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"INELASTIC COLLAPSE" WITH OR WITHOUT EXTERNAL FIELD

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Abstract: Though elastic collision increase a system's entropy and disturb the order, inelastic collision dissipates the internal energy to environment as heat. As a result, inelastic system tends to make order automatically. The prototipical example of the phenomenon is found by the author's group in late 1980s, and named "inelastic collapse" by McNamara and Young. This tendency is found by computer simulation through our study for planetary ringlet formation. And the author also reports the new phenomenon that inelastic particls in a spherical shell makes a ring on the equator.

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Key Words: inelastic collapse, ringlet formation, computer simulation

1. Introduction

Assume some particles stand in a row in one-dimensional space, and a particle are driving into the raw. No boundary nor external field. If the particles are elastic, all particle are scattered at last. But if they are inelastic, and the nuber of particles are larger than a threshold which is determined by the restitition of coefficient, vast majority particle cannot escape and collapse into a few clusters in which both the relative velocities and distance between particle is becoming into infinitecimal, [6] (see Figure 1). Computer simulation cannot exceed the time when the first cluster formed. To avoid the infinite collision, when the

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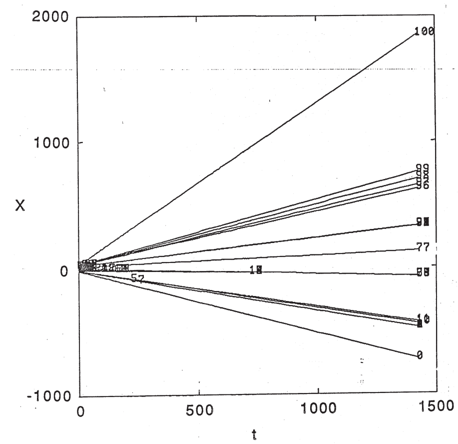

Figure 1: A handred particles on one-dilensional space, which shown as the vertical axis, are colliding and fusing each other and the number of independent particle or cluster decrease with time. Rebound coefficient is 0.5, and initial velocities are uniform random. This figure is reproduced from [6].

distances and relative velocities with in the cluster become enough small (for example, 10^{-6}), we may coalesce the particles with one particle. The simpler experiment is shown in Figure 2. Quite similar figure is shown in McNamara and Young [3].

If the particles are completely inelastic, the expected number of final cluster made of n particles is $1/n$ which was proved mathmatically after the numerical discovery, see [5]. Moreover, the velocity distribution of final clusters is $1/v$ in the interval $N^{-1/2} \ll v/\sigma \ll 1$, where σ is the initial rms velocity and N is the initial number of particles [1]. Survival-time distribution of inelastic particle is investigated by Swift and Bray [8].

Such clusters are also found in two- and three- space dimension with and without boundary or external field. Clusters in two-dimensinal free space had been reported in [4]. I also had done the same simulations and got positive result which has been shown in my thesis (1992), but not published.

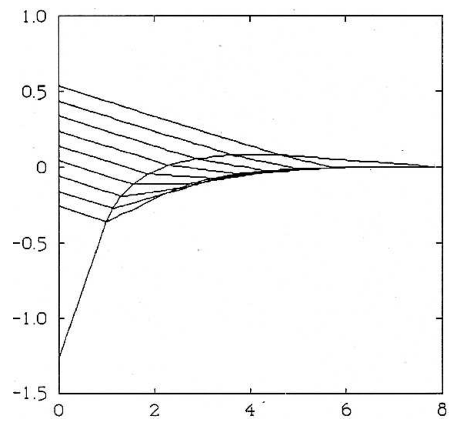

Figure 2: Trajectries of motion. $x - t$ graph of a simulation result with parameters $N = 101$, $e = 0.5$. Until $t = 1425.2$ when the last, 3642-nd collision has occured. Rebound coefficient is 0.5. This figure is reproduced from [6].

2. Planetary Ringlets Formation

Planetary ringlets is an application of "inelastic collapse" and formed in computers. All of the giant planets, Saturn, Jupiter, Uranus, and Neptune have narrow ringlets. Especially in Saturn, there are handred of such ringlets which looks like a disk. Assume particles are sphere which have finite diameter, two parameters were used to specify inelasitc collisions: the normal and tangential rebound coefficient, e_n and e_t. All particle are the same, and initially they orbits around a planet by the gravitational central force. All orbits share a common angular momentum, but inclinations and eccentricities are different.

When a certain condition of e_n, e_t is satisfied, particles under the gravitational influence of a planet are shown to evolve from a three-dimensional torus to a two-dimensional disc, and finally to an one-dimensional ringlet [3, 4]. When the angular momenta are not normalized, groups of frequently colliding particle are formed, and muptiple concentric circular ringlets appear according to the angular moments of the group. A three-dimensional case with 2000-body was then simulated. The ringlets might be formed by themself even with no shepherding satellites nor gravitational resonance.

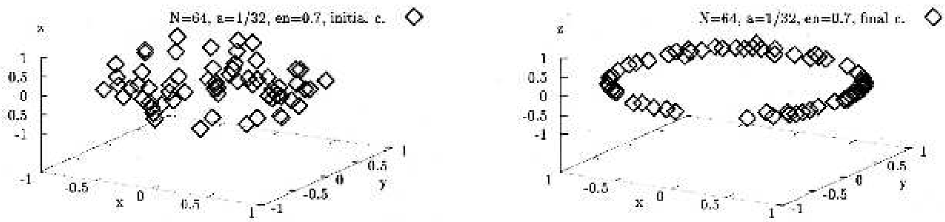

Figure 3: The case of spherical container. Particles form a ring on the equator. Particle configuration: The initial condition (left), and the final (right). $r = 1$, $N = 64$, total $50N$-pair collisions.

3. Elastic Sphere Shell

There are sphere particles with finite size within a sphere shell. There are no external field. Assume particle-particle collisions are inelastic, and particle-wall collisions are elastic. Initially poistion of particles are put in the sphere uniform randomly. Initial velocity of particles are generated to be omni-directional random Gaussian. Total angular momentum vector is set to z-axis for convenience by transformation of coordinate system.

At the final state, all particles rotate on the equator in a row. The explanation is below. Both P-W and P-P collisions conserve total angular momentum L_0. Total kinetic energy $K > 0$ monotonically decrease,

$$\Delta k_m = \frac{1}{2}\mu[(1 - e_n^2)(\Delta v_{ij} n)^2 + (1 - e_t^2)(\Delta v_{ij} t)^2] > 0,$$

by m-th collision. Ergodicity guarantees infinite collisions. Δk_m must converge because the infinite sequence of energy K is monotonically decreasing and bound. This means that the relative velocities at collision converges to zero after enough time. Only the largest cross section (the equator) is stable and zero relative velocity orbit. At the final state, all particles have almost the same velocity and share the equator.

Figures 3 shows the initial and final configuration of the particles in a sphere. We see a ring consist of all particles on the equator. If total angular momentum is artificially set to 0, the system tends to freeze keeping random configuration.

Figures 4 show the orbal shell case instead of sphere. The system keeps random configuration because P-W collision does not conserve L_0.

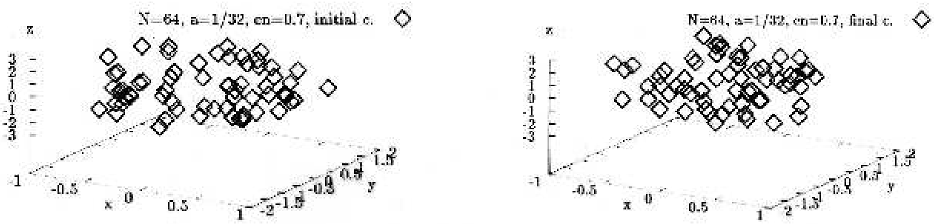

Figure 4: The case of obal container. Particle configuration keeps random throught the simulation. The left is initial and the right is final condition. The radii are (1,2,3).

4. Discussion and Conclusion

If the central force is Fook's Law spring, $f = -kx$, and the mass and k are common, it is almost self-evident that only one cluster remains at the final state. Because all particles have the same period, and if a pair of particles collide each other, they must continue to collide eternally. This means the origin is the attracter. "Inelastic Collapse" is quite ordinary phenomenon when there is some mechanism to keep collision going.

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