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REFLECTION AND DEFORMATION IN MAGNETO-THERMO-MICROSTRETCH ELASTIC SOLID

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Abstract: The present investigation deals with the reflection and deformation in thermo-microstretch elastic solid in the presence of a transverse magnetic field, at the boundary surface. The generalized theories of thermoelasticity developed by Lord and Shulman [6] and Green and Lindsay [4] have been used to investigate the problem. Thermal and magnetic effects on the amplitude ratios of various reflected waves with the angle of incidence have been depicted graphically. Fourier transform technique is used to study the deformation due to time harmonic distributed thermomechanical sources. Uniformly and linearly distributed sources have been taken to illustrate the utility of the apporach. The integral transform has been inverted by using a numerical technique to obtain the components of normal force stress, tangential couple stress, microstress, temperature distribution and induced magnetic field. The resulting quantities have been depicted graphically for different sources to depict the thermal and magnetic effects.

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1. Introduction

Eringen [1], [2] developed a theory of thermo-microstretch elastic solids and

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fluids. Different authors discuss different types of problems in these theories.

2. Basic Equations

The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous and perfectly conducting elastic solid are the following:

$$\begin{cases} \epsilon_{ijk}h_{k,j} - (J_i + \epsilon_0 \dot{E}_i) = 0, \quad \epsilon_{ijk}E_{j,i} + \mu_0 \dot{h}_k = 0, \\ E_i + \mu_0(\epsilon_{ijk}\dot{u}_j H_{0k}) = 0, \quad h_{i,i} = 0\,. \end{cases} \tag{1}$$

Maxwell stress components are given by

$$T_{ij} = \mu_0(H_i h_j + H_j h_i - H_k h_k \delta_{ij})\,. \tag{2}$$

The above equations (1) are supplemented by the field of equations of motion and constitutive relations in the theory of generalized thermo-microstretch elastic solid, taking into account the Lorentz force, are

$$(\lambda+\mu)u_{j,ij}+(\mu+K)u_{i,jj}+K\epsilon_{ijk}\phi_{k,j}-\nu\,(T_{,i}+\tau_1\dot{T}_{,i})+\lambda_0\phi^*_{,i}+F_i-\rho\,\ddot{u}_i=0, \tag{3}$$

$$(\alpha+\beta)\phi_{j,ij}+\gamma\phi_{i,jj}+K\epsilon_{imn}u_{n,m}-2K\phi_i-\rho\jmath\ddot{\phi}_i=0, \tag{4}$$

$$\alpha_0\phi^*_{,rr}+\nu_1(T+\tau_1\dot{T})-\lambda_1\phi^*-\lambda_0 u_{j,j}-\frac{1}{2}\rho\jmath_0\ddot{\phi}^*=0, \tag{5}$$

$$\rho\,c^*(\dot{T}+\tau_0\ddot{T})+\nu_1 T_0(\dot{\phi}^*+n_0\tau_0\ddot{\phi}^*)+\nu\,T_0(\dot{u}_{j,j}+n_0\tau_0\ddot{u}_{j,j})-K^*T_{,rr}=0, \tag{6}$$

$$\sigma_{ij}=\lambda\,u_{r,r}\delta_{ij}+\mu(\,u_{i,j}+\,u_{j,i})+K(\,u_{j,i}-\epsilon_{ijr}\phi_r)-\nu(T+\tau_1\dot{T})\delta_{ij}, \tag{7}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}, \tag{8}$$

$$\lambda_k=\alpha_0\phi^*_{,k}+b_0\epsilon_{klm}\phi_{l,m}, \tag{9}$$

where the symbols have their usual meaning and the Lorentz force is given by

$$F_i=\mu_0\epsilon_{ijk}J_j H_{0k} \tag{10}$$

and for L-S theory, $\tau_1 = 0$, $n_0 = 1$; for G-L theory, $\tau_1 > 0$, $n_0 = 0$. The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for GL theory only.

3. Formulation of the Problems

For two dimensional problem, we assume the displacement vector u_i and microrotation vector ϕ_i as

$$u_i=(u_1,u_2,0) \quad \text{and} \quad \phi_i=(0,0,\phi_3) \tag{11}$$

and the initial magnetic field $H_{0i} = (0, 0, H_0)$, where h is the induced magnetic field.

We define the non-dimensional quantities as

$$\begin{cases} x_i' = \frac{\overline{\omega}}{c_1} x_i, \quad a' = \frac{\overline{\omega}}{c_1} a, \quad u_i' = \frac{\rho c_1 \overline{\omega}}{\nu T_0} u_i, \quad T' = \frac{T}{T_0}, \\ \sigma_{ij}' = \frac{\sigma_{ij}}{\nu T_0}, \quad T_{ij}' = \frac{T_{ij}}{\nu T_0}, \quad m_{ij}' = \frac{\overline{\omega}}{c_1 \nu T_0} m_{ij}, \quad \lambda_k' = \frac{c_1}{\nu T_0 \overline{\omega}} \lambda_k, \\ t' = \overline{\omega} t, \quad \tau_1' = \overline{\omega} \tau_1, \quad \tau_0' = \overline{\omega} \tau_0, \quad h' = \frac{h}{H_0}, \quad E_i' = \frac{E_i}{\mu_0 H_0 c_1}, \\ \phi_3' = \frac{\rho c_1^2}{\nu T_0} \phi_3, \quad \phi^{*'} = \frac{\rho c_1^2}{\nu T_0} \phi^*, \quad P_1' = \frac{P_1}{\nu T_0}, \quad P_2' = \frac{c_1}{\overline{\omega}} \frac{P_2}{T_0}, \end{cases} \tag{12}$$

where

$$\overline{\omega} = \frac{\rho c^* c_1^2}{K^*}, \quad c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \quad c_2^2 = \frac{\mu + K}{\rho}, \quad c^2 = \frac{1}{\mu_0 \epsilon_0}, \quad a_0^2 = \frac{\mu_0 H_0^2}{\rho}.$$

The expressions relating u_1, and u_2 to the scalar potential functions ψ_1 and ψ_2 are

$$u_1 = \frac{\partial \psi_1}{\partial x_1} + \frac{\partial \psi_2}{\partial x_2}, \quad u_2 = \frac{\partial \psi_1}{\partial x_2} - \frac{\partial \psi_2}{\partial x_1}. \tag{13}$$

4. Problem-I (Reflection)

We consider a longitudinal displacement (LD) wave is incident on the free surface $x_2 = 0$, of the semi-infinite medium $x_2 \geq 0$, then five waves are reflected namely LD wave, thermal (T) wave, longitudinal microstretch (LM) wave, coupled transverse (CD-I) and (CD-II). To solve the resulting equations, we assume

$$\{\psi_1, T, \phi^*, \psi_2, \phi_3\} = \{\overline{\psi}_1, \overline{T}, \overline{\phi^*}, \overline{\psi}_2, \overline{\phi_3}\} e^{\imath\{k(x \sin\theta - y\cos\theta) - \omega t\}}. \tag{14}$$

Using the equations (1), (3)-(6) and (10)-(14), we obtain

$$V^6 + AV^4 + BV^2 + C = 0, \quad \text{and} \quad V^4 + DV^2 + E = 0, \tag{15}$$

where $V = \omega/k$ is the velocity of the reflected wave. A, B, C, D and E are given in Appendix. In view of equation (14), we assume

$$\begin{cases} \{\psi_1, T, \phi^*\} = \sum_{i=1}^{3} \{1, \eta_i, \xi_i\} [A_{01} e^{\imath\{k_1(x\sin\theta_0 - y\cos\theta_0) - \omega_1 t\}} + P_i], \\ \{\psi_2, \phi_3\} = \sum_{j=4}^{5} \{1, \eta_j\} P_j, \end{cases} \tag{16}$$

where

$$P_i = A_i e^{\imath\{k_i(x\sin\theta_i + y\cos\theta_i) - \omega_i t\}}, \quad P_j = B_j e^{\imath\{k_j(x\sin\theta_j + y\cos\theta_j) - \omega_j t\}}.$$

A_{01} is the amplitude of the incident longitudinal displacement (LD) wave and A_i, B_j are the amplitudes of the corresponding reflected waves; η_i, ξ_i and η_j are given in Appendix.

4.1. Boundary Conditions

The boundary conditions are

$$\sigma_{22} + T_{22} = 0, \quad \sigma_{21} = 0, \quad m_{23} = 0, \quad \lambda_2 = 0, \quad \frac{\partial T}{\partial y} = 0\,. \tag{17}$$

Snell's law is given as

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_i}{V_i}\,, \text{ where } k_i V_i = \omega \text{ at} x_2 = 0 \quad (i = 1, 2, .., 5)\,. \tag{18}$$

With the help of (16)-(18), we get a system of five non-homogeneous equations as

$$\sum_{i=1}^{5} a_{ij} Z_j = Y_i \quad (j = 1, 2, ..., 5), \tag{19}$$

where

$$Z_i = \frac{A_i}{A_{01}}, \quad Z_j = \frac{B_j}{A_{01}}, \quad i = 1, 2, 3\,, \; j = 4, 5 \tag{20}$$

where a_{ij} and Y_i are given in Appendix and $V_0 = V_1$. In the absence of magnetic field and microstretch effect, our results tally with the results of Singh and Kumar [7], by changing the dimensionless quantities into the physical quantities.

5. Problem-II (Deformation)

Thermomechanical source is applied on the boundary of the assumed half space. Assuming time harmonic behavior as

$$\{\psi_1, T, \phi^*, \psi_2, \phi_3\}(x, y, t) = \{\psi_1, T, \phi^*, \psi_2, \phi_3\}(x, y) e^{\imath\omega\, t}\,. \tag{21}$$

Applying the Fourier transform with respect to "x" defined by

$$\{\tilde{\psi_1}, \tilde{T}, \tilde{\phi^*}, \tilde{\psi_2}, \tilde{\phi_3}\}(\xi, y, \omega) = \int_{-\infty}^{\infty} \{\psi_1, T, \phi^*, \psi_2, \phi_3\}(x, y, \omega) e^{\imath\xi\, x} dx. \tag{22}$$

Making use of (21) and (22) in the resulting equations, yields the same equation (15), with $-k^2$ replaced by $q^2 = \frac{d^2}{dy^2} - \xi^2$ and the solutions satisfying the radiation conditions have been taken.

5.1. Boundary Conditions

The boundary conditions are

$$\sigma_{22} + T_{22} = -P_1 f_1(x, t), \;\; \sigma_{21} = 0, \;\; m_{23} = 0, \;\; \lambda_2 = 0, \;\; \frac{\partial T}{\partial y} = P_2 f_2(x, t), \tag{23}$$

where P_1 is the magnitude of the force and P_2 is the constant temperature applied on the boundary. With the aid of (2), (7)-(9), (11)-(13), (22) and (23), we obtain the components of normal force stress, tangential couple stress, microstress, temperature distribution, and induced magnetic field.

5.2. Uniformly or Linearly Distributed Source

As an application, we take

$$\{f_1(x,t), f_2(x,t)\} = e^{-\imath\omega t}\begin{cases} 1 & \text{if } |x| \le a, \\ 0 & \text{if } |x| > a, \end{cases} \tag{24}$$

or

$$\{f_1(x,t), f_2(x,t)\} = e^{-\imath\omega t}\begin{cases} (1-\frac{|x|}{a}) & \text{if } |x| \le a, \\ 0 & \text{if } |x| > a. \end{cases} \tag{25}$$

The corresponding solutions are obtained by substituting the value of $\tilde{f}_1(\xi,\omega)$ and $\tilde{f}_2(\xi,\omega)$, after applying Fourier transform on (24) and (25) in the resulting equations.

5.3. Inversion of the Transforms

To obtain the solution of the problem in the physical domain, the inversion of the Fourier transform is used by the method given in [5].

6. Numerical Results and Discussion

Following Gauthier [3], the values of micropolar constants are

$$\begin{cases} \lambda = 7.59\times10^9 Nm^{-2}, \quad \mu = 1.89\times10^9 Nm^{-2}, \quad K = .0149\times10^9 Nm^{-2}, \\ \rho = 2.638\times10^3 Kg\,m^{-3}, \quad \gamma = 2.63\times10^3 N, \quad \jmath = 0.196\times10^{-6} m^2, \end{cases}$$

and other parameters are taken

$$\begin{cases} c^* = 0.9614\times10^3 Jkg^{-1}Kelvin^{-1}, \quad K^* = 2.502 Jm^{-1}sec^{-1}Kelvin^{-1}, \\ \alpha_{t_1} = 0.5\times10^{-3}Kelvin^{-1}, \quad \alpha_{t_2} = 0.5\times10^{-3}Kelvin^{-1}, \\ \lambda_0 = 0.5\times10^9 Nm^{-2}, \quad \lambda_1 = 0.5\times10^9 Nm^{-2}, \quad \jmath_0 = 0.185\times10^{-6}m^2, \\ \alpha_0 = .9\times10^3 N, \quad b_0 = .91\times10^3 N, \quad \tau_0 = .2, \quad \tau_1 = .4 \\ \delta_1^2 = 1.3, \quad \delta_2^2 = 1.2, \quad \omega/\overline{\omega} = 10, \quad T_0 = 298 Kelvin. \end{cases}$$

The solid line and small dashes line represent magneto-thermo-microstretch elastic medium for LS-theory-MMT1 and for GL-theory-MMT2 respectively. The solid line and small dashes line with centre symbols represent thermo-microstretch elastic medium for LS-theory-MT1 and for GL-theory-MT2 respectively.

The computations for the second problem are carried out for the non-dimensional time $t = 0.5$ in the range $0 \leq x \leq 10$.

6.1. Problem-I (Incident LD Wave)

It is observed that the trends of variations of amplitude ratios $Z(1)$-$Z(5)$ are similar for LS theroy (MMT1 and MT1) and GL theory (MMT2 and MT2) for all values of θ_0 with difference in magnitude, except the behavior of variations of $Z(1)$, which is opposite for MT2 in comparison to MMT2 when $0 \leq \theta_0 \leq 80$.

6.2. Problem-II (Uniformly Distributed Source)

The behavior of variations of normal force stress σ_{22} and tangential couple stress m_{23} is similar with deference in magnitude, for the epicentral distance x. Similar behavior of variations of microstress λ_2 and temperature distribution T for LS theroy (MMT1 and MT1) and GL theory (MMT2 and MT2) respectively with difference in magnitude, for all values of x is obtained. The behavior of variations of induced magnetic field h is oscillatory for both LS and GL theories in the whole range.

7. Conclusion

Appreciable magnetic effect have been observed on reflection coefficients and σ_{22}, m_{23}, λ_2, T respectively. The problem though theoretical, is of physical interest in the field of seismology, geophysics and earthquake engineering, etc.

References

[1] A.C. Eringen, Theory of thermo-microstretch elastic solids, *Int. J. Eng. Sci.*, **28** (1990), 1291-1301.

[2] A.C. Eringen, Theory of thermo-microstretch fluids and bubbly liquids, *Int. J. Eng. Sci.*, **28** (1990), 133-143.

[3] R.D. Gauthier, *Mechanics of Micropolar Media* (Ed-s: O. Brulin, R.K.T. Hsieh), World Scientific, Singapore (1982).

[4] A.E. Green, K.A. Lindsay, Thermoelasticity, *J. Elasticity*, **2** (1972), 1-7.

[5] R. Kumar, P. Ailawalia, Moving load response of micropolar elastic half-space with voids, *J. of Sound and Vibration*, **280** (2005), 837-848.

[6] H. Lord, Y. Shulman, A generalized dynamical theory of thermo-elasticity, *J. Mech. Phys. Solid*, **15** (1967), 299-309.

[7] B. Singh, R. Kumar, Reflection of plane waves from the flat boundary of a micropolar generalized thermoelastic half-space, *Int. J. Eng. Sci.*, **36** (1998), 865-890.

Appendix

$$\begin{cases} A = \dfrac{-1}{\delta_2^2 d_2}[\delta_2^2 d_1 + \delta_1^2 d_2 - \imath\omega\tau_{10}\tau_{30}\{a_1 a_5 + \dfrac{1}{\omega^2}(a_2 a_3 - a_1 a_6)\} \\ \quad -\dfrac{\lambda_0}{\rho\, c_1^2}(\dfrac{\imath}{\omega} a_1 a_4 \tau_{10}\tau_{30} - \dfrac{1}{\omega^2} a_3)], \\ B = \dfrac{1}{\delta_2^2 d_2}[\delta_1^2 d_1 + \delta_2^2 - \imath\omega\, a_1\tau_{10}\tau_{30} + \dfrac{\lambda_0}{\rho\, c_1^2}(\dfrac{1}{\omega^2}a_3)], \quad C = -\dfrac{\delta_1^2}{\delta_2^2 d_2}, \\ D = \dfrac{1}{d_3}[\dfrac{1}{\omega^2}(2a_7 + \dfrac{K}{\rho\, c_2^2}) - (\dfrac{c_1^2}{c_2^2}\delta_2^2 + a_8)], \quad E = \dfrac{1}{d_3}. \end{cases}$$

$$\begin{cases} a_1 = \dfrac{\nu^2 T_0}{\rho^2 c_1^2 c^*}, \quad a_2 = \dfrac{\nu\nu_1 T_0}{\rho^2 c_1^2 c^*}, \quad a_3 = \dfrac{\lambda_0 K^{*2}}{\alpha_0 \rho^2\, c^{*2} c_1^2}, \quad a_4 = \dfrac{\nu_1 K^{*2}}{\nu\alpha_0 \rho\, c^{*2}}, \\ a_5 = \dfrac{\rho\jmath_0 c_1^2}{2\alpha_0}, \quad a_6 = \dfrac{\lambda_1 K^{*2}}{\alpha_0\rho^2\, c^{*2} c_1^2}, \quad a_7 = \dfrac{KK^*}{\gamma\rho^2 c^{*2} c_1^2}, \quad a_8 = \dfrac{\rho\jmath}{\gamma} c_1^2, \\ \delta_1^2 = 1 + \dfrac{a_0^2}{c_1^2}, \quad \delta_2^2 = 1 + \dfrac{a_0^2}{c^2}. \end{cases}$$

$$\begin{cases} d_1 = a_5 - \dfrac{a_6}{\omega^2} + \tau_{20}, \quad d_2 = a_5\tau_{20} - \dfrac{1}{\omega^2} a_6 \tau_{20} + \dfrac{\imath}{\omega} a_2 a_4 \tau_{10}\tau_{30}, \\ d_3 = \dfrac{c_1^2}{c_2^2}\delta_2^2 (a_8 - 2\dfrac{a_7}{\omega^2}), \quad \tau_{10} = \tau_1 + \imath\omega^{-1}, \quad \tau_{20} = \tau_0 + \imath\omega^{-1}, \\ \tau_{30} = \tau_0 n_0 + \imath\omega^{-1},. \end{cases}$$

$$\begin{cases} \{\eta_i, \xi_i\} = (1 \cdot \{(a_1\omega^2\tau_{30}(-1 + a_5 V_i^2 - \dfrac{1}{\omega^2} a_6 V_i^2) + a_2 a_3 \tau_{30} V_i^2), \\ (\imath\, a_1 a_4 \tau_{10}\tau_{30}\omega\, V_i^2 - a_3(-1 + \tau_{20} V_i^2))\}) \\ \times((-1 + \tau_{20}V_i^2)(-1 + a_5 V_i^2 - \dfrac{1}{\omega^2} a_6 V_i^2) + \dfrac{\imath}{\omega} a_2 a_4 \tau_{10}\tau_{30} V_i^4)^{-1}, \\ \eta_j = \dfrac{-a_7}{-1 + a_8 V_j^2 - \dfrac{2}{\omega^2} a_7 V_j^2} \quad (i = 1, 2, 3\,, \quad j = 4, 5). \end{cases}$$

$$
\begin{cases}
a_{1i} = -r_1 k_i^2 - r_2 k_i^2 \cos^2\theta_i + \imath\omega_i \tau_{10}\eta_i + r_3\xi_i, \quad a_{1j} = r_2 k_j^2 \sin\theta_j \cos\theta_j, \\
a_{2i} = -(2+r_4) k_i^2 \sin\theta_i \cos\theta_i, \\
a_{2j} = k_j^2 \sin^2\theta_j - (1+r_4) k_j^2 \cos^2\theta_j + r_4\eta_j, \\
a_{3i} = -\imath\, r_5 \xi_i k_i \sin\theta_i, \quad a_{3j} = \imath\eta_j k_j \cos\theta_j, \\
a_{4i} = \imath\xi_i k_i \cos\theta_i, \quad a_{4j} = \imath\, r_6 \eta_j k_j \sin\theta_j, \\
a_{5i} = \imath\, k_i \cos\theta_i, \quad a_{5j} = 0, \quad (i = 1,2,3 \quad \& \quad j = 4,5) \\
r_1 = \dfrac{\lambda}{\rho\, c_1^2} + \delta_1^2 - 1, \quad r_2 = \dfrac{2\mu + K}{\rho\, c_1^2}, \quad r_3 = \dfrac{\lambda_0}{\rho\, c_1^2}, \\
r_4 = \dfrac{K}{\mu}, \quad r_5 = \dfrac{b_0}{\gamma}, \quad r_6 = \dfrac{b_0}{\alpha_0},
\end{cases}
$$

and

$$
Y_1 = -a_{11}, \quad Y_2 = a_{21}, \quad Y_3 = -a_{31}, \quad Y_4 = a_{41}, \quad Y_5 = a_{51}\,.
$$