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TWO AGENT BASED MODELS AND MARKET STYLIZED FACTS

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Abstract: The paper revisits two agent based models of the stock market. We find that the generalized Ising model exhibits the index of variation being close to that for the Russian trading system. For Sato–Takayasu model we introduce a generalization that, becoming more stable with respect to the input data, reproduces the market stylized facts.

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1. Introduction

The traditional economy describes financial markets as the equilibrium of the system of rational homogeneous agents operating with the same information [2, 4]. However this approach hardly explains the stylized facts observed numerically.

We list the main stylized facts. Firstly, the distribution of the log-returns has heavy tails [10, 5], while the central part of the distribution admits an exponential approximation with sufficient accuracy [12]. Secondly, market numerical

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characteristics have some fractal properties [7]. Thirdly, financial systems exhibit a long memory determined in terms of the auto-correlation function and the volatility of time series [6]. Some other stylized facts can be found in [9, 14].

A certain disagreement of the classical theory with the stylized facts gives rise to new approaches. Conceptual papers [8, 1] give evidence for the hypothesis according to which financial markets are complex systems with a big number of the degree of freedom. Then financial markets can be studied by methods of the dynamical systems [13].

In this paper we focus on agent based models reproducing the stylized facts. The generalized Ising model [15] is discussed in Section 2. We use the variation index defined in [3] to compare the model behaviour with the index of RTS (Russian trading system). Section 3 deals with a generalization of the Sato–Takayasu model. Section 4 concludes.

2. Generalized Ising Model and Index RTS

2.1. The Dynamics

The paper [15] introduces the generalized Ising model of financial markets. For sake of completeness we remind the model dynamics. The model involves the system of N agents. At any moment t each agent is supposed to be a buyer or a seller. The function $s_i(t)$ is equal to 1 if the i-th agent is a seller and -1 otherwise. It is supposed that the decision to sell or to buy for any agent i at the next time moment $t+1$ depends on the following factors:

1. the expectation $E_i(s_j)(T)$ formed by agent i on what will be the decision of agent j;

2. the impact of the external news $G(t)$;

3. the agent's private information $\varepsilon_i(t)$.

The equation of the dynamics is supposed to be as follows:

$$s_i(t) = \operatorname{sign}\left(\sum_{j=1}^{N} K_{ij}(t)E(s_j)t + g_i G(t) + \varepsilon_i(t)\right). \tag{1}$$

The set of functions $\varepsilon_i(t)$, $E_i(s_j)(t)$, $K_{ij}(t)$ determines the behaviour of any agent i. The functions K_{ij} in (1) is not equal to 0 only for some agents j really related with agent i. Representing heterogeneity of agents, g_i is the realization of the uniformly distributed in $(0, \sigma_{max})$ random variable. The random variable

$\varepsilon_i(t)$ has the normal distribution with zero mean and agent depended standard deviation σ_i, $\sigma_i = \sigma_0 + u_i$, where numbers u_i are uniformly distributed in $(0, 0.1)$ and fixed at the beginning of the simulation. It is supposed that $K_{ij}(t) = \beta_i + \beta K_{ij}(t-1) + \beta_G G(t-1)$. The interpretation of the constants β can be found in [15]. The market price $p(t)$ is defined as $p(t) = p(t-1)\exp\big(r(t)\big)$, where $r(t) = \sum s_i(t)/(\lambda N)$, and λ measures the market depth of liquidity.

The model dynamics depends on the parameters of the evolution. We mention only the stylized facts reproduced by the model [15]. The distribution of log-returns has mono-modal shape with heavy tails converging to Gaussian distribution at large time scale. Log-returns demonstrates a short-time correlation, while one can see a long memory of volatility. Finally, the model exhibits a rich multifractal structure described in terms of log-returns' auto-correlation function. The combination of the parameters leading to the listed stylized facts are given in [15]

2.2. Variation Index

Following [3], we define a variation index. Let $y = f(t)$ be a function defined on $[a, b]$. The interval $[a, b]$ is divided into m equal sub-intervals of length $\delta = (b-a)/m$. Put

$$V_f(\delta) = \sum_{i=1}^{m} \left(\max_{[t_{i-1}, t_i]} f(t) - \min_{[t_{i-1}, t_i]} f(t) \right),$$

where $t_i = a + i\delta$, $i = 0, \ldots, m$. If $V_f(\delta) \sim \delta^{-\mu}$ for $\delta \to 0$ then μ is called *a variation index*. In [3] μ is compared with the Hurst index H and the fractal dimension D. It is shown that $\mu = D - 1$ if both the sides are well defined. While the numerical procedures for the estimation of the fractal dimension usually converge too slowly, the variation index can be found with acceptable accuracy. Estimating the variation index for the RTS during 09.12.1996–27.10.2006 (1024 days) we have $\mu(t) \in [0.30, 0.37]$. Then we use the variation index as an additional criteria to compare the dynamics of Zhou–Sornette model [15] with that of RTS. Table 1 reports the appropriate values of the parameters.

CV	$\mu(t)$	
	lower	upper
0.53	0.30	0.37
0.50	0.27	0.36
0.55	0.35	0.40

Table 1: Values of CV reproducing the variation index of RTS; $\beta_{max} = 0.3$, $\sigma_{max} = 0.03$

3. Generalized Sato–Takayasu Model

3.1. Market Mechanism

Paper [11] introduces an agent-based model imitating a simple trade between a seller and a buyer. The model involves N agents. The number of sellers is $N_s(t)$, and the number of buyers is $N_b(t)$. A buyer cannot become a seller and otherwise until he/she trades. Two functions $s_i(t)$ and $p_i(t)$ describe the position of the agents. Let $s_i(t) = 1$ if agent i is a seller and $s_i(t) = -1$ if agent i is a buyer. Let, further, $p_i(t)$ be the ask/bid price of agent i at time t. Without loss of generality one can consider agents 1, ..., $N_s(t)$ as the sellers and $N_s(t)+1$, ..., N as the buyers such that $p_1 \leq \ldots \leq p_{N_s}$, $p_{N_s+1} \leq \ldots \leq p_N$. The trading condition is $p_N \leq p_1$. Violation of this inequality fails the trading at the moment t and keeps the market price: $P(t) = P(t-1)$. If the trading condition is satisfied then $P(t) = (p_1(t) + p_N(t))/2$.

3.2. Step of Agent Dynamics

At the considered time moment t the trade occurs for such pairs $(i, N-i)$, $i \leq N_s$, which satisfy inequality $p_i \leq p_{N-i}$. If agent i does not make a trade he/she changes the price p_i to the direction of the market price: $p_i(t+1) = p_i(t) - \alpha_i(t)s_i(t)$, where

$$\alpha_i(t) = \big|1 + c_i\big(P(t) - P(t-1)\big)\big|a_i\,. \qquad (2)$$

The constants a_i and c_i determined at the beginning of the experiment are uniformly distributed in $[0, a]$ and $[-c, c]$ respectively.

The original model of [11] connects the position of an agent having made a trade with the last market changes only. Looking for more realistic dynamics we introduce a long memory in the model. If any agent makes a trade, he/she determines a new position comparing short-run and long-run trends. Formally, let k and K be linear trends calculated on time intervals $(t-\delta, t)$ and $(t-\Delta, t)$ respectively ($\Delta \approx 5\delta$ in the experiments). Then

$$s_i(t+1) = \begin{cases} -1, & \text{if } k > K\,, \\ 1, & \text{if } k \leq K\,. \end{cases} \tag{3}$$

New bid/ask prices are defined as

$$p_i(t+1) = P(t+1) + \Lambda s_i(t+1), \tag{4}$$

where $\Lambda > 0$ is a pre-defined constant.

3.3. Initial Conditions and Parameters

Formulae (2) and (3) need a correction during initial steps. It is fixed $P(-1) = 0$ in (2). Instead of (3) during first Δ steps the decision to buy or to sell is made at random. In this paper we put $N = 200$, $\Delta = 0.02$, $a = 0.015$, $c = 100$, $\delta = 40$, $\Delta = 200$, $P(0) = 520$. However the dynamics demonstrates the similar behaviour for a significant domain of the parameters.

3.4. Distribution of Log-Returns

We compare the distributions $\rho(r)$ of log-returns r for the model prices and DJ. The daily data for DJ is found at www.yahoo.com. The daily model data has to be determined using output time series. It is supposed that z model steps constitute one working day. Appropriate values of z are in $[10, 40]$. We fix $z = 20$.

At Figure 1 a central part of the histogram for the distributions is presented. A linear part in linear-logarithmic scale gives evidence of the exponential decay for both the distributions. This observation for the real data was announced as a stylized fact in [12].

Figure 2 shows a “power” part of $\rho(r)$ for sufficiently big $|r|$. This phenomenon is well known as heavy tails. According to Figures 1 and 2, $\rho(x) > \rho(-x)$ for “middle” positive x ($|x \in [0.02, 0.03]$, $|\xi_\pm| < 0.5$). This observation stays in correspondence with the fact that the financial markets rather grow than fall. The breakdown of symmetry may be considered as the consequence of (2), which is not symmetric with respect to positive and negative ΔP. However formula (2) apriori does not lead to such symmetry violation, which can be observed for the real data.

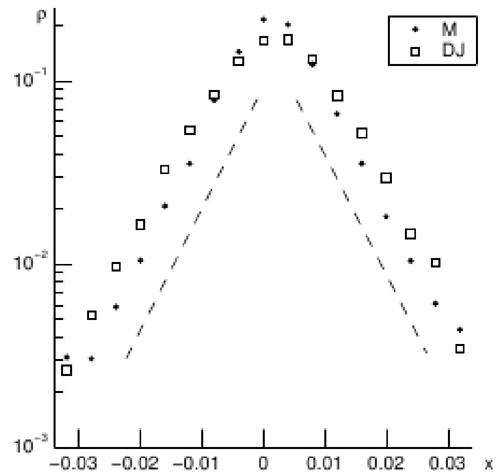

Figure 1: Histogram (central part) of log-returns for the model (M) and DJ; $\rho(x)$ is the fraction of such working days that $r \in [x-\Delta x, x+\Delta x)$, $\Delta x = 0.002$

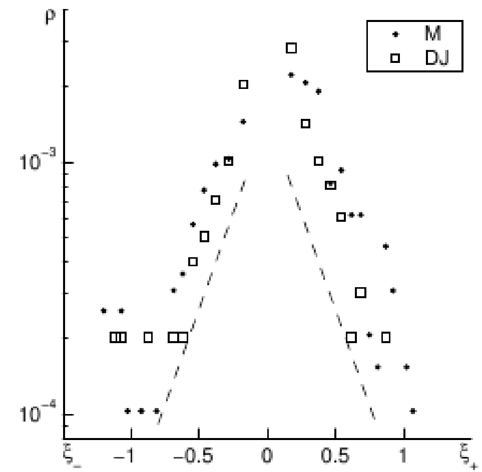

Figure 2: Histogram (tails) of log-returns for the model (M) and DJ; horizontal axis ξ is the image of x by the map $\xi_\pm = \log(\pm x) - (\pm \log(0.03))$, the number 0.03 separates the central part and the tails

4. Conclusion

Two agent based models of the financial market are investigated. For the generalized Ising model, [15], we find such parameters, which reproduce the variation index calculated for the RTS. Starting from Sato–Takayasu model [11] we introduce a memory in the agent's decision mechanism. This generalization generates the market with stylized features being typical for real financial markets.

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