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AN APPLICATION OF THE DOMAIN DECOMPOSITION METHOD TO THREE-DIMENSIONAL LARGE-SCALE MAGNETIC FIELD ANALYSES

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Abstract: REVOCAP_Magnetic is a module developed in the project of "Revolutionary Simulation Software" for analysis of 3D magnetic fields. The hierarchical domain decomposition method is implemented for computing in parallel environments using PC clusters or network-connected computers. It is confirmed to be able to analyze problems with fifty million degrees of freedom by REVOCAP_Magnetic.

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1. Introduction

There are many machines or devices where the electromagnetic phenomena are applied such as a computer, a cell phone, a transformer, and a Magnetic Resonance Imaging (MRI), etc. In order to analyze these engineering or physical phenomena, computer simulation is a reliable and yet economical approach.

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Moreover, a computational object is made to a large scale and complicated for numerical analysis recently. In addition, subdivision of the mesh is performed for the improvement of accuracy. Therefore, large-scale computations is increasingly important in electromagnetic field problems. To reply this requirement, we have developed REVOCAP_Magnetic.

In this paper, we outline REVOCAP_Magnetic and show some computational results.

2. Ountline of REVOCAP_Magnetic

REVOCAP_Magnetic is a module developed in the project of "Revolutionary Simulation Software", see [1]. The Hierarchical Domain Decomposition Method (HDDM) (see [5] [6]) together with the data handling type "Parallel processor mode (P-mode)" (see [3]) is implemented for computing in parallel environments using PC clusters or network-connected computers.

Then, REVOCAP_Magnetic supports two analyses.

The first is 3D non-linear magnetostatic analysis using the Newton method to solve the simultaneous non-linear equations and the A method with the continuity of the electric current density that uses the magnetic vector potential A as an unknown function, see [4]. In this capability, Conjugate Gradient (CG) method is used as the symmetric solver. In each subdomain, the CG method is again used as the solver for the symmetric system arising in approximations. Then, a shifted incomplete Cholesky factorization is used as the preconditioner.

The second is 3D time-harmonic eddy current analysis using the A-ϕ method that uses the magnetic vector potential A and the electric scalar potential ϕ as unknown functions, and the A method, see [3]. These formulations are used with the continuity of the excitation current density. Conjugate Orthogonal Conjugate Gradient (COCG) method is used as the complex symmetric solver. In each subdomain, the COCG method is again used as the solver for the complex symmetric system arising in approximations. Then a similar shifted incomplete Cholesky factorization is again used as the preconditioner.

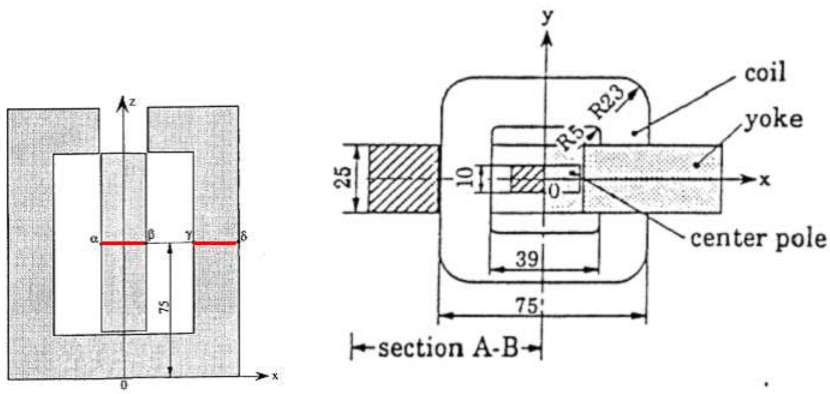

Figure 1: TEAM workshop problem 20

3. Numerical Examples

3.1. Non-Linear Magnetostatic Analysis

Testing Electromagnetic Analysis Methods (TEAM) Workshop Problem 20 (see [2]) is considered, which consists of a center pole, a yoke and a coil (see Figure 1). The center pole and the yoke are made of SS400, and the coil is made of polyimide electric wire. The electric current in the coil is 1,000[A]. The magnetic reluctivity is a positive constant in each element such that in the region of air and coil, the value is $1/\left(4\pi \times 10^{-7}\right)$[m/H]. The B-H curve is used for the ν-B curve in the center pole and the yoke. Table 1 shows numbers of elements, Degrees of Freedom (DOF) and subdomains.

Mesh	elements	DOF	subdomains
Mesh(1)	26,813,542	31,286,845	32×8,400
Mesh(2)	34,917,602	40,722,854	32×11,200
Mesh(3)	43,141,979	50,295,288	32×14,000

Table 1: Numbers of elemets, DOF and subdomains

The Newton iteration is stopped by $\left\|x^{n+1} - x^n\right\| / \left\|x^{n+1}\right\| < 1.0 \times 10^{-3}$, where x^{n+1} is the solution vector, and $\left\|\cdot\right\|$ denotes the Euclidean norm, respectively. In the initial computation of the Newton iteration, the magnetic reluctivity of the center pole and the yoke is 100[m/H]. A simplified block diag-

Mesh	iteration counts	CPU time [s]	Memory per CPU [MB]
Mesh(1)	2	16,441	392
Mesh(2)	2	23,371	512
Mesh(3)	2	31,111	633

Table 2: Iteration counts of the Newton method, CPU time and amount of memory

Mesh	Magnetic flux density B_z[T]			Relative error [%]	
	Mesured				
	I	II	Computed	vs. I	vs. II
Mesh(1)	0.72	0.71	0.665	7.6	6.3
Mesh(2)			0.670	6.9	5.6
Mesh(3)			0.672	6.7	5.4

Table 3: The magnetic flux density on the α-β face

onal scaling is used as the preconditioner in the CG procedure on the interface. Each process is stopped when the residual norm becomes less than 10^{-3}. In each subdomain, the resultant linear equations are solved by the shifted ICCG method, namely the CG method that uses the shifted incomplete Cholesky factorization as the preconditioner (the shift value is 1.2), and the ICCG method is stopped when the residual norm becomes less than 10^{-9}.

Computation was performed by 32 CPUs using *Pentium 4 HT 630* (3.0 GHz / 64bit / 2MB L2). Table 2 shows iteration counts of the Newton method, CPU time and amount of memory. Table 3 and Table 4 show comparisons between measured values (see [2]) and our computed values on α-β (-12.5[mm] $\leq x \leq$ 12.5[mm], -5.0[mm] $\leq y \leq$ 5.0[mm], $z = 75.0$[mm]) and γ-δ (38.5[mm] $\leq x \leq$ 63.5[mm], -12.5[mm] $\leq y \leq$ 12.5[mm], $z = 75.0$[mm]) faces (see Figure 1). From comparisons of the magnetic flux density in Table 3 and Table 4, we can conclude that our results are suitable.

3.2. Time-Harmonic Eddy Current Analysis

We consider a simple model for the accuracy verification of the eddy current analysis using the solenoidal coil with unlimited length (see Figure 2), [3]. The radius of the conductor is 0.1m, and the height of z-axis is 0.1m. The magnetic reluctivity is $1/\left(4\pi \times 10^{-7}\right)$[m/H]. The conductivity in the conductor is $7.7 \times$

Mesh	Magnetic flux density B_z[T]			Relative error [%]	
	Mesured				
	I	II	Computed	vs. I	vs. II
Mesh(1)	-0.14	-0.13	-0.1298	7.3	0.15
Mesh(2)			-0.1305	6.8	0.38
Mesh(3)			-0.1310	6.4	0.77

Table 4: The magnetic flux density on the γ-δ face

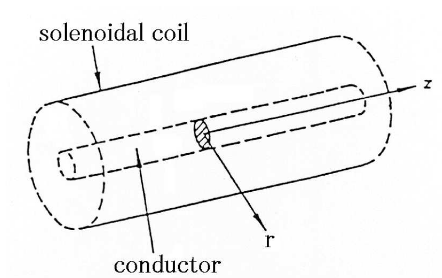

Figure 2: Solenoidal coil with unlimited length

Mesh	elements	DOF	subdomains
Mesh(1)	17,065,354	21,470,601	32×5,000
Mesh(2)	25,917,735	32,537,036	32×7,500
Mesh(3)	34,814,775	43,546,445	32×10,000

Table 5: Numbers of elemets, DOF and subdomains

10^6[S/m]. The angular frequency is $2\pi \times 60$[rad/s]. The absolute value of real (or imaginary) part of the excitation current density $|J_r|$ (or $|J_i|$) in the coil is 50 (or 0) [A/m^2]. Dirichlet boundary conditions of $A \times n = 0$ and $\phi = 0$ are given on the surfaces of $\theta = 0°$ and $\theta = 20°$. Table 5 shows numbers of elements, DOF and subdomains.

In this section, A-ϕ method is used for computing. A simplified block diagonal scaling is used as the preconditioner in the COCG procedure on the interface. Each process is stopped when the residual norm becomes less than 10^{-3}. In each subdomain, the COCG method is used as the solver for the complex symmetric (not Hermitian) system arising in approximations. A shifted incomplete Cholesky factorization is used as the preconditioner with the accelerative parameter 1.2. The COCG method in each subdomain is stopped

Mesh	CPU time [s]	iteration counts	Memory per CPU [MB]
Mesh(1)	6,789	735	512
Mesh(2)	12,400	891	685
Mesh(3)	17,228	935	913

Table 6: CPU time, number of iterations and amount of memory

Mesh	Theoretical [A/m]	Computed [A/m]	Relative error [%]
Mesh(1)		9.87e-01	1.29
Mesh(2)	1.0e+00	9.89e-01	1.10
Mesh(3)		9.90e-01	1.03

Table 7: The real part of the magnetic field H[A/m] at r=0.1m

when the preconditioned residual norm becomes less than 10^{-9}. Computation was performed by 32 CPUs using Pentium 4 HT 630 (3.0 GHz / 64bit / 2MB L2). Table 6 shows CPU time, iteration counts to solve interface problems and amount of memory per CPU. Table 7 compares theoretical values with computed values at $r = 0.1$[m]. From the comparison of the magnetic field in Table 7, we can conclude that our results are suitable.

4. Conclutions

To enable a large-scale 3D magnetic field analysis, we have developed REVOCAP_Magnetic and have shown the possibility of large-scale analysis in magnetic field problems. Moreover, we have verified the accuracy of our results.

In future research, it is very important for us to reduce number of iterations and computational time. As one possibility, we are trying to reexamine the solution strategy of FEA (Finite Element Analysis) in subdomains.

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