International Journal of Pure and Applied Mathematics

Volume 42 No. 2 2008, 161-168

UNIFORM PAGE MIGRATION ON GENERAL NETWORKS

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Abstract: In this paper we consider the page migration problem in the uniform model. We give a $2+\sqrt{2}$-competitive deterministic online algorithm on general networks. We also show an improved lower bound of 3.1639 for general networks and an explicit lower bound of 3.1213 for ring networks.

AMS Subject Classification: 90C35
Key Words: page migration, data management, online algorithm

1. Introduction

The page migration problem is, given a graph $G=(V,E)$ with the weight $w: E \to \mathbb{R}^+$, an integer $D \geq 1$, and nodes $s_0, r_1, \dots, r_k \in V$, to compute $s_1, \dots, s_k \in V$ so that the cost function $\sum_{i=1}^{k}(d(s_{i-1}, r_i) + D \cdot d(s_{i-1}, s_i))$ is minimized, where $d(u,v)$ is the minimum sum of weights of edges of a path connecting u and v. The setting of $D=1$ is often called the *uniform model*. An *online* page migration algorithm computes s_i without any information of $r_{i+1}, \dots, r_k$ for $1 \leq i < k$. We denote by $c_A(\sigma)$ the cost of a page migration algorithm A for an instance $\sigma = (G, D, s_0, r_1, \dots, r_k)$. An online page migration algorithm ALG is *ρ-competitive* if there exists a value α independent of k such that $c_{\text{ALG}}(\sigma) \leq \rho \cdot c_{\text{OPT}}(\sigma) + \alpha$ for an optimal offline algorithm OPT and for any σ. The online page migration has network applications such as memory management in a shared memory multiprocessor system and Peer-to-Peer applications on Internet. In the context of certain network applications, $s_0, \dots, s_k$ and $r_1, \dots, r_k$ are called *servers* and *clients*, respectively.

Received: August 17, 2007

It is mentioned in [4] that a naive deterministic algorithm which always moves the server to the client is $2D+2$-competitive. A currently best deterministic algorithm for general networks given by Bartal, Charikar, and Indyk [1] is 4.086-competitive. Black and Sleator [2] gave a 3-competitive deterministic algorithm for trees, uniform networks, and Cartesian products of those networks, including grids and hypercubes. Besides, for $D=1$, a 3-competitive deterministic algorithm on arbitrary 3-node networks was given in [3]. The tightness of the competitive ratio of 3 for deterministic algorithms was shown in [2] by proving that no deterministic algorithm has competitive ratio less than 3 for one link networks. As for lower bounds of deterministic algorithms for other networks, a lower bound of $\frac{85}{27} \simeq 3.1481$ for general networks, which was previously known as the best lower bound, was given in [3]. It was also mentioned in [3] that the lower bound for ring networks (i.e., single cycles) is greater than 3, but neither explicit value nor written proof was given.

In this paper we give a $2+\sqrt{2}$-competitive deterministic algorithm on general networks for $D=1$. Moreover, we show an improved lower bound of 3.1639 for general networks and an explicit lower bound of 3.1213 for ring networks.

2. Algorithm on General Networks

In this section we show the following theorem.

Theorem 1. *For $D=1$, there exists a $2+\sqrt{2}$-competitive deterministic page migration algorithm on general networks.*

Proof. Let $\rho = 2+\sqrt{2}$ and $r_0 = s_0$. We prove that an algorithm which sets $s_i = s_{i-1}$ if $d(s_{i-1}, r_i) - d(r_{i-1}, r_i) \le \frac{\rho-2}{\rho} d(s_{i-1}, r_{i-1})$, $s_i = r_{i-1}$ otherwise $(i \ge 1)$ is ρ-competitive.

Let $t_1, \ldots, t_k$ be a solution computed as $s_1, \ldots, s_k$ by an optimal offline algorithm. Let $\Phi(s_i, r_i, t_i) = \frac{\rho}{2}(d(s_i, t_i) + d(r_i, t_i)) + (\frac{\rho}{2} - 1) d(s_i, r_i)$. To prove the theorem, it suffices to show

$$\begin{aligned}\Delta_1 \equiv & d(s_{i-1}, r_i) + d(s_{i-1}, s_i) + \Phi(s_i, r_i, t_{i-1}) - \Phi(s_{i-1}, r_{i-1}, t_{i-1}) \\ & - \rho \cdot d(t_{i-1}, r_i) \le 0, \text{ and} \\ \Delta_2 \equiv & \Phi(s_i, r_i, t_i) - \Phi(s_i, r_i, t_{i-1}) - \rho \cdot d(t_{i-1}, t_i) \le 0\end{aligned}$$

for each $i \ge 1$.

First, $\Delta_2 = \frac{\rho}{2}(d(s_i, t_i) - d(s_i, t_{i-1}) + d(r_i, t_i) - d(r_i, t_{i-1})) - \rho \cdot d(t_{i-1}, t_i) \le \frac{\rho}{2}(d(t_{i-1}, t_i) + d(t_{i-1}, t_i)) - \rho \cdot d(t_{i-1}, t_i) = 0$. Next, if $s_i = s_{i-1}$, then $\Delta_1 =$

$d(s_{i-1}, r_i) + \frac{\rho}{2}(d(r_i, t_{i-1}) - d(r_{i-1}, t_{i-1})) + (\frac{\rho}{2} - 1)(d(s_{i-1}, r_i) - d(s_{i-1}, r_{i-1})) - \rho \cdot d(t_{i-1}, r_i) \leq \frac{\rho}{2}(d(s_{i-1}, r_i) - d(r_{i-1}, r_i)) - (\frac{\rho}{2} - 1)d(s_{i-1}, r_{i-1})$, which is at most 0 by the definition of the algorithm. Finally, if $s_i = r_{i-1}$, then $\Delta_1 = d(s_{i-1}, r_i) + d(s_{i-1}, r_{i-1}) + \frac{\rho}{2}(d(r_{i-1}, t_{i-1}) - d(s_{i-1}, t_{i-1}) + d(r_i, t_{i-1}) - d(r_{i-1}, t_{i-1})) + (\frac{\rho}{2} - 1)(d(r_{i-1}, r_i) - d(s_{i-1}, r_{i-1})) - \rho \cdot d(t_{i-1}, r_i) \leq -(\frac{\rho}{2} - 1)(d(s_{i-1}, r_i) - d(r_{i-1}, r_i)) - (\frac{\rho}{2} - 2)d(s_{i-1}, r_{i-1}) < (-\frac{(\rho-2)^2}{2\rho} - (\frac{\rho}{2} - 2))d(s_{i-1}, r_{i-1})) = 0$. □

3. Lower Bound for General Networks

In this section we show the following theorem.

Theorem 2. *There exists no ρ-competitive deterministic page migration algorithm for general networks if $\rho < 3.1639$.*

A lower bound of the competitive ratio of $\frac{85}{27} \simeq 3.1481$ for general networks was given in [3] by showing the following lemmas:

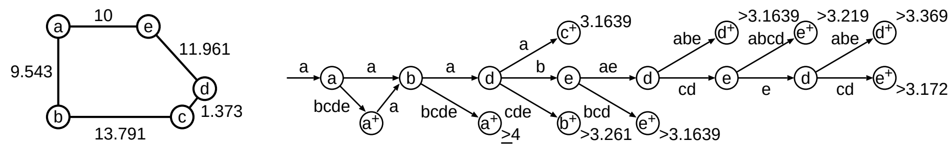
Lemma A. *For any deterministic online page migration algorithm* ALG *and any optimal offline algorithm* OPT*, there exists an instance $\sigma = (G, D, s_0, r_1, \ldots, r_k)$ such that $c_{\text{ALG}}(\sigma) \geq \frac{85}{27} c_{\text{OPT}}(\sigma) > 0$ and that both* ALG *and* OPT *set $s_k = r_k$.*

Lemma B. *For any deterministic online page migration algorithm* ALG *and any optimal offline algorithm* OPT*, if there exists an instance $\sigma = (G, D, s_0, r_1, \ldots, r_k)$ such that $c_{\text{ALG}}(\sigma) \geq \rho \cdot c_{\text{OPT}}(\sigma) > 0$ and that both* ALG *and* OPT *set $s_k = r_k$, then there exists an instance $\sigma' = (G', D, s'_0, r'_1, \ldots, r'_{k'})$ such that $c_{\text{ALG}}(\sigma') \geq \rho \cdot c_{\text{OPT}}(\sigma') + \alpha$ for any α independent of k'.*

Lemma A was proved in [3] by giving a 4-node ring and an adversary's strategy which satisfy the conditions of the lemma. We modify the ring and the strategy of [3] and obtain the following lemma.

Lemma 1. *For any deterministic online page migration algorithm* ALG *and any optimal offline algorithm* OPT*, there exists an instance $\sigma = (G, D, s_0, r_1, \ldots, r_k)$ such that $c_{\text{ALG}}(\sigma) \geq 3.1639 c_{\text{OPT}}(\sigma) > 0$ and that both* ALG *and* OPT *set $s_k = r_k$.*

Proof. We define G as a 5-node ring R_1 shown in Figure 1(a) and a strategy for an adversary ADV to generate clients on R_1 as shown in Figure 1(b). We set $D = 1$ and $s_0 = a$. The strategy is illustrated by a tree-like DAG, in



(a) R_1 (b) ADV's strategy

Figure 1: Ring R_1 and ADV's strategy on R_1

servers of ALG	clients of ADV	servers of OPT	$c_{\text{ALG}}/c_{\text{OPT}} \geq$
$aaaac$	$abdc^+$	$abccc$	$78.172/24.707 > 3.1639$
$aaab[ae][abe]d$	$abdedd^+$	$abddddd$	$116.016/36.668 > 3.1639$
$aaab[ae][cd][abcd]e$	$abdedee^+$	$aaeeeeee$	$139.938/43.465 > 3.219$
$aaab[ae][cd]e[abe]d$	$abdededd^+$	$abddddddd$	$163.86/48.629 > 3.369$
$aaab[ae][cd]e[cd]e$	$abdedede^+$	$aaeeeeeee$	$175.821/55.426 > 3.172$
$aaab[bcd]e$	$abdee^+$	$aaeeee$	$99.676/31.504 > 3.1639$
$aaa[cde]b$	$abdb^+$	$abbbb$	$80.59/24.707 > 3.261$
$aa[bcde]a$	aba^+	$aaaa$	$38.172/9.543 \geq 4$

Table 1: Cost ratios of OPT and ALG

which each edge represents a server set by an online algorithm ALG, and each node represents a client chosen by ADV. An edge with more than one servers denotes that ALG moves its server to one of the nodes. A client followed by a plus sign denotes that ADV repeats the client until ALG moves the server to the client. In response to the moves of ALG, an online game between ALG and ADV proceeds along a path from the unique source node to a sink node on the DAG. Table 3 shows the cost ratios of ALG and OPT for all the paths except the paths preceded by the nodes aa^+, which clearly increase only the cost of ALG. By Table 3, the minimum cost ratio is 3.1639. □

By Lemmas B and 1, we have Theorem 2.

The precise weight of R_1 is obtained from the conditions that the four cost ratios for ALG's servers (ADV's clients, respectively) $aaaac$ ($abdc^+$), $aaabbe$ ($abdee^+$), $aaabde$ ($abdee^+$), $aaabeed$ ($abdedd^+$) are the same and that $d(a, c)$ is exactly half of the total weight, maximizing the cost ratio for ALG's servers $aaaac$ (ADV's clients $abdc^+$).

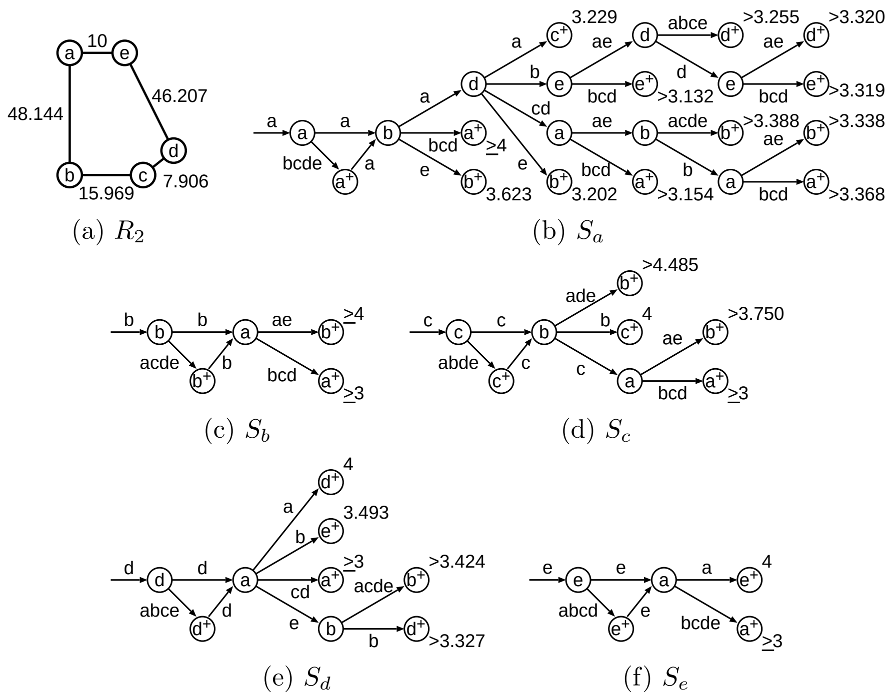


Figure 2: Ring R_2 and ADV's partial strategies on R_2

4. Lower Bound for Ring Networks

The proof of Theorem 2 requires a sufficiently large tree-of-rings network due to Lemma B. In this section we give a lower bound for ring networks.

Theorem 3. *There exists no ρ-competitive deterministic page migration algorithm for ring networks if $\rho < 3.1213$.*

Proof. We show that for any deterministic online page migration algorithm ALG, there exists an instance $\sigma = (G, D, s_0, r_1, \ldots, r_k)$ such that G is a ring and that $c_{\text{ALG}}(\sigma) \geq 3.1213 c_{\text{OPT}}(\sigma) + \alpha$ for any α independent of k. To show this, we define G as a 5-node ring R_2 shown in Figure 2(a) and a strategy for an adversary ADV to generate arbitrarily long sequence of clients on R_2 such that $\frac{c_{\text{ALG}}(\sigma)}{c_{\text{OPT}}(\sigma)} \geq 3.1213$ with an arbitrarily large $c_{\text{OPT}}(\sigma)$. The strategy consists of partial strategies S_a, S_b, S_c, S_d, and S_e (Figure 2). We set $D = 1$. By an argument similar to the proof of Lemma 1 and by the cost ratios for

servers of ALG	clients of ADV	servers of OPT	$c_{\mathrm{ALG}}/c_{\mathrm{OPT}} \geq$
$aaaac$	$abdc^+$	$abccc$	$232.577/72.019 > 3.229$
$aaab[ae][abce]d$	$abdedd^+$	$abddddd$	$384.915/118.226 > 3.255$
$aaab[ae]d[ae]d$	$abdeded^+$	$abdddddd$	$546.025/164.433 > 3.320$
$aaab[ae]d[bcd]e$	$abdedee^+$	$aaeeeeee$	$499.818/150.558 > 3.319$
$aaab[bcd]e$	$abdee^+$	$aaeeee$	$326.927/104.351 > 3.132$
$aaa[cd][ae][acde]b$	$abdabb^+$	$abbbbbb$	$407.167/120.163 > 3.388$
$aaa[cd][ae]b[ae]b$	$abdabab^+$	$abbbbbbb$	$561.836/168.307 > 3.338$
$aaa[cd][ae]b[bcd]a$	$abdabaa^+$	$aaaaaaaa$	$513.692/152.495 > 3.368$
$aaa[cd][bcd]a$	$abdaa^+$	$aaaaaa$	$329.179/104.351 > 3.154$
$aaaeb$	$abdb^+$	$abbbb$	$230.639/72.019 > 3.202$
$aa[bcd]a$	aba^+	$aaaa$	$192.576/48.144 = 4$
$aaeb$	abb^+	$abbb$	$174.432/48.144 > 3.623$
$bb[ae]b$	bab^+	$bbbb$	$192.576/48.144 = 4$
$bb[bcd]a$	baa^+	$baaa$	$144.432/48.144 = 3$
$cc[ade]b$	cbb^+	$cbbb$	$71.625/15.969 > 4.485$
$ccbc$	cbc^+	$cccc$	$63.876/15.969 = 4$
$ccc[ae]b$	$cbab^+$	$cbbbb$	$240.483/64.113 > 3.750$
$ccc[bcd]a$	$cbaa^+$	$cbaaa$	$192.339/64.113 = 3$
$ddad$	dad^+	$dddd$	$224.828/56.207 = 4$
$ddbe$	dae^+	$deee$	$196.37/56.207 > 3.493$
$dd[cd]a$	daa^+	$daaa$	$168.621/56.207 = 3$
$dde[acde]b$	$dabb^+$	$dbbbb$	$246.609/72.019 > 3.424$
$ddebd$	$dabd^+$	$ddddd$	$266.452/80.082 > 3.327$
$eeae$	eae^+	$eeee$	$40/10 = 4$
$ee[bcde]a$	eaa^+	$eaaa$	$30/10 = 3$

Table 2: Cost ratios of of OPT and ALG

the partial strategies shown in Table 4, for each node v of R_2 and any online algorithm A, there exists a sequence χ_A^v of clients such that $c_A((R_2, 1, v, \chi_A^v)) \geq 3c_{\mathrm{OPT}}((R_2, 1, v, \chi_A^v)) > 0$ and that both A and OPT set their servers on the last client of χ_A^v. As done in the proof of Lemma 1, we omit to consider the sequences beginning with vv^+ in S_v.

We set $s_0 = a$. ADV generates clients in phases: The i-th phase ($i \geq 1$) is defined as $\chi_{A_i}^{v_i}$, where A_i is the algorithm performed by ALG in the i-th phase, and v_i is the server of ALG (also of OPT) just before the i-th phase. Let $\sigma_i = (R_2, 1, v_i, \chi_{A_i}^{v_i})$. The theorem is proved by observing that $\frac{\sum_i c_{A_i}(\sigma_i)}{\sum_i c_{\mathrm{OPT}}(\sigma_i)} \geq 3.1213$. By Table 4, all the sequences of clients in the partial strategies yield the cost ratios greater than 3.1213 except for baa^+, $cbaa^+$, daa^+, and eaa^+. Assume that there exists $j > 1$ with $v_j \in \{b, c, d, e\}$ and $\chi_{A_j}^{v_j} \in \{baa^+, cbaa^+, daa^+, eaa^+\}$.

$\chi_{A_j}^{v_j}$	$\chi_{A_{j-1}}^{v_{j-1}}$ ended with v_j^+	$\frac{c_{A_{j-1}}(\sigma_{j-1})+c_{A_j}(\sigma_j)}{c_{\text{OPT}}(\sigma_{j-1})+c_{\text{OPT}}(\sigma_j)} \geq$
baa^+	$abdabb^+$	$(407.167+144.432)/(120.163+48.144) > 3.277$
	$abdabab^+$	$(561.836+144.432)/(168.307+48.144) > 3.262$
	$abdb^+$ or $dabb^+$	$(230.639+144.432)/(72.019+48.144) > 3.1213$
	abb^+ or bab^+	$(174.432+144.432)/(48.144+48.144) > 3.311$
	cbb^+	$(71.625+144.432)/(15.969+48.144) > 3.369$
	$cbab^+$	$(240.483+144.432)/(64.113+48.144) > 3.428$
$cbaa^+$	$abdc^+$	$(232.577+192.339)/(72.019+64.113) > 3.1213$
	cbc^+	$(63.876+192.339)/(15.969+64.113) > 3.199$
daa^+	$abdedd^+$	$(384.915+168.621)/(118.226+56.207) > 3.173$
	$abdeded^+$	$(546.025+168.621)/(164.433+56.207) > 3.238$
	dad^+	$(224.828+168.621)/(56.207+56.207) = 3.5$
	$dabd^+$	$(266.452+168.621)/(80.082+56.207) > 3.192$
eaa^+	$abdedee^+$	$(499.818+30)/(150.558+10) > 3.299$
	$abdee^+$	$(326.927+30)/(104.351+10) > 3.1213$
	dae^+	$(196.37+30)/(56.207+10) > 3.419$
	eae^+	$(40+30)/(10+10) = 3.5$

Table 3: Combined cost ratios for the $(j-1)$st and j-th phases

If there exists no such j, then the theorem is immediate. Table 4 shows that $\frac{c_{A_{j-1}}(\sigma_{j-1})+c_{A_j}(\sigma_j)}{c_{\text{OPT}}(\sigma_{j-1})+c_{\text{OPT}}(\sigma_j)} \geq 3.1213$ for every possible combination of $\chi_{A_j}^{v_j}$ and $\chi_{A_{j-1}}^{v_{j-1}}$ ended with v_j^+. Therefore, we have the theorem. □

The precise weight of R_2 are obtained from the conditions that the four combined cost ratios for ALG's servers (ADV's clients, respectively) *aaaac-cccba* (*abdc*$^+$-*cbaa*$^+$), *aaabbe-eeea* (*abdee*$^+$-*eaa*$^+$), *aaabde-eeea* (*abdee*$^+$-*eaa*$^+$), *aaaeb-bbba* (*abdb*$^+$-*baa*$^+$) are the same and that $d(a,c)$ is exactly half of the total weight.

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