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A 3D MODEL FOR A STOCHASTIC SET OF SMALL SCALE PLUMES IN OPEN SEA

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Abstract: A stochastic model for the Lagrangian dynamics of sea dense water plumes, generated by a surface stochastic buoyancy forcing due to wind bursts, is here presented. The entrainment and mixing effect, the dependence of scaling laws on the statistics of the external events, and the enhancing perturbative effect of rotation are discussed.

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1. Introduction

A mathematical model for generation of a set of small scale ($a \leq O(1Km)$) plumes in a very weakly stratified ($10^{-6}s^{-2} > N^2 > 10^{-8}s^{-2}$), mesoscale ($L \simeq O(50Km)$) region on open sea during the first convective phase of a deep water formation process is here presented. A comparison with the inferred by dimensional arguments Marshall scale laws, see [3], will be made. The plumes source (Bouché [1], [2]) is given by a not homogeneous surface buoyancy flux B_0, such that a set of random, not homogeneous buoyancy spots is generated on the surface. Each time, a variability of the surface buoyancy spots is given, such that: $\mathbf{db_s(x,y,z)} = \mathbf{d}(-\mathbf{g}\frac{\rho_\mathbf{s}-\rho_\mathbf{0}(\mathbf{z})}{\rho_\mathbf{0}}) = \mathbf{e}^{-\lambda\mathbf{z}}\mathbf{e}^{-\frac{\mathbf{y^2}}{\mathbf{2L^2}}}\mathbf{d}(\mathbf{b_a}\sum_\mathbf{k}\mathbf{e}^{-\frac{\mathbf{(x-nka)^2}}{\mathbf{2a^2}}})$; along the crosswise winds x direction, a short space correlation lenght is assumed in order to have separate plumes (Bouché [2]): a set of periodical axes,centres of such independent plumes, is defined. If $\gamma = \frac{a}{\sqrt{2}L} \ll 1$ and $\bar{y} = \frac{y}{a}$ are defined, a

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slow $Y = \gamma \bar{y}$ and a fast $y' = \bar{y}$ variables are identified along the winds y direction. The dynamics, defined by the 3D Navier-Stokes and the continuity equations in Boussinesq and hydrostatic approximation, for times $t < f^{-1}$ can be described (Bouché [2]), through a multiple space scale method, by a coupled set of 2D equations for the vorticity $\nabla_\perp \psi(x,z)$ and the buoyancy $b(x,z)$. For times $t \simeq O(f^{-1})$, at $O(\gamma = 0)$ this 2D system is coupled to an equation for the transverse velocity $v(x,z)$ through the Coriolis terms, as at $O(\gamma)$ the large scale 3D process becomes effective.

2. Solutions for $t < f^{-1}$

If $s_1 = -\frac{\partial b}{\partial x} = -\frac{\partial}{\partial x}\frac{-g(\rho(x,z)-\rho_0(z))}{\rho_0}$, $s_2 = -\frac{N^2}{g\rho_0}\frac{\partial p}{\partial x} = -\frac{N^2}{g\rho_0}\frac{\partial \int_0^z \rho(x,z')dz')}{\partial x}$, the 2D equations set is (Bouché [2]):

$$\frac{D_\perp \nabla_\perp^2 \psi}{Dt} = \sum_{i=1}^{3} s_i(x,z)\,, \quad \frac{D_\perp b}{Dt} - N^2 w = \frac{db_s}{dt}(x,z,t)\,, \tag{1}$$

where $\frac{D_\perp}{Dt}$ is the material derivative on the (x,z)-plane, to be solved in $\Omega \equiv (-L < x < L; 0 \leq z \leq H; \lambda^{-1} \ll H \ll L)$ with the initial and boundary conditions $\psi(t=0) = 0$, $b(t=0) = 0$, $\rho(t=0) = \rho_0(z)$, $\psi(\partial\Omega) = 0$. If $db_s/dt = \delta(t)b_s$ (instantaneous source), for short times the vorticity equation in (1) is a linear Dirichelet-Poisson problem solvable by the 'image charge' method: the symmetry properties of b_s allow 'octupole' vorticity sources to be identified, generating the 2D logaritmic Green function $G(x,z;x_0,z_0)$ in Ω; the integration all over the space, with the given boundary conditions, gives $\frac{\partial\psi}{\partial t}(x,z)$. The periodicity properties of b_s allow the 'octupoles' to be transformed into parallel 'quadrupoles' (each one centred on each axis) so that, by a Galileian translation $x'_k = x_0 - nka$, it is: $\frac{\partial\psi}{\partial t}(x,z) = \sum_k \frac{\partial\psi_k}{\partial t}(x,z)$. Inside each $\Omega_k \equiv (-\frac{na}{2} \leq x'_k \leq \frac{na}{2}; 0 \leq z \leq H)$ each vorticity source generates closed streamfunctions, having a hyperbolic form near the origin: their integration is essentially additive there, where the buoyancy is essentially concentrated. So $\frac{\partial\psi_k}{\partial t}(x,z,t) \propto \sum_{i=1}^{2}\int_{q_{ik}} dq_{ik}(x'_k,z_0) \cdot \frac{x'_k(t)z_0(t)}{(z_0^2(t)+x'^2_k(t))^2}|x-nka|z$: integrated, conserved in time, "quadrupole charges" $q_{1k} = \int_{q_{1k}} |dq_{1k}(x'_k,z_0)| = \frac{|b_a|}{\lambda}(\beta(0)-\beta(\frac{na}{2}))(\beta(x) = \sum_k e^{\frac{-x'^2_k}{2a^2}})$, $|q_{2k}| = \frac{N^2 H}{g}|q_{1k}| \ll |q_{1k}|$ can be defined in Ω_k^+. 'Charges barycentre' positions $\bar{z}_1 = \frac{\zeta_1}{2} = \lambda^{-1} log2 \gg \bar{z}_2 = \frac{\zeta_2}{2} = \frac{H}{2}$; $\pm\bar{x}_{i=1,2} = \pm\frac{\xi(na/2)}{2}$ can be defined so that: $\int_0^{\bar{z}_i} dz \int_0^{\frac{na}{2}} dq_i(x',z) = \frac{q_i}{2}$;$b(\bar{x}_i,\bar{z}_i) = \frac{b(\frac{na}{2},\bar{z}_i)+b(0,\bar{z}_i)}{2} = b_a\bar{\beta}$. These positions will be the initial condition for their following evolution. Convective motion is possible only if $1 \geq \Delta\beta_{Mm} = \beta(0) - \beta(\frac{l_\lambda}{2}) \gg 0; \bar{\beta} < 1$; $na \geq 4a\sqrt{-2log\bar{\beta}}$. In time, within a Lagrangian rapresentation, this hyperbolic form is maintained, so that the barycentres positions sink with the law:

$$\frac{d^2\zeta_1}{dt^2} = \frac{dw_0}{dt} = \frac{d}{dt}\Big(\frac{\partial\psi_k}{\partial x}\Big) \propto \sum_{i=1}^{2} q_{ik}\frac{\xi\zeta_i}{(\zeta_i^2+\xi^2)^2}z|_{z=\zeta_1/2} \sim C_1q_1[\frac{1}{\zeta_1^3} - \epsilon_N] \qquad (2)$$

($\epsilon_N = \frac{q_2}{q_1\zeta_2^3} = \frac{8N^2}{H^{*2}g^*}$) in the limit $\zeta_1 \gg \xi > 0$, but $\zeta_1\xi = C$. This law is scale invariant if $\zeta = \zeta_1\lambda$ and, for times such that $\zeta \gg 1$, has solution $\zeta \simeq (1/\epsilon_N)(\sqrt{1+2\epsilon_N(C_1q_1)^{1/2}t} - 1) \simeq (C_1q_1)^{1/2}t(1 - \frac{\epsilon_N(C_1q_1)^{1/2}t}{2})$, corresponding (Bouché[1],[2]) to $\zeta \simeq (C_1 < \Delta B_0 >_{z,t} t^3)^{1/2}$.

If a random variability is allowed in time on b_a and on na so that $db_s(t) = db_a(t) \cdot \sum_k \beta_k + b_a d(\sum_k \beta_k)$, $\frac{db_s}{dt} = \frac{d\int_0^t db_s(t')}{dt}$, the assumption of a short correlation lenght on $dn_k(x'_k, t) = \sqrt{\eta_k}dg(t)e^{-\frac{4|x'_k|}{a}}(1 + 2\sqrt{\frac{2}{a}}\int_0^{|x'_k|} e^{\frac{4x''}{a}} dW_k(x''))$ and of statistical independence between different $\Omega_k \equiv (0 \leq x' \leq (na + nda(k) + adn_k)/2; 0 \leq z_0 \leq H)$ domains, allows $d\psi(x,z) = \sum_k d\psi_k(x,z)$, $\frac{\partial d\psi_k}{\partial t}(x,z) = \sum_{i=1}^2 \int_0^H dz_0 e^{-\lambda z_0} \int_0^{\frac{l_\lambda(k)}{2}} dx'_k ds_i(x'_k) log\frac{r_{ik}^{+-}r_{ik}^{-+}}{r_{ik}^{++}r_{ik}^{--}}$; $< \frac{\partial d\psi}{\partial t} >= 0$, $< (\frac{\partial d\psi}{\partial t})^2 >^{1/2} = \sum_{k,k'} < \frac{\partial d\psi_k}{\partial t}\frac{\partial d\psi_{k'}}{\partial t} >^{1/2} = \sum_k < (\frac{\partial d\psi_k}{\partial t})^2 >^{1/2}$ to be written and different sinking plumes to be generated. On physical grounds it is possible to assume small independent white noises $da(t) = \sqrt{\epsilon_1}f(t)$ around the average a ($< da >= 0$), $dg(t) \propto db_a^k(t)$, so that all the process is led by this one. In the limit $\sqrt{\eta_k} \simeq \sqrt{\epsilon_1} \ll 1$, $\zeta_1 \gg \xi > 0$, but $\zeta\xi = C$, it is possible to write at the $O(\sqrt{\eta_k})$:

$$\frac{d^2\int_0^t d\zeta_{1k}}{dt^2} = \sum_{i=1}^{2}\frac{C\int_0^t dq_{ik}}{\zeta_{ik}^3}\,, \quad dq_{2k}/\zeta_{2k}^3 = -dq_{1k}\epsilon_N\,. \qquad (3)$$

The plumes evolution is defined by the statistics of dq_{1k} i.e. by the statistics of $db_a^{(k)}(t)$, $d(na)$: the first defines the time random variability of the buoyancy absolute maxima, the second defines the time variability of their distance and of the buoyancy horizontal gradient. In order to mathematically assure the statistical independence among plumes in the following linearization process, it is useful to put $db_a^{(k)}(t) = f_k db_a(t)$, where f_k is a random number $0.9 \leq |f_k| \leq 1$ taken on indipendent uniform distributions, such that $< f_k f_{k'} >= 0$. Some trial statistics have been introduced in Bouché [1], [2]: a) a Wiener process $db_a \sim \varepsilon^{1/2}d\mathbf{W}(t)$; b)a multiplicative white process $db_a \sim b_a\varepsilon^{1/2}d\mathbf{W}(t)$; c) a Ornstein-Uhlenbeck (red) process $db_a = -kb_a dt + \varepsilon^{1/2}dW(t)$ ($k^{-1}$is a correlation time). If $\varepsilon \ll 1$(or $\ll k$), it is $\zeta_1 \simeq \zeta_0 + \sqrt{\varepsilon}\zeta_1^1 + O(\varepsilon)$: so equation (2) can be split into a set of equations, one for each order; at $O(0)$:$\frac{d^2\int_0^t d\zeta_0}{dt^2} = 0$, whose only solution is the initial $\zeta_0 = \bar{z}_1$. At the first order, it is:

$$\frac{\mathbf{d^2 \int_0^t d\zeta_1^{(1)}}}{\mathbf{dt^2}} = \mathbf{C\Delta}\beta_{\mathbf{mM}}\frac{\int^{\mathbf{t}} \mathbf{db_a(t)}}{\zeta_{\mathbf{0}}^{\mathbf{3}}}(\mathbf{1}-\epsilon_{\mathbf{N}}) = \mathbf{C\Delta}\beta_{\mathbf{mM}}\int^{\mathbf{t}} \mathbf{db_a(t)}(\mathbf{1}-\epsilon_{\mathbf{N}}),$$

whose average value is zero, but whose RMS $(<\zeta_1^2> - <\zeta_1>^2)^{1/2}$ is:

$$\mathbf{RMS} = \mathbf{C\Delta}\beta_{\mathbf{mM}}$$
$$< \int_{\mathbf{0}}^{\mathbf{t}} \mathbf{dt'} \int_{\mathbf{0}}^{\mathbf{t'}} \mathbf{dt''} \int_{\mathbf{0}}^{\mathbf{t'''}} \mathbf{db_a(t''')} \int_{\mathbf{0}}^{\mathbf{t}} \mathbf{dt_1} \int_{\mathbf{0}}^{\mathbf{t_1}} \mathbf{dt_2} \int_{\mathbf{0}}^{\mathbf{t_2}} \mathbf{db_a(t_3)} >^{\mathbf{1/2}}.$$

This integration gives RMS $\simeq \varepsilon^{1/2}t^{5/2}$ for each considered statistics. In the opposite limit $\varepsilon \gg k$, $b_a(0) = b_a(-\infty) = 0$ (steady statistics) limit, a $k^{-1}t^{3/2}$ law is given in the process c). If $\varepsilon \gg 1$ the processes b) and c) show a $(<\zeta^2>^{1/2})^{-3}$ law (similar to the former instantaneous plume law); the introduction of a scaled time $t = \varepsilon^{-1/4}\bar{t}$ and a $1/\varepsilon$ expansion in the process a) gives again the same $t^{5/2}$ law. In Bouché [1] it is shown how, by introducing a time scale τ between buoyancy variations externally generated, such that $t = t/\tau$, and by introducing a scaled buoyancy flux, depending on the statistics, defined by dimensional arguments (Bouché [1], [2]),this corresponds to laws $<\zeta^2> - <\zeta>^2]^{1/2} \propto [<B_0^2> - <B_0>^2]^{1/2}t^{5/2}$ if $\varepsilon \ll 1$, and $<\zeta^2> - <\zeta>^2]^{1/2} \propto [<B_0^2> - <B_0>^2]^{1/2}t^{3/2}$ (like instantaneous plumes process) if $\varepsilon \gg 1$, or $\varepsilon \gg k$.

However, for long times, during the sinking motion, two processes become effective: a) localized plumes mixing, due to stirring processes in the limit $\xi \ll \zeta$; b) entrainment, introduced as soon as $Ri_p = N^2\xi^{*^2}/\dot{\zeta}^2 \simeq N^2\xi^{*^2}/C_1q_1 \ll 1$, when the plumes become turbulent on their boundary; this has an enhancing effect on the process a). It is to note how the former defined constant C can be considered proportional to the 2D one-plume volume. So, for entrainment, this constant has to change (Morton [4]):

$$\frac{\mathbf{dV_p}}{\mathbf{dt}} = \int Ewdz \simeq E\zeta\frac{d\zeta}{dt} = \frac{E(\zeta)}{2}\frac{d\zeta^2}{dt} \propto \frac{dC}{dt} + O(\epsilon_D), \quad 0.1 \le E < 0.3,$$
$$\frac{\mathbf{d<b_p>}}{\mathbf{dt}} = -\frac{1}{V}\frac{dV}{dt} < b_p > -N^2\frac{d\zeta}{dt},$$
$$k_\rho^* = k_\rho^*(\dot{\zeta}) \simeq k_\rho^*(t_0) + (\alpha + E)\dot{\zeta}\zeta,$$
$$\alpha \simeq O(10^{-5}), \quad k_\rho(t_0) \simeq 1.0cm^2/s,$$

$V_p \propto \zeta\xi^{1\pm\epsilon_D}$ ($D = 2 \pm \epsilon_D$ is the fractal dimension of the entraining plume; $\epsilon_D \ll 1$). For $D = 2$ ($E_1 \simeq O(E/\pi)$), it is $C(t) \simeq C + E_1(\zeta^2 - \zeta_E^2)/2$; $< b_p(t) > \simeq \frac{<b_p(t_0)>C}{C+E(\zeta^2-\zeta E_1^2)}[1 - N^2[C(\zeta - \zeta_E) + E_1(\zeta^3 - \zeta_E^3)/3]$; $e^{-\int_{t_0}^t dt' k_\rho^* \zeta^2(t')/C^2} \simeq$

$[1 + \frac{E_1}{2C}(\zeta^2 - \zeta_E^2)]^{-2E/E_1^2} e^{-\frac{E}{E_1 C}\frac{(\zeta^2-\zeta_E^2)(E_1\zeta_E^2/2C-1)}{1+E_1(\zeta^2-\zeta_E^2)/2C}}$. At last it is:

$$\frac{\mathbf{d^2}\zeta}{\mathbf{dt^2}} \sim [\frac{\mathbf{C_1 q_1 + f(\zeta_E, E, C)}}{\zeta^\mathbf{3}} + \frac{\mathbf{E_1}}{\mathbf{2}}\frac{\mathbf{q'}}{\zeta} - < b_p(t_0) > CN^2(\frac{C + E_1\zeta_E^2/2}{\zeta^2} + \frac{E_1}{6})](\mathbf{1} - \epsilon_\mathbf{N}\zeta^\mathbf{3}) .$$

As soon as $E \neq 0$ (when $\zeta = \zeta_E$) a crossover from a $C\zeta^{-3}$ to a $E_1(q'\zeta^{-1} - N^2)$ law is evident, enhancing the effect of the variability and of the small stratification, so that,in the instantaneous and in the $\varepsilon \gg 1$ stochastic plumes model, it is $\dot{\zeta} \to \dot{\zeta_E} - \frac{<b_p(t_0)>CN^2E_1}{6}(t - t_E)$. If $\varepsilon \ll 1$, it is, as before, $\zeta_0 = 1$, so that the former scale laws do not change for entrainment, but for the constant $E_1 < C_1$. Corrections due to the introduction of $\epsilon_D \ll 1$, at $O(\epsilon_D)$, do not seem, at the moment, to be meaningful.

3. Solutions for $t \simeq O(f^{-1})$

For $t \geq O(f^{-1})$ a time scale $t = tf$ and dimensionless variables $1 < \mathcal{N} = \frac{N^2}{f^2} < 10$,$\mathcal{E} = \frac{\nu\lambda^2}{f} \leq O(10^{-1})$,$\mathcal{B} = \frac{b\lambda}{f^2}$, $P = \frac{p\lambda}{g\rho_0}$, $\sigma = \frac{\nu}{k} \simeq 1$ can be introduced, so that, at $O(\gamma = 0)$, if $s_3 = \frac{\partial v}{\partial z}$ and $\mathcal{B}_s = (1 - Y^2 + e^{-\gamma^2 y'^2})\mathcal{B}_s^\perp = (1 - Y^2 + e^{-\gamma^2 y'^2})\mathcal{B}_a(t)e^{-\lambda z}\sum_{k=0}^{k_{max}} \beta_k$, it is: $\sigma(\frac{D_\perp \mathcal{B}}{Dt} - \mathcal{N}w) - \mathcal{E}\nabla_\perp^2\mathcal{B} = \sigma\frac{d\mathcal{B}_s^\perp}{dt}$; $\vec{\nabla}_\perp \cdot \vec{u} = 0$ and:

$$\frac{D_\perp \nabla_\perp^2 \psi}{Dt} = \sum_{i=1}^{3} s_i(x, z) + \mathcal{E}\nabla_\perp^4 \psi; \quad \frac{D_\perp \nabla_\perp^2 v}{Dt} = \frac{\partial \nabla_\perp^2 \psi}{\partial z} + \mathcal{E}\nabla_\perp^4 v \qquad (4)$$

to be solved with the conditions: $\mathcal{B}(t_0) = \mathcal{B}(x, z, \xi(t_0), \zeta(t_0))$,$\xi(t_0) \ll \zeta(t_0)$, $\psi(t_0) = \psi(x, z, \xi(t_0), \zeta(t_0))$, $\dot{\zeta} = \dot{\zeta}(t_0)$, $v(t_0) = 0$,$\frac{\partial v}{\partial z}(z = 0) = 0$. For times $t \simeq O(f^{-1})$, the dissipative terms dependent on $\mathcal{E}$ can be ignored till the steady state is got. From equation (4), the Coriolis deviation, in Lagrangian rapresentation, is visible, so that in time it is: $v(x, z, t) = -\int_{t_0}^t u(x_0, z_0, \tau)d\tau + v(x_0 + \int_{t_0}^t u(x_0, z_0, \tau)d\tau, z_0 + \int_{t_0}^t w(x_0, z_0, \tau)d\tau, t)$; $\vec{u} \equiv (u, w) = \vec{u}(x, z, \xi(t), \zeta(t))$. Buoyancy si driven along the y direction, along which a horizontal shear, satisfying the necessary Fjordoff conditions for instability, is generated. But the slow dependence of $\mathcal{B}$ on y allows a buoyancy flux $B_y(Y) = B_y(Y')$ for $\forall Y, Y'$, so that $\mathcal{B}$ remains unchanged on each vertical plane. In time, around each plume, shear changes sign along a surface ($z = z_r(x) > \zeta$): so two separate, concentric regions tend to rotate into two opposite directions; vertical turbulent lines along the y direction are generated causing formation of small scale vortices for Fjordoff instability. On the vertical plane two new, non linear, dependent on time, vorticity sources $s_{3i} = s(v(u(q_i)))$ $(i = 1, 2)$, are generated. For an instantaneous or stochastic $\varepsilon \gg 1$ buoyancy source, their integrated value is: $|q_{31}(t)| = \int dx \int^{z_r} dz(\frac{\partial v}{\partial z}) = \int dx \int_1^t dt' \int_{t_i}^{t'} dt'' \frac{8q_1 Cx}{(x^2-\zeta^2)^2} = \int_1^t dt' \int_{t_i}^{t'} dt'' \frac{4q_1 C}{\zeta^2(t'')}$; $|q_{32}(t)| = \int_1^t dt' \int_{t_i}^{t'} dt'' \frac{q_1 C\epsilon_N H^2}{4\zeta(t'')}$. Non linear integrations have been got because $\zeta = \zeta(q_1, q_2, q_{31}, q_{32})$, but $t \simeq O(1)$,$\Delta t \ll 1$, so that non linear terms can be ignored, and:$|q_{31}| \simeq \Delta t + \log t$, $|q_{32}| \simeq \sqrt{Cq_1} H^2 \epsilon_N t(1 - \log t)$. As before, 'barycentre positions' can be defined for them: $(\xi_{c1} = \zeta; \zeta_{c1} = 0.88\zeta)$ $(\xi_{c2} = \frac{H}{2}; \zeta_{c2} = 0.88\frac{H}{2})$. Final 'barycentre positions' for opposite vorticity sources are: $(\xi^+(t) = \frac{q_1\xi + q_{32}(t)H/2}{q_1 + q_{32}(t)} > \xi; \zeta^+(t) = \frac{q_1\zeta + q_{32}(t)0.88H/2}{q_1 + q_{32}(t)} \simeq O(\zeta) < \zeta)$ and $(\xi^-(t) = \frac{q_2\xi + q_{31}(t)\zeta}{q_1 + q_{31}(t)} > \xi; \zeta^-(t) = \frac{q_2 H/2 + q_{31}(t)0.88\zeta}{q_1 + q_{31}(t)} \simeq O(H/2) < H/2)$. Horizontal propagation of vorticity results. Nevertheless the buoyancy remains essentially trapped in the hyperbolic region driving the sinking motion: the equation of motion for the sinking plume is, at last:

$$\frac{d^2\zeta}{dt^2} \simeq C_1 q_1 [\frac{1}{\zeta^3} - \epsilon_N] - |q_{31}(t)| \frac{0.88}{\zeta} + |q_{32}| \frac{7}{H^2}\zeta.$$

Sources are dependent on themself, but integration is possible by steps for interesting times $\Delta t \ll 1; t \simeq O(1)$; for $t > O(1)$ a steady state will be got. It is to note how Coriolis sources remain very small during interesting times, so that the former time power laws $t^{3/2} \to f^{-3/2}$will be obtained. The enhancing environmental perturbation lets detrainment effect and mixing be generated and keep from entrainment. For a stochastic $\epsilon \ll 1$ buoyancy source, $\int dq_{3i} \simeq \int dt \int dt' db_a + \int dt \int dt' \int dt_1 \int dt_2 db_a + ... \simeq \int dt \int dt' db_a$ for $\Delta t \ll 1$. At $O(\sqrt{\varepsilon}$, it is: $< \zeta^2 > - < \zeta >^2 = < \zeta^2(1) > - < \zeta(1) >^2 + < \int_1^t dt' \int_0^{t'} dt'' \int_1^t dt_1 \int_0^{t_1} dt_2 \int_0^{t''} db_a(t'') \int_0^{t_2} db_a(t_2) + ... > \simeq < \zeta^2(1) > - < \zeta(1) >^2$ and $< q_{31}^2 >^{1/2} \simeq < \zeta^2 >^{1/2} \simeq < \xi^2 >^{1/2}$. The former horizontal broadening of fluctuactions is confirmed; plumes sink with a law, scaling like $f^{-5/2}$; vorticity is driven along horizontally enlarged streamlines, allowing detrainment and plumes mixing. The full coupled more plumes behaviour can be analysed at $O(\gamma)$.

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