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CAVITATION, INDENTATION AND PENETRATION

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Abstract: Elastoplastic deformation patterns, induced by expanding cavities, provide useful analytical models in applied mechanics and engineering. Applications range from material hardness indentation tests to dynamic penetration of projectiles. Recent advances in deriving analytical solutions for steady state expanding cavities enable the construction of highly accurate formulae for cavitation pressure, accounting for strain hardening and elastic compressibility. For dynamic fields, solutions are described by Mach number (relative to medium wave velocity) power series with coefficients that depend on elastic and plastic moduli. Cylindrical quasi-static cavitation solutions have been obtained for both Mises and Tresca solids.

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1. Cavitation

Cavitation phenomena in elastoplastic solids, originally observed by Bishop et al[1], are widely used in simulating penetration, Masri[9], and indentation, Durban and Masri[4], processes. Early results, Hill[6], for elastic/perfectly-plastic solids, are given by the cavitation pressure (p_{c}) formulae

spherical cavity (Mises and Tresca)

$$p_{\mathrm{c}} = \frac{2}{3} Y \left[1 + \ln \frac{E}{3(1-\nu)Y} \right] , \tag{1}$$

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cylindrical cavity (Mises)

$$p_c = \frac{1}{\sqrt{3}} Y \left[1 + \ln \frac{\sqrt{3}E}{(5-4\nu)Y} \right] , \quad (2)$$

cylindrical cavity (Tresca)

$$p_c = \frac{1}{2} Y \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right] . \quad (3)$$

Here, Y is the yield stress, E – elastic modulus, ν – Poisson ratio. For a pressurized spherical cavity both Mises and Tresca yield conditions coincide due to spherical symmetry of the stress field. However, they differ in plane-strain axially-symmetric fields implying the difference between (2) and (3).

Cavitation pressure serves as an extreme measure of strength of elastoplastic solids. The dynamic version of p_c has been successfully applied to impact penetration, Masri[9], in the spirit of the methodology suggested by Goodier[5]. A detailed review of earlier work on dynamic cavitation by Hopkins[7] concentrates on incompressible elastic/perfectly-plastic material. An elegant expression due to Hill is available for spherical cavities, namely,

$$P_c = \frac{p_c}{E} = \frac{2}{3} \Sigma_y \left[1 + \ln \frac{2}{3\Sigma_y} \right] + \frac{3}{2} m^2 , \quad (4)$$

where $\Sigma_y = Y/E$ and $m = \dot{A}/\sqrt{E/\rho_0}$, $\dot{A}$ is the cavity expansion velocity and ρ_0 the density.

A possible analogue of (4) for plane-strain cylindrical dynamic cavitation in incompressible Mises solids has been given recently by Masri and Durban[11] in the form

$$P_c = \frac{\Sigma_y}{\sqrt{3}} + \left(\frac{\Sigma_y}{\sqrt{3}} + \frac{m^2}{2} \right) \ln \left(\frac{1}{\sqrt{3}\Sigma_y + \frac{3}{2} m^2} \right) . \quad (5)$$

This solution however is not perfect as small stresses are required in the far field to sustain the expansion process.

2. Indentation

Both experimental data, Johnson[8], and numerical analysis, Casalas and Alcala[2], indicate the existence of a spherical cavity expansion field beneath rigid axisymmetric indenters (Figure 1).

The general result for quasi-static spherical cavitation pressure, Durban and

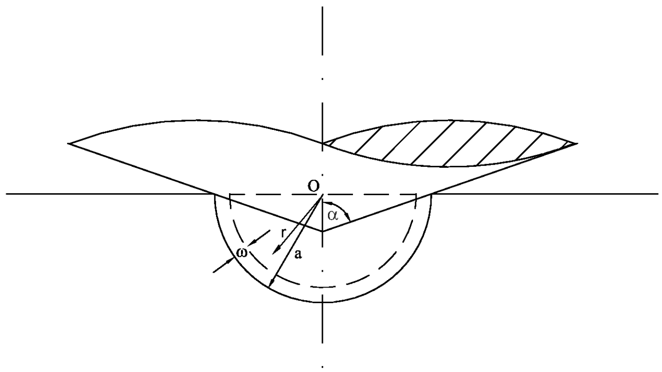

Figure 1: Conical indentation. Included angle is 2α, imprint radius – a, hypothetical radial displacement – w. Inner core is bounded by $r = a$.

Baruch [3], is

$$P_{\mathrm{c}} = \int_0^\infty \frac{\Sigma\left(d\epsilon + \beta d\Sigma\right)}{\exp(\frac{3}{2}\epsilon - \frac{\beta}{2}\Sigma) - 1 + 2\beta\Sigma} \tag{6}$$

and for quasi-static, plain strain, cylindrical cavitation of compressible Mises and Tresca solids we have the general results, Masri and Durban [12], respectively

$$P_{\mathrm{c}} = \int_0^\infty \frac{\Sigma\left(d\epsilon + \frac{1-2\kappa}{3}\beta d\Sigma\right)}{\exp\left(\sqrt{3}\epsilon - \frac{\beta}{\sqrt{3}}\Sigma\right) - 1 + \frac{2(1-\kappa)}{\sqrt{3}}\beta\Sigma} \quad \text{with} \quad \kappa = -0.4725, \tag{7}$$

and

$$P_{\mathrm{c}} = \int_0^\infty \frac{\Sigma\left[d\epsilon - \left(\frac{1-\beta}{2}\right)^2 d\Sigma\right]}{\exp(2\epsilon - \frac{1+\beta}{2}\Sigma) - 1 + \frac{3-\beta}{2}\beta\Sigma}. \tag{8}$$

Here the total strain ϵ is a known function of the effective Mises stress Σ (nondimensionalized with respect to E) and $\beta = 1 - 2\nu$ is a compressibility measure. Formulae (6-8) generalize approximations (1-3) to include strain hardening. In the absence of elastic compressibility ($\beta = 0$) it is possible, Masri and Durban [13], to derive for power hardening materials the close approximations

$$P_{\mathrm{c}} = \frac{2}{3}\Sigma_{\mathrm{y}}\left[1 + \left(\frac{2}{3\Sigma_{\mathrm{y}}}\right)^n F(n) - \frac{1}{n}\right], \tag{9}$$

$$P_\mathrm{c} = \frac{1}{\sqrt{3}}\Sigma_\mathrm{y}\left[1 + \left(\frac{1}{\sqrt{3}\Sigma_\mathrm{y}}\right)^n F(n) - \frac{1}{n}\right], \tag{10}$$

$$P_\mathrm{c} = \frac{2}{4-\eta^T}\Sigma_\mathrm{y}\left\{1 + \left(1-\eta^T\right)\ln\left(\frac{4}{3}\right) + \left(\frac{1}{2\Sigma_\mathrm{y}}\right)^n F(n) - \frac{1}{n}\right\}, \tag{11}$$

with

$$\eta^T = \frac{\left(\frac{1}{2\Sigma_\mathrm{y}}\right)^n F(n) - \frac{1}{n} - \ln\left(\frac{1}{2\Sigma_\mathrm{y}}\right)}{\left(\frac{1}{2\Sigma_\mathrm{y}}\right)\frac{\pi^2}{6} - 1 - \ln\left(\frac{1}{2\Sigma_\mathrm{y}}\right)} \quad \text{and} \quad F(n) = \zeta(1+n)\Gamma(1+n), \tag{12}$$

where n is the hardening exponent and ζ and Γ denote the Zeta and Gamma functions, respectively.

Durban and Masri[4] have examined the relation between the average indentation pressure of standard conical indenter, identified with material hardness H, and quasi-static cavitation pressures. It turns out that for power law response, in plastic range, there is a good agreement between P_c of (8) and the hardness (H/E).

3. Penetration

Following Goodier[5] it is assumed that the pressure on the penetrator is given by the local dynamic cavitation pressure. Considering steady-state dynamic expansion of a spherical cavity, Masri and Durban [10], we introduce the non-dimensional coordinate $\xi = R/A$, where R is the radial coordinate and A the instantaneous cavity radius. For self similar expansion fields we use the transformation

$$\dot{(\;)} = \dot{\xi}\frac{d(\;)}{d\xi} = \frac{\dot{A}}{A}(V-\xi)\frac{d(\;)}{d\xi}, \text{ where } V = \dot{R}/\dot{A}. \tag{13}$$

With the J_2 theory of plasticity it is possible to reduce the governing equations to the non-linear couple

$$\beta\Sigma_\mathrm{r}' + \frac{\beta}{2}\Sigma' + \frac{1}{2}\epsilon' = \frac{1}{\xi}\left(1 - \mathrm{e}^{-\frac{\beta}{2}\Sigma + \frac{3}{2}\epsilon}\right), \tag{14}$$

$$\Sigma_\mathrm{r}' - \frac{2}{\xi}\Sigma = m^2\xi^2\left(\beta\Sigma_\mathrm{r}' + \beta\Sigma' - \epsilon'\right)\mathrm{e}^{-3\beta\Sigma_\mathrm{r} - \beta\Sigma - 3\epsilon}, \tag{15}$$

where radial stress Σ_r and effective stress Σ are nondimensionalized with respect to E and a superposed prime denotes differentiation with respect to ξ. Numerical solutions of (14)-(15) provide the dependence of P_c on m. However, for elastic/perfectly-plastic solids it is instructive to peruse a power expansion

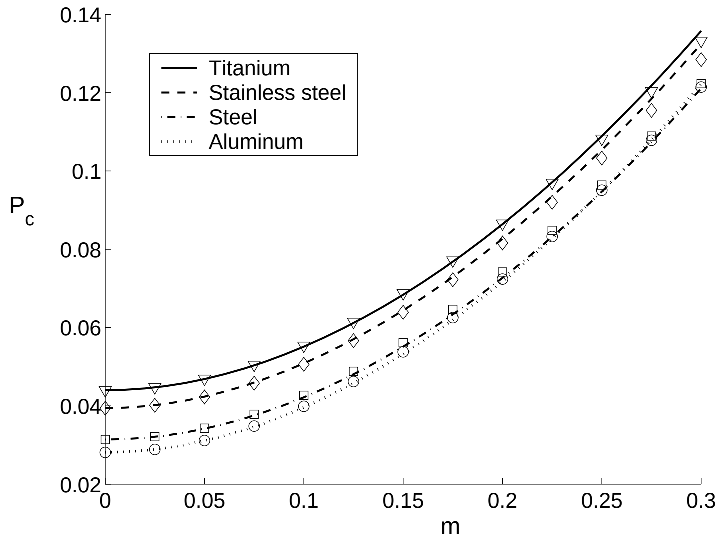

Figure 2: Variation of cavitation pressure P_c with expansion velocity m for four metals. The different markers represent the power expansion solution (16) with Σ_y^* instead of Σ_y.

solution, valid for the practical range $m^2 \ll 1$, and the dynamic cavitation pressure follows, to the third order, as

$$P_c = P_0 + P_1 m + P_2 m^2 + P_3 m^3 \,, \tag{16}$$

where coefficient P_0 is the quasi-static cavitation pressure (1) normalized by E, $P_1 = 0$ (no linear term) and

$$P_2 = \frac{3}{2} - \left(\frac{2}{3}\right)^{\frac{2}{3}} \frac{(7 + \frac{5}{3}\beta)}{(1+\beta)^{\frac{5}{3}}} \beta \Sigma_y^{\frac{1}{3}} \,, \tag{17}$$

$$P_3 = -\frac{2}{9}\sqrt{\beta \left[\frac{(3-\beta)}{(1+\beta)}\right]^5} . \tag{18}$$

Relation (16) applies also to strain hardening solids if Σ_y is replaced by an equivalent cavitation yield stress Σ_y^* obtained by equating (6) with (1), where in the latter Σ_y is replaced with Σ_y^*. Comparison between numerical solutions of (14)-(15) and the power expansion (16), for several metals, is shown in Figure 2 revealing excellent agreement.

4. Conclusions

Cavitation driven processes like axially symmetric indentation and dynamic penetration can be analyzed with the aid of plastic cavitation fields. A range of basic solutions is now available for spherical and cylindrical patterns accounting for strain hardening and elastic compressibility. Future research will include cavitation phenomena in pressure sensitive solids, strain gradient effects and influence of temperature and loading rate.

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