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APPROXIMATE SOLUTION FOR 1-D COMPRESSIBLE VISCOUS MICROPOLAR FLUID MODEL IN DEPENDENCE OF INITIAL CONDITIONS

Ivan Dražić[1 §], Nermina Mujaković[2]

[1]Department of Applied Mathematics
Faculty of Engineering
University of Rijeka
Vukovarska 58, Rijeka, 51000, CROATIA
e-mail: idrazic@riteh.hr

[2]Department of Mathematics
Faculty of Philosophy
University of Rijeka
Omladinska 14, Rijeka, 51000, CROATIA
e-mail: mujakovic@inet.hr

Abstract: We consider a model for nonstationary 1-D flow of a compressible viscous heat-conducting micropolar fluid which is thermodynamically perfect and polytropic. A corresponding initial-boundary value problem has a unique strong solution on $]0,1[\times]0,T[$, for each $T>0$ and for sufficiently small T this solution is a limit of approximate solutions which we get by implementing the Faedo-Galerkin method. Using the initial functions in the form of Fourier expansions we analyze the numerical approximate solutions in dependence of number of terms in Fourier series.

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1. Introduction

In this paper we consider nonstationary 1-D flow of a compressible viscous and

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§Correspondence author

heat-conducting micropolar fluid being in a thermodynamical sense perfect and polytropic. A corresponding initial-boundary value problem has a unique strong solution on $]0,1[\times]0,T[$ for each $T>0$ (see Mujaković [2]) which analytic form is not known. For sufficiently small T this solution is a limit of approximate solutions which we get by the Faedo-Galerkin method (see Mujaković [1]). In Mujaković et al [4] the method for obtaining this solution in the case of initial conditions with finite number of terms in Fourier expansion was given. In this work we use the same method in the case of initial functions that have infinite number of terms in Fourier expansion. We also analyze the convergence of solutions in dependence of number of terms in Fourier expansion.

2. Mathematical Model

Let ρ, v, ω and θ denote, respectively, the mass density, velocity, microrotation velocity and temperature of the fluid in the Lagrangean description. The problem we consider has the formulation as follows (see Mujaković [1]):

$$\frac{\partial \rho}{\partial t}+\rho^2\frac{\partial v}{\partial x}=0, \tag{1a}$$

$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\left(\rho\frac{\partial v}{\partial x}\right)-K\frac{\partial}{\partial x}\left(\rho\theta\right), \tag{1b}$$

$$\rho\frac{\partial \omega}{\partial t}=A\left[\rho\frac{\partial}{\partial x}\left(\rho\frac{\partial \omega}{\partial x}\right)-\omega\right], \tag{1c}$$

$$\begin{aligned}\rho\frac{\partial \theta}{\partial t}=&-K\rho^2\theta\frac{\partial v}{\partial x}+\rho^2\left(\frac{\partial v}{\partial x}\right)^2+\rho^2\left(\frac{\partial \omega}{\partial x}\right)^2\\&+\omega^2+D\rho\frac{\partial}{\partial x}\left(\rho\frac{\partial \theta}{\partial x}\right)\end{aligned} \tag{1d}$$

in $]0,1[\times]0,T[, T>0$, where K, A and D are positive constants. Equations (1) are, respectively, local forms of the conservations laws for the mass, momentum, momentum moment and energy. We take the homogeneous boundary conditions:

$$v(0,t)=v(1,t)=0, \quad \omega(0,t)=\omega(1,t)=0, \tag{2a}$$

$$\frac{\partial \theta}{\partial x}(0,t)=\frac{\partial \theta}{\partial x}(1,t)=0 \tag{2b}$$

for $t\in]0,T[$ and non-homogeneous initial conditions:

$$\rho_0(x)=1, \quad v_0(x)=x-x^2, \tag{3a}$$

$$\omega_0(x) = x^2 - x^4, \quad \theta_0(x) = 2x^3 - 3x^2 + 2 \tag{3b}$$

for $x \in \Omega =]0,1[$. It should be noticed that (3) satisfy the conditions of the local and global existence theorems (see Mujaković [1], [2]). Because of the simplicity we take $K = A = D = 1$.

3. Approximate Solutions

In Mujaković [1] is introduced the sequence $\{(\rho^n, v^n, \omega^n, \theta^n),\ n \in \mathbf{N}\}$ of approximate solutions to the problem (1)-(3) as follows:

$$\rho^n(x,t) = \rho_0(x)(1 + \rho_0(x)\sum_{i=1}^{n}(\pi i)\cos(\pi i x) z_i^n(t))^{-1}, \tag{4a}$$

$$v^n(x,t) = \sum_{i=1}^{n} v_i^n(t)\sin(\pi i x), \quad \omega^n(x,t) = \sum_{j=1}^{n} \omega_j^n(t)\sin(\pi j x), \tag{4b}$$

$$\theta^n(x,t) = \sum_{k=0}^{n} \theta_k^n(t)\cos(\pi k x), \tag{4c}$$

where

$$\{(v_i^n, \omega_j^n, \theta_k^n, z_r^n); i,j,r = 1,...,n, k = 0,...,n\} \tag{5}$$

is the solution to the following Cauchy problem:

$$\dot{v}_i^n(t) = \Phi_i^n(t, v_1^n, ...v_n^n, \omega_1^n, ..., \omega_n^n, \theta_0^n, \theta_1^n, ..., \theta_n^n, z_1^n, ..., z_n^n), \tag{6a}$$

$$\dot{\omega}_j^n(t) = \Psi_j^n(t, v_1^n, ...v_n^n, \omega_1^n, ..., \omega_n^n, \theta_0^n, \theta_1^n, ..., \theta_n^n, z_1^n, ..., z_n^n), \tag{6b}$$

$$\dot{\theta}_k^n(t) = \lambda_k \Pi_k^n(t, v_1^n, ...v_n^n, \omega_1^n, ..., \omega_n^n, \theta_0^n, \theta_1^n, ..., \theta_n^n, z_1^n, ..., z_n^n), \tag{6c}$$

$$\dot{z}_r^n(t) = v_r^n(t), \tag{6d}$$

$$v_i^n(0) = v_{0i},\ \omega_j^n(0) = \omega_{0j},\ \theta_k^n(0) = \theta_{0k},\ z_r^n(0) = 0, \tag{6e}$$

$$\lambda_0 = 1;\ \lambda_k = 2,\ k = 1,2,...,n \tag{6f}$$

with

$$\Phi_i^n = 2\int_0^1 \left[\frac{\partial}{\partial x}\left(\rho^n \frac{\partial v^n}{\partial x}\right) - K\frac{\partial}{\partial x}(\rho^n\theta^n)\right]\sin(\pi i x)dx, \tag{7a}$$

$$\Psi_j^n = 2\int_0^1 A\left[\frac{\partial}{\partial x}\left(\rho^n \frac{\partial \omega^n}{\partial x}\right) - \frac{\omega^n}{\rho^n}\right]\sin(\pi j x)dx, \tag{7b}$$

$$\Pi_k^n = \int_0^1 \left[-K\rho^n\theta^n\frac{\partial v^n}{\partial x} + \rho^n\left(\frac{\partial v^n}{\partial x}\right)^2 + \rho^n\left(\frac{\partial \omega^n}{\partial x}\right)^2\right.$$

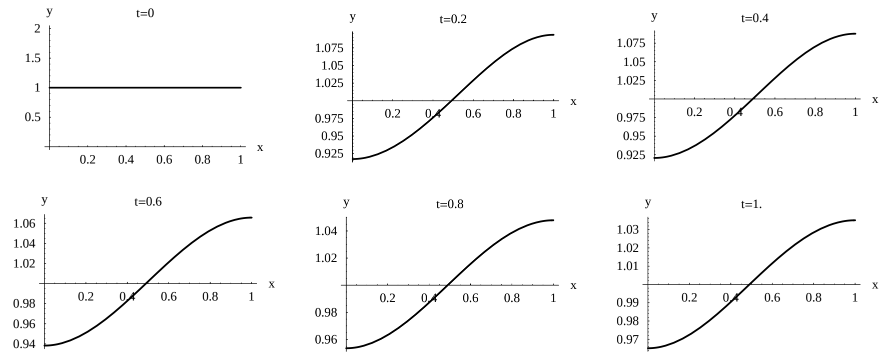

Figure 1: Graph ($n = 8$, $m = 20$) for the approximation ρ^n of the function $\rho(x,t)$ for some values of t

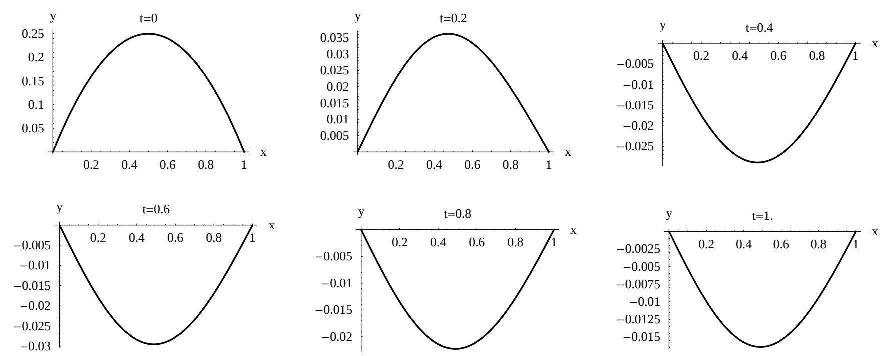

Figure 2: Graph ($n = 8$, $m = 20$) for the approximation v^n of the function $v(x,t)$ for some values of t

$$+\frac{(\omega^n)^2}{\rho^n} + D\frac{\partial}{\partial x}\left(\rho^n \frac{\partial \theta^n}{\partial x}\right)\Bigg] \cos(\pi k x)dx, \tag{7c}$$

where v_{0i}, ω_{0j} and θ_{0k} are the coefficients in Fourier expansions of the functions v_0, ω_0 and θ_0, respectively.

Here, as in Mujaković et al [4], we approximate the integrals in (7) with Gaussian-Legendre quadrature formula and solve the system (6) using the solver from Wolfram *Mathematica*.

n	$\lVert\rho^n - \rho^{n-1}\rVert_\infty$	$\lVert v^n - v^{n-1}\rVert_\infty$	$\lVert\omega^n - \omega^{n-1}\rVert_\infty$	$\lVert\theta^n - \theta^{n-1}\rVert_\infty$
5	$4.80049 \cdot 10^{-6}$	$1.76009 \cdot 10^{-6}$	0.00280	$3.31076 \cdot 10^{-6}$
6	$2.56834 \cdot 10^{-6}$	0.00049	0.00098	0.00017
7	$1.21175 \cdot 10^{-6}$	$2.87314 \cdot 10^{-7}$	0.00089	$5.90355 \cdot 10^{-7}$
8	$3.81200 \cdot 10^{-6}$	0.00029	0.00056	0.00002
9	$4.33901 \cdot 10^{-7}$	$4.06259 \cdot 10^{-8}$	0.00014	$1.77150 \cdot 10^{-7}$
10	$1.51461 \cdot 10^{-7}$	0.00015	0.00008	0.00001

Table 1: Analysis of convergence in L_∞-norm for $m = 20$

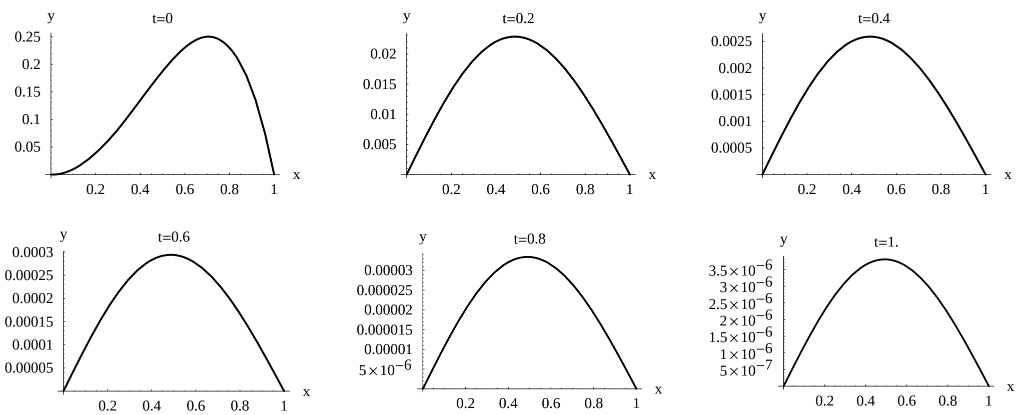

Figure 3: Graph ($n = 8,\ m = 20$) for the approximation ω^n of the function $\omega(x,t)$ for some values of t

4. Numerical Experiments

Analysis of the convergence is shown in Table 1. Our calculations were done by taking the time variable t from $[0, 1]$.

In Table 1, $n \in \mathbf{N}$ is the number of terms in Fourier expansion of initial functions (3). Approximate functions ρ^n, v^n, ω^n and θ^n are presented in Figures 1, 2, 3 and 4 for some values of variable t. It is also important to notice that by increasing the values of t we must expect our results in accordance with the results from Mujaković [3]. It means that our solution $(\rho^n, v^n, \omega^n, \theta^n)$ converges to $(1, 0, 0, 1.53)$ when $t \to \infty$.

In our experiment we get that for $t > 2$ the difference between solution $(\rho^n, v^n, \omega^n, \theta^n)$ and $(1, 0, 0, 1.53)$ is less than 10^{-2} in sup-norm for $n > 5$.

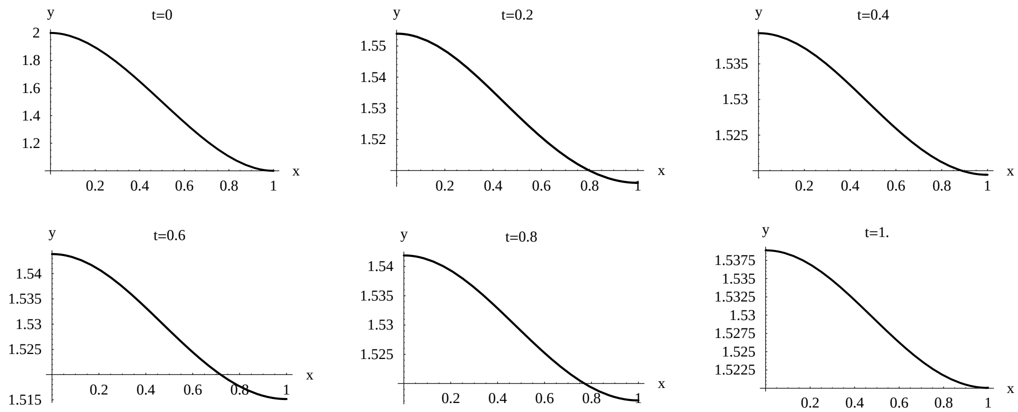

Figure 4: Graph ($n = 8,\ m = 20$) for the approximation θ^n of the function $\theta(x,t)$ for some values of t

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