*(y(t-\tau_1) - Y^*),\\
\frac{d\ln y(t)}{dt} &= -a(y(t) - Y^*) - c(x_2(t-\tau_2) - X_2^*).
\end{aligned}$$

Let us define

$$V(t) = \max\{|x_1(t) - X_1^*|,\ |x_2(t) - X_2^*|,\ |\ln y(t) - \ln Y^*|\}\,.$$

From (H4), we can choose $\xi > 1$ and $\delta > 0$ such that

$$r_1+\omega > \xi\alpha + \delta, \quad r_2 + \eta X_2^* > \xi(\omega + bX_2^*) + \delta, \quad a > \xi c + \delta\,.$$

Note that

$$\begin{aligned} D^+|x_1(t)-X_1^*| &\le \alpha|x_2(t)-X_2^*|-(r_1+\omega)|x_1(t)-X_1^*|,\\ D^+|x_2(t)-X_2^*| &\le \omega|x_1(t)-X_1^*|-r_2|x_2(t)-X_2^*|-\eta X_2^*|x_2-X_2^*|\\ &\quad +bX_2^*|y(t-\tau_1)-Y^*|,\\ D^+|\ln y(t)-\ln Y^*| &\le -a|y(t)-Y^*|+c|x_2(t-\tau_2)-X_2^*|]. \end{aligned}$$

It is easy to see

$$D^+V(t)\le -\delta V(t),$$

if $V(t+\theta)\le \xi V(t)$ for $(-\tau^*\le\theta\le 0)$, where $\tau^*=\max\{\tau_1,\ \tau_2\}$, therefore the positive equilibrium is asymptotically stable and attracts all positive solutions, that is the positive equilibrium $(X_1^*,\ X_2^*,\ Y^*)$ of system (1.1) is globally asymptotically stable. The proof is completed. □

From Theorem 2.2 and Theorem 2.3, we know $E_1,\ E_2$ is locally asymptotically stable if (H2) holds. Now we consider the stability of the positive equilibrium E^* under the assumption (H2) holds.

Theorem 4.2. *If (H2) holds, then the positive equilibrium E^* of system (2.1) is unstable for any $\tau\ge 0$.*

Proof. By putting $\tau=0$ in (3.3), we obtain the characteristic equation

$$\lambda^3+A\lambda^2+(B-D)\lambda+(C-D)=0\,. \tag{4.1}$$

From (3.5) and (H2), we have $C-D<0$. Therefore the equation (4.1) has a positive characteristic root. Then E^* is unstable when $\tau=0$.

Let $\lambda=i\sigma,\ \sigma\in R$, the characteristic equation (3.3) reduces to

$$-i\sigma^3-A\sigma^2+B\sigma i+C=D[(\sigma\cos(\sigma\tau)-\sin(\sigma\tau)]\,.$$

Hence

$$\begin{cases} -\sigma^3+B\sigma=D[\sigma\cos(\sigma\tau)-\sin(\sigma\tau)]\,,\\ -A\sigma^2+C=D[\cos(\sigma\tau)+\sigma\sin(\sigma\tau)]\,. \end{cases} \tag{4.2}$$

It follows by taking the sum of squares, we have

$$\sigma^6+(A^2-2B)\sigma^4+(B^2-D^2-2AC)\sigma^2+C^2-D^2=0. \tag{4.3}$$

This is a cubic equation about σ^2. By using (H2), (3.4) and (3.5), it is easy to obtain $A^2-2B>0$ and $C^2-D^2<0$. Hence the equation (4.3) has an unique positive root, denotes σ_0^2 $(\sigma_0>0)$. Therefore the characteristic equation (4.1) has a pair of pure imaginary roots $\lambda=\pm i\sigma_0$.

By (3.3), we have

$$\frac{d(Re\lambda)}{d\tau}=\frac{-\lambda(\lambda+1)De^{-\lambda\tau}}{3\lambda^2+2A\lambda+B-De^{-\lambda\tau}+D\tau(\lambda+1)e^{-\lambda\tau}}=\frac{P}{Q},$$

from (3.4) and (4.2), we have

$$\begin{aligned}
\text{sign}\left\{\frac{d(Re\lambda)}{d\tau}\right\}_{\lambda=i\sigma_0} &= \text{sign}\left\{RePReQ + ImPImQ\right\}_{\lambda=i\sigma_0} \\
&= \text{sign}\left\{D\sigma_0(\cos(\sigma_0\tau) - \sigma_0\sin(\sigma_0\tau))(-3\sigma_0^2 + B) \right.\\
&\quad \left. - D\sigma_0(\cos(\sigma_0\tau) + \sigma_0\sin(\sigma_0\tau))2A\sigma_0 - D^2\sigma_0^2\right\} \\
&= \text{sign}\left\{\sigma_0(-\sigma_0^3 + B\sigma)(-3\sigma^2 + B) - \sigma_0(-A\sigma_0^2 + C)2A\sigma_0 - D^2\sigma_0^2\right\} \\
&= \text{sign}\left\{\sigma_0^2[3\sigma_0^4 + 2(A^2 - 2B)\sigma_0^2 + (B^2 - 2AC - D^2)]\right\} = 1\,.
\end{aligned}$$

That is

$$\frac{d(Re\lambda)}{d\tau}|_{\lambda=i\sigma_0} > 0\,.$$

Thus, the positive equilibrium E^* of system (2.1) is unstable for any $\tau \geq 0$ provided that (H2) holds. □

Remark. From the above analysis, we can see, our results show that the time delays $\bar{\tau}_1$, $\bar{\tau}_2$ have no effect on global stability of the positive equilibrium under some conditions, that is, the time delays are harmless or "useless".

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