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ANALYSIS OF FRICTION-INDUCED SELF-GENERATED VIBRATIONS ORIGINATED FROM MODE-COUPLING IN CLUTCHES

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Abstract: The present study deals with an audible disturbance known as automotive clutch squeal noise. A two-degrees-of-freedom phenomenological model is exposed highlighting a non conservative coupling due to the friction forces, as well as gyroscopic effects. Then, a parametric study of the stability domain is performed by application of the Hartman-Grobman Theorem, analytically and numerically. Important information is obtained on the role of the structural damping regarding the fluttering destabilization, relatively to the coupling actions and especially the gyroscopic coupling. Furthermore, a non-linear integration method is proposed in order to determine the limit cycle arising beyond the Hopf's bifurcation point and allowing characterizing the steady vibrations. Finally, some practical design recommendations are provided in order to reduce the propensity of clutches to squeal.

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1. Introduction

During the clutch engagement, vibrations can arise and lead to noise generation. The squeal noise is associated to vibrations larger than the static modal response to the engine. However, a mode-coupling mechanism can be evocated, leading to a flutter instability, see North [5]. A model is exposed in the first part, and the stability is investigated in the second part. Important information regarding damping is obtained, in agreement with Hoffmann et al [3] and Sinou et al [6], and extended to partially gyroscopic systems as in Kirillov [4] and Hervé et al [2]. Finally, a non-linear identification of the limit cycles is introduced.

2. Phenomenological Model

The model depicted in Figure 1 includes a disc (A) rotating about **Oz** and free to swing along **Ox** and **Oy**. **Oxyz** is the principal inertia frame of (A). Flexible parts characterized by stiffness k_θ and k_ϕ, and damping d_θ and d_ϕ, exert restoring torques on these swinging motions. (A) is assumed to be in permanent contact with a second disc, (B), rotating about **Oz** too. This contact is simplified into four flexible elements equally distributed, and characterized by equal stiffness k_c and damping d_c. The distance between the swinging axis and the contact surface is denoted h. A constant Coulomb's friction law is considered at the end of each contact element, with a friction coefficient μ. It is assumed that the rotation speeds are constant, and that the sliding speeds never vanish. Finally, a unique stationary state is considered represented on Figure 1.

(A) and (B) represent the clutch disc and engine flywheel respectively. The swinging motions of (A) represent its first flexural modes. The contact elements represent the flexibility of the contact area, and h is related to the thickness of the clutch disc.

3. Stability Analysis of the Equilibrium

Consider equation (1) as the equation of motion of a dynamical system,

$$\ddot{\mathbf{X}} + \mathbf{f}(\mathbf{X}, \dot{\mathbf{X}}) = \mathbf{0}\,. \tag{1}$$

The dot indicates a derivation with respect to time and f is a C^1 function. The Hartman-Grobman Theorem states that the linearization of equation (1)

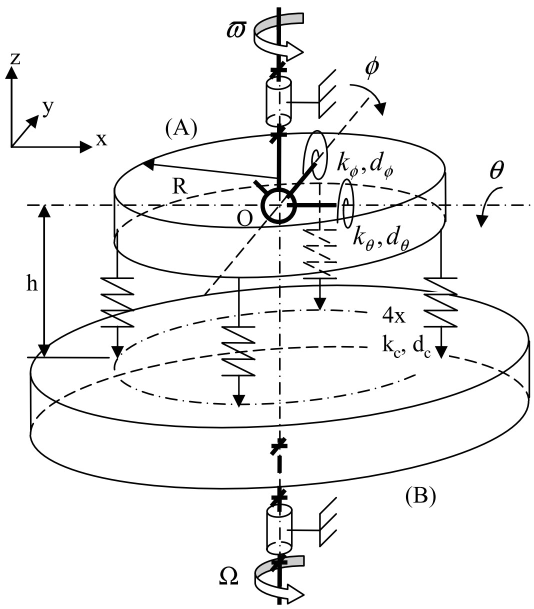

Figure 1: Model of friction-induced mode-coupling

preserves the stability nature of an equilibrium.

For a detailed study of the model, see Hervé et al [2]. We report the linearized equation in equation (2). We note ξ the reference damping factor, α and β the ratios of the natural pulsations and damping factors respectively, φ and ρ the nondimentional circulatory and gyroscopic actions respectively and τ a new time scale.

$$\frac{d^2}{d\tau^2}\begin{pmatrix}\theta\\ \phi\end{pmatrix} + \begin{pmatrix}2\xi & \rho\\ -\rho & 2\alpha\beta\xi\end{pmatrix}\frac{d}{d\tau}\begin{pmatrix}\theta\\ \phi\end{pmatrix} + \begin{pmatrix}1 & \varphi\\ -\varphi & \alpha^2\end{pmatrix}\begin{pmatrix}\theta\\ \phi\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}. \tag{2}$$

In an eigenbase of the stiffness matrix equation (2) gives equation (3)-(6),

$$\ddot{\mathbf{\Psi}} + \mathbf{C}\dot{\mathbf{\Psi}} + \mathbf{K}\mathbf{\Psi} = \mathbf{0}\,, \tag{3}$$

$$\mathbf{K} = \begin{pmatrix} e_1 = \dfrac{\alpha^2+1}{2} - \sqrt{\left(\dfrac{\alpha^2-1}{2}\right)^2 - \varphi^2} & 0 \\ 0 & e_2 = \dfrac{\alpha^2+1}{2} + \sqrt{\left(\dfrac{\alpha^2-1}{2}\right)^2 - \varphi^2} \end{pmatrix}, \tag{4}$$

$$\mathbf{\Delta} = \left(\left(\frac{\boldsymbol{\alpha^2 - 1}}{\mathbf{2}}\right)^{\mathbf{2}} - \boldsymbol{\varphi^2}\right)^{\mathbf{1/2}} \left(\varphi\rho\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right.$$
$$\left. + \xi\left((\alpha^2-1)\begin{pmatrix}1 & 0\\ 0 & \alpha\beta\end{pmatrix} - \frac{\varphi^2(\alpha\beta+1)}{e_2-1}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right)\right)\,, \quad (5)$$

$$\mathbf{\Gamma} = \left(\left(\frac{\boldsymbol{\alpha^2 - 1}}{\mathbf{2}}\right)^{\mathbf{2}} - \boldsymbol{\varphi^2}\right)^{\mathbf{1/2}} \left(\frac{\boldsymbol{\alpha^2-1}}{\mathbf{2}}\rho - (\alpha\beta - \mathbf{1})\xi\varphi\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\right)\,. \quad (6)$$

Equation (4) expresses the coalescence phenomenon. Equation (5) exhibits a destabilizing combination of the coupling actions and a stabilizing effect of damping. Lastly, equation (6) shows the gyroscopic action and a term related to the distribution of damping. We call ξ_t the total damping amount. Figure 2 shows the influence of its distribution.

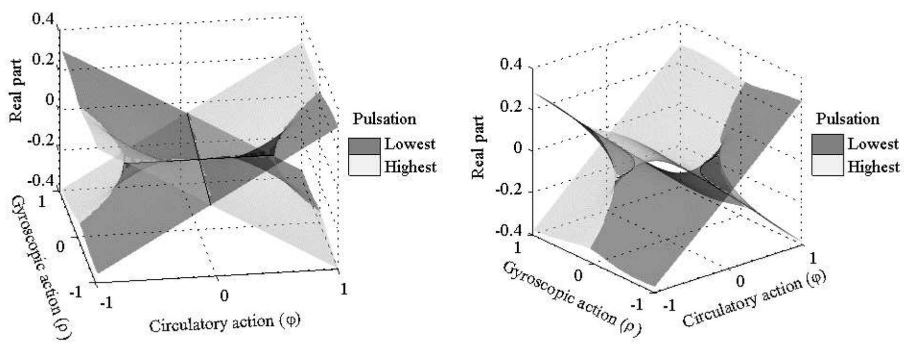

Figure 2: Real part of the eigenvalues for $\alpha = 1.5$ and $\xi_t = 0.1$, iso-damped system ($\alpha\beta = 1$, left) and $\beta = 0$ (right)

The additional gyroscopic effect deforms the map. To illustrate its influence on stability, the eigenvalues of a purely circulatory system are reported on Figure 3 for various distributions.

Both imaginary and real parts are split and the iso-distribution is near an optimum for the stability range. Thus, a strong effect of the distribution appears and an optimization is practicable.

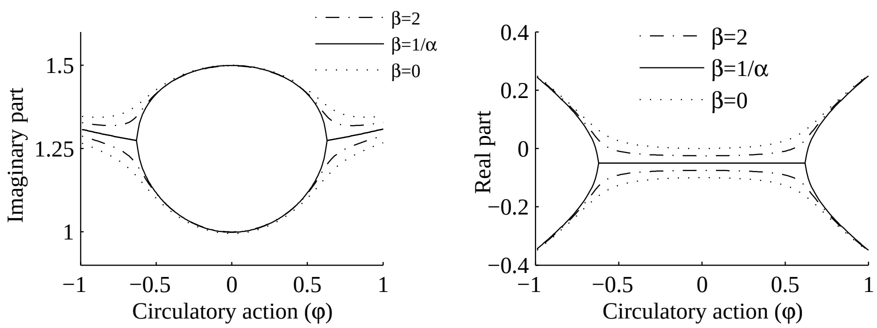

Figure 3: Circulatory system, eigenvalues for $\alpha = 1.5$, $\xi_t = 0.1$ and various distributions, imaginary part (left) and real part (right)

4. Investigation of Limit Cycles

Equation (1) is assumed to admit a periodic solution in the form of equation (7),

$$\mathbf{X}(\mathbf{t}) = \mathbf{A_0} + \sum_{\mathbf{n}\geq\mathbf{1}} \mathbf{A_n}\mathbf{cos}(\mathbf{n}\omega\mathbf{t}) + \mathbf{B_n}\mathbf{sin}(\mathbf{n}\omega\mathbf{t})\,. \tag{7}$$

An optimization method is proposed as in Hervé et al [1], according to the target function equation (8), and constraints equation (9)-(11), where the subscript i indicates the i-th component of a vector,

$$(\mathbf{A_n}, \mathbf{B_n}, \omega) \rightarrow \int\limits_{\mathbf{t_0}}^{\mathbf{t_0}+\mathbf{2}\pi/\omega} \|\ddot{\mathbf{X}}(\mathbf{t}) + \mathbf{f}(\mathbf{X}(\mathbf{t}), \dot{\mathbf{X}}(\mathbf{t}))\|^{\mathbf{2}} \mathrm{d}\mathbf{t}\,, \tag{8}$$

$$\int\limits_{t_0}^{t_0+2\pi/\omega} \ddot{\mathbf{X}}(t) + \mathbf{f}(\mathbf{X}(\mathbf{t}), \dot{\mathbf{X}}(\mathbf{t}))\mathrm{d}\mathbf{t} = \mathbf{0}\,, \tag{9}$$

$$\int\limits_{t_0}^{t_0+2\pi/\omega} |\ddot{\mathbf{X}}_i(t)| - |\mathbf{f}_i(\mathbf{X}(\mathbf{t}), \dot{\mathbf{X}}(\mathbf{t}))|\mathrm{d}\mathbf{t} = 0\,, \tag{10}$$

$$\int\limits_{t_0}^{t_0+2\pi/\omega} \left(\ddot{\mathbf{X}}_i(t) + \mathbf{f}_i(\mathbf{X}(\mathbf{t}), \dot{\mathbf{X}}(\mathbf{t}))\right) \dot{\mathbf{X}}_i(t) / \sqrt{\dot{\mathbf{X}}_i^2(t) + \ddot{\mathbf{X}}_i^2(t)}\mathrm{d}t = 0\,. \tag{11}$$

Figure 4 shows a typical result obtained from equation (12), which is merely a numerical example with no relation to experimental data.

$$\ddot{\mathbf{X}} + \begin{pmatrix} 0.1 + 3\mathbf{X_1^2} & 0 \\ 0 & 0.1 + 3\mathbf{X_1^2} \end{pmatrix} \dot{\mathbf{X}} + \begin{pmatrix} 1 + 8\mathbf{X_1^2} & 0.53 - 5\mathbf{X_2} \\ -0.53 + 5\mathbf{X_2} & 2 + 2\mathbf{X_2} + \mathbf{8X_2^2} \end{pmatrix} \mathbf{X} = \mathbf{0}\,. \quad (12)$$

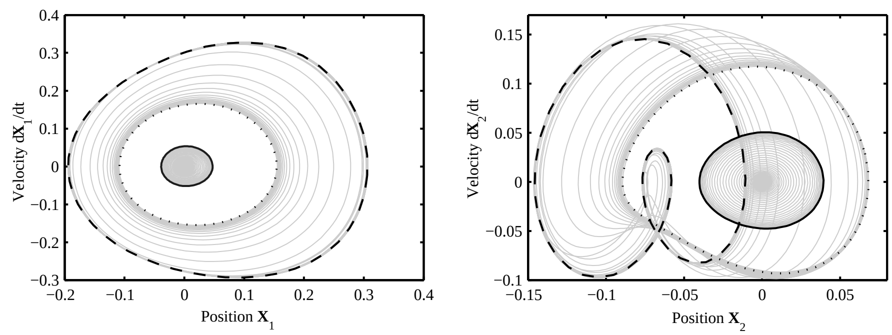

Figure 4: Trajectories in the phase planes, 1-st DOF (left) and 2-nd DOF (right), time integration (grey) and identification (black)

A good agreement is obtained between direct time integration and the method, which requires less computing resources.

5. Conclusion

The destabilization paradox, see Kirillov [4], has been illustrated with general coupling actions. Assuming a particular relationship between the coupling actions, it appears that the stability range can be optimized. Moreover, a non-linear method is proposed for estimating the steady motion arising from an unstable stationary state. Therefore, not only the rise of vibrations is predicted but their amplitude and frequency are also estimated. Thus a complete understanding of the experimental observations is allowed.

This approach may be an useful tool in order to investigate squealing systems and establish design rules. The iso-distribution of damping in quasi-purely circulatory systems whose benefit has been illustrated is a significant example of such a design rule.

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