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ADJACENT VERTEX DISTINGUISHING TOTAL COLORING OF P_n AND C_n DOUBLE GRAPH

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Abstract: A total coloring is called adjacent vertex distinguishing if every two adjacent vertices are incident to different sets of colored vertex and incident edge with vertex. The minimum number of colors required for a adjacent vertex distinguishing proper total coloring, a simple graph G is denoted by $\chi_{at}(G)$.

Let $G(V, E)$ be a simple graph. If $V(D(G)) = V(G) \cup V(G')$, $E(D(G)) = E(G) \cup E(G') \cup \{v_i v_j' | v_i \in V(G), v_j' \in V(G')$ and $v_i v_j \in E(G)\}$, then we call $D(G)$ is the double graph of G graph,where G' is the copy of G. The paper studies the adjacent vetex distinguishing total chromatic number of C_n of $D(G)$.

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1. Introduction

In computer science and information science, we know a series of concepts of new coloring of graphs, see [2], [8], [5], [6], [7]. In this paper, we get the adjacent vertex distinguishing total chromatic numbers of path and cycle of double graph.

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Definition 1. (see [9]) Let $G(V,E)$ be a simple graph. If $V(D(G)) = V(G) \cup V(G')$, $E(D(G)) = E(G) \cup E(G') \cup \{v_i v'_j | v_i \in V(G), v'_j \in V(G')$ and $v_i v_j \in E(G)\}$, we call that D(G) is the double graph of G.

Definition 2. (see [5]) Let $G(V,E)$ be a connect graph of which the order is at least 2, k is an positive integer and f is the mapping from $V(G) \cup E(G)$ to $\{1, 2, \cdots, k\}$. For any $v \in V(G)$, if:

1. for any $uv, vw \in E(G), u \neq w$, there is $f(uv) \neq f(vw)$;

2. for any $uv \in E(G), u \neq v$, there is $f(u) \neq f(v), f(u) \neq f(uv), f(v) \neq f(uv)$;

3. for any $uv \in E(G), u \neq v$, there is $C(u) \neq C(v)$.

Here $C(u) = \{f(u)\} \cup \{f(uv) | uv \in E(G)\}$. Then f is called a k-adjacent vertex-distinguishing of coloring of graph G (in brief, denoted by k-AVDTC) and $\chi_{at}(G) = \min\{k | G \quad \text{has} \quad k\text{-AVDTC}\}$ is called the adjacent vertex-distinguishing total chromatic number of graph G.

It is obvious that for any graph $G(|V(G)| \geq 2)$, $\chi_{at}(G)$ exists.

Conjecture 2. (see [9]) *For any connected graphs $G(V,E)$, if $|V(G)| \geq 6$, then $\chi_{at}(G) \leq \Delta(G) + 3$, where $\Delta(G)$ is the maximum degree of G.*

The paper does not define the terminologies and signs (see [1] or [4]).

2. Main Results

Lemma. (see [3]) *For graph G, if $uv \in G$ and $d(u) = d(v) = \Delta(G)$, then $\chi_{at}(G) \geq \Delta(G) + 2$.*

Theorem 1. *Suppose P_n is a path with order n $(n \geq 2)$, then when $n \equiv 2$, $\chi_{at}(D(P_n)) = 4$; when $n \equiv 3$, $\chi_{at}(D(P_n)) = 5$; when $n \geq 4$, $\chi_{at}(D(P_n)) = 6$.*

Proof. When n=2. According to Lemma, it is easy to know that $\chi_{at}(D(P_n)) \geq 4$. So, we only need to give a 4-AVDTC of $D(P_n)$. Suppose $C = \{1, 2, 3, 4\}, P_2 = u_1 u_2$, $P'_2 = v_1 v_2$.

Let f be as follows:

u_1, u_2 and v_1, v_2. are colored with colors 1, 2;

$u_1 u_2$ and $v_1 v_2$. are colored with colors 3;

$u_1 v_2$ and $v_1 u_2$. are colored with colors 4.

Obviously, f is a 4-AVDTC of $D(P_n)$.

When n=3. It is easy to know that $\chi_{at}(D(P_n)) \geq 5$. So, we only need to give a 5-AVDTC of $D(P_n)$. Suppose $C = \{1, 2, 3, 4, 5\}, P_3 = u_1u_2u_3,\ P_3' = v_1v_2v_3$.

Let f be as follows:

u_1, u_2, u_3 and v_1, v_2, v_3. are colored with colors 1, 2, 3;

u_1u_2, u_2u_3 and v_1v_2, v_2v_3. are colored with colors 3, 1;

u_1v_2, v_2u_3 and v_1u_2, u_2v_3. are colored with colors 4, 5.

Obviously, f is a 5-AVDTC of $D(P_n)$

When $n \geq 4$. According to Lemma, it is easy to know that $\chi_{at}(D(P_n)) \geq 6$. So, we only need to give a 6-AVDTC of $D(P_n)$. Suppose $C = \{1, 2, 3, 4, 5, 6\}$, $P_n = u_1u_2 \cdots u_n,\ P_n' = v_1v_2 \cdots v_n$.

Let f be as follows:

$u_1, u_2 \cdots u_n$ and $v_1, v_2 \cdots v_n$. are colored with colors 1, 2, 3, 4, 5, 6 alternately;

$u_1u_2, u_2u_3 \cdots u_{n-1}u_n$ and $v_1v_2, v_2v_3 \cdots v_{n-1}v_n$. Are colored with colors 6, 1, 2, 3, 4, 5 alternately;

$u_1v_2, v_2u_3 \cdots v_{n-1}u_n$ and $v_1u_2, u_2v_3 \cdots u_{n-1}v_n$. Are colored with colors 4, 5, 6, 1, 2, 3 alternately.

Obviously, f is a 6-AVDTC of $D(P_n)$. So, the theorem is true. □

Theorem 2. *Suppose C_n is a cycle with order n $(n \geq 2)$, When $n \equiv 0(mod 3), n \equiv 0(mod 4)$ or $n \equiv 0(mod 5)$, then $\chi_{at}(D(C_n)) = 6$.*

Proof. According to Lemma, it is easy to know that $\chi_{at}(D(P_n)) \geq 6$. So, we only need to give a 6-AVDTC of $D(C_n)$. Suppose $C = \{1, 2, 3, 4, 5, 6\}$, $C_n = u_1u_2 \cdots u_nu_1,\ C_n' = v_1v_2 \cdots v_nv_1$.

Case 1. When $n \equiv 0(mod 3)$, we only need to give a 6-AVDTC of $D(C_n)$.

Let σ be as follows:

$u_1, u_2 \cdots u_n$ and $v_1, v_2 \cdots v_n$ are colored with colors 1, 2, 3 alternately;

$u_1u_2, u_2u_3 \cdots u_{n-1}u_n, u_nu_1$ and $v_1v_2, v_2v_3 \cdots v_{n-1}v_n, v_nv_1$ are colored with colors 6, 1, 2 alternately.

When $n \equiv 0(mod 2)$:

$u_1v_2, v_2u_3 \cdots u_{n-1}v_n, v_nu_1$ and $v_1u_2, u_2v_3 \cdots v_{n-1}u_n, u_nv_1$ are colored with colors 3, 4, 5 alternately.

When $n \equiv 1(mod 2)$:

$u_1v_2, v_2u_3 \cdots v_{n-1}u_n, u_nv_1$ and $v_1u_2, u_2v_3 \cdots u_{n-1}v_n, v_nu_1$ are colored with colors 3, 4, 5 alternately.

Obviously,σ is a 6-AVDTC of $D(C_n)$.

Case 2. When $n \equiv 0(mod4)$, we only need to give a 6-AVDTC of $D(C_n)$.

Let σ be as follow:

$u_1, u_2 \cdots u_n$ and $v_1, v_2 \cdots v_n$. are colored with colors 1,4,2,3 alternately.

$u_1u_2, u_2u_3 \cdots u_{n-1}u_n, u_nu_1$ and $v_1v_2, v_2v_3 \cdots v_{n-1}v_n, v_nv_1$ are colored with colors 6, 1, 6, 2 alternately.

$u_1v_2, v_2u_3 \cdots u_{n-1}v_n, v_nu_1$ and $v_1u_2, u_2v_3 \cdots v_{n-1}u_n, u_nv_1$ are colored with colors 3, 5, 4, 5 alternately.

Obviously,σ is a 6-AVDTC of $D(C_n)$.

Case 3. When $n \equiv 0(mod5)$, we only need to give a 6-AVDTC of $D(C_n)$.

Let σ be as follows:

$u_1, u_2 \cdots u_n$ and $v_1, v_2 \cdots v_n$. are colored with colors 1, 2, 1, 2, 3 alternately.

$u_1u_2, u_2u_3 \cdots u_{n-1}u_n, u_nu_1$ and $v_1v_2, v_2v_3 \cdots v_{n-1}v_n, v_nv_1$ are colored with colors 6, 5, 6, 1, 2 alternately.

When $n \equiv 0(mod2)$,

$u_1v_2, v_2u_3 \cdots u_{n-1}v_n, v_nu_1$ and $v_1u_2, u_2v_3 \cdots v_{n-1}u_n, u_nv_1$ are colored with colors 3, 4, 3, 4, 5 alternately.

When $n \equiv 1(mod2)$,

$u_1v_2, v_2u_3 \cdots v_{n-1}u_n, u_nv_1$ and $v_1u_2, u_2v_3 \cdots u_{n-1}v_n, v_nu_1$ are colored with colors 3, 4, 3, 4, 5 alternately.

Obviously σ is a 6-AVDTC of $D(C_n)$.

So, the theorem is true. □

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