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A LOWER BOUND ON THE PERFORMANCE OF SIMPLIFIED LINEAR PRECODING FOR VECTORED VDSL

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Abstract: Mutual interference between lines is the dominant noise in modern DSL systems such as ADSL and VDSL. This mutual interference is termed crosstalk. Next generation VDSL systems will use vector precoding techniques to suppress crosstalk. In recent years many crosstalk cancelation methods based on matrix computation theory have been proposed to cancel crosstalk. However, the large complexity involved in crosstalk cancelation is still the major obstacle in implementing these techniques. In this paper we study a simplified linear precoding scheme for FEXT cancelation in VDSL downstream transmission. Theoretical tight lower bound of the achievable rate based on second order precoding method is derived. This bound together with the lower bound based on first order precoding method allow us to predict the performance of the proposed algorithms on each tone. We end up with testing the theoretical bounds based on both theoretic channels and empirical channels.

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1. Introduction

Crosstalk is the major limiting noise in very high bit rate digital subscriber line (VDSL) systems in terms of bit-rate and reach of service. For downstream transmission, the receiving modems for different users are located at different locations. Hence, crosstalk cancelation cannot be implemented at the receiver side. Since, the transmitting modems are co-located at the central office (CO) or optical network unit (ONU), crosstalk precoding can be applied at the transmitter side.

Several crosstalk cancelers based on matrix computation theory have been proposed recently. Ginis and Cioffi [5] proposed a Tomlinson Harashima type FEXT precoding with high complexity due to nonlinear processing. Cendrillon et al [3] have noted that it is sufficient to use a linear precoding due to the diagonal dominance property of the FEXT channel matrix, still their zero-forcing (ZF) solution requires matrix inversion at each tone of the multichannel DSL system. Leshem and Li [7] proposed a simplified linear precoding scheme by approximating the matrix inversion using power series expansion. This approach has a much lower implementation complexity. Theoretic lower bound of achievable rate for first order precoding method is also derived there.

In this paper, we derive a tight lower bound of achievable rate for second order precoding method. This bound with the lower bound based on first order precoding method are very useful for predicting the performance of the proposed algorithm on each tone. Computer simulation results based on both real measured and theoretic crosstalk channel data are provided to verify our analysis results.

The structure of the paper is as follows: In Section 2 we describe a mathematical model for multi-pair DSL systems. In Section 3 we review two common precoding schemes for downstream transmission. In Section 4 we derive the lower bound on the achievable rates of second precoding method. In Section 5 we describe simulated experiments verifying the performance of the proposed methods. We end up with some conclusions.

2. Model and Notations

In this section we describe the signal model of a multichannel precoded system. We concentrate on discrete multitone (DMT) systems where the transmission is done independently over many narrow sub-bands, see [2].

Assume that we have a binder consisting of p twisted pairs. Typical binders include 25, 28, 50 or 100 pairs. Such telephone binders are well modeled by as a multi-dimensional linear time invariant system. Further information on the statistics of these transfer function is provided in [8], [6]. Let the transfer function of the system be given by

$$\mathbf{H}(z) = \begin{bmatrix} h_{11}(z) & \dots & h_{1p}(z) \\ \vdots & \ddots & \vdots \\ h_{p1}(z) & \dots & h_{pp}(z) \end{bmatrix}.$$

Assuming no coordination between transmitters the received signal at pair i, $1, \leq i \leq p$ can be written as

$$x_i(t) = h_{ii} * s_i(t) + \sum_{l \neq i} h_{il} * s_l(t) + \nu_i(t)\,, \tag{1}$$

where h_{ii} is the i-th pair channel impulse response, h_{il} are the FEXT transfer functions from pair l to pair i, $\nu_i(t)$ is zero mean circularly symmetric additive white Gaussian noise with covariance matrix $E\left(\boldsymbol{\nu}\boldsymbol{\nu}^*)\right) = \sigma^2\mathbf{I}$ (typically the AWGN power is assumed to have a flat power spectral density (PSD) of -140 dBm/Hz in DSL applications) and $*$ denotes convolution. Translating to the frequency domain we obtain that

$$x_i(f) = h_{ii}(f)s_i(f) + \sum_{l \neq i} h_{il}(f)s_l(f) + \nu_i(f)\,. \tag{2}$$

In vector form we can represent the received signal by

$$\mathbf{x}(f_k) = \mathbf{H}(f_k)\mathbf{s}(f_k) + \boldsymbol{\nu}(f_k)\,, \tag{3}$$

where

$$\mathbf{H}(f_k) = \begin{bmatrix} h_{11}(f_k) & \cdots & h_{1p}(f_k) \\ \vdots & \ddots & \vdots \\ h_{p1}(f_k) & \cdots & h_{pp}(f_k) \end{bmatrix}$$

is the channel frequency response, $\mathbf{s}(f_k) = [s_1(f_k), \dots, s_p(f_k)]^T$ are the frequency domain representations of the signals transmitted by the system, and $\mathbf{x}(f_k) = [x_1(f_k), ..., x_p(f_k)]^T$ is the vector of received signals.

When the specific frequency processed is not relevant for the discussion we shall suppress the explicit dependence on f_k and use the following notation

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \boldsymbol{\nu}\,. \tag{4}$$

The achievable rate on the i-th channel under no coordination is now given

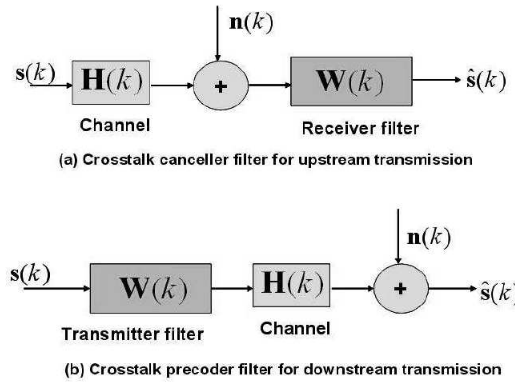

Figure 1: Crosstalk cancelation for up and downstream transmission

by [4]

$$C_i = \int_f \log_2 \left(1 + \frac{P_i(f)\, |h_{ii}(f)|^2}{\mathbf{\Gamma} \left(\sum_{l \neq i} P_l(f)\, |h_{il}(f)|^2 + |v_i(f)|^2 \right)} \right) df \,, \tag{5}$$

where $\mathbf{\Gamma}$ is the gap to capacity which for coded QAM with BER 10^{-7} is $9.8 - c_g + m_g$, where c_g is the coding gain and m_g is the margin. When the gap is ($\mathbf{\Gamma} = 1$) we obtain the Shannon capacity, under Gaussian noise. The leading term in the denominator of (5) is the crosstalk term. In the next section we will discuss linear precoding of the transmitted signal, i.e., replacing the vector $\mathbf{s}$ by a linear transformation $\mathbf{Ws}$ such that crosstalk is either eliminated or significantly reduced so that the achievable rate is significantly increased.

3. Crosstalk Precoding for Downstream Transmission

A typical scenario of a VDSL system with crosstalk canceler for upstream and downstream transmission are shown respectively in Figure 1 (a) and (b). For upstream transmission, we design the receiver matrix $\mathbf{W}(k)$ in the receiver side to cancel the crosstalk, and for downstream transmission, our purpose is to design a crosstalk precoding matrix $\mathbf{W}(k)$ in the transmitter side to eliminate the crosstalk in the receiver side. We only consider downstream transmission in this paper.

We now review several simplified linear precoding techniques.

3.1. Zero-Forcing Precoder

The zero-forcing precoder (see [3]) estimates the transmitted symbols by multiplying the transmitted symbol vector in the transmitter side with the following filter matrix $\mathbf{W}$,

$$\mathbf{W} = \mathbf{H}^{-1}\mathbf{D}\,. \tag{6}$$

ZF precoder cancels the crosstalk completely, and we can estimate the symbols in the receiver side from

$$\hat{s_k} = \mathbf{D}\mathbf{s}_k + \mathbf{v}_k\,. \tag{7}$$

Assuming linear ZF precoding is used in the transmitter side, the capacity is now given by [7]

$$C_i = \int_f \log_2\left(1 + \frac{P_i(f)\,|h_{ii}(f)|^2}{\mathbf{\Gamma}\,|v_i(f)|^2}\right) df\,. \tag{8}$$

As we will see in simulations, in typical cases crosstalk precoding for downstream transmission leads to substantial increase in rate.

The main disadvantage of the zero-forcing precoder is the high complexity of matrix inversion at each tone as well as the need to store the coefficients of the matrix $\mathbf{W}$. Typical VDSL systems use up to 4096 tones, and storing 4096 100×100 matrices at the required accuracy turns out to be significant in terms of silicon implementation complexity.

3.2. Diagonal Line Precoder

The basic idea of the diagonal line precoder (see [7]) is to split the crosstalk channel matrix into diagonal matrix plus a matrix with zero elements on the main diagonal line. The inverse is then approximated by a power series expansion.

Using the diagonal dominance property of matrix $\mathbf{H}$ (see [7]) and matrix splitting $\mathbf{H} = \mathbf{D}+\mathbf{E}$, the inversion of $\mathbf{H}$ has the following power series expansion:

$$\mathbf{H}^{-1} = \mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1} + \left(\mathbf{D}^{-1}\mathbf{E}\right)^2\mathbf{D}^{-1} + \ldots\,. \tag{9}$$

For first order precoder based on diagonal line splitting, we have

$$\mathbf{W} = (\mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1})\mathbf{D}\,. \tag{10}$$

Since $\mathbf{D}$ is a diagonal matrix, it is inversion requires only N divisions. Compared to ZF precoder, the diagonal line precoder has much lower computational and implementation complexity.

The first order precoder estimates the received symbols by:

$$\hat{s_k} = \mathbf{D}\mathbf{s}_k - \mathbf{E}\mathbf{D}^{-1}\mathbf{E}\mathbf{s}_k + \mathbf{v}_k \,. \tag{11}$$

As $\mathbf{E}\mathbf{D}^{-1}\mathbf{E}\mathbf{s}_k$ is non zero, this precoder does not cancel the crosstalk completely, causing some loss as compared to ZF precoder.

For second order precoder based on diagonal line splitting, obtain that

$$\mathbf{W} = (\mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1})\mathbf{D} \,. \tag{12}$$

With second order precoder, the estimated symbols at the receiver side are given by:

$$\hat{s_k} = \mathbf{D}\mathbf{s}_k + \mathbf{E}(\mathbf{D}^{-1}\mathbf{E})^2\mathbf{s}_k + \mathbf{v}_k \,. \tag{13}$$

The second order precoder can easily be implemented in Horner form:

$$\mathbf{W}\mathbf{s} = \left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{E}\left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{E}\right)\right)\mathbf{s}. \tag{14}$$

Simplifying we obtain

$$\mathbf{W}\mathbf{s} = \mathbf{s} - \mathbf{D}^{-1}\mathbf{E}\left(\mathbf{s} - \mathbf{D}^{-1}\mathbf{E}\mathbf{s}\right). \tag{15}$$

In this form only $\mathbf{D}^{-1}, \mathbf{E}$ should be stored, and no matrix inversion is required. The price is increases slightly in the steady state complexity due to extra matrix vector multiplication. According to [7] this results in significant reduction in storage and initialization complexity of the precoded vectored system.

4. Lower Bounds of the Precoding Method for Downstream Transmission

We now turn to analyze the performance of the second order approximations to the ZF precoder given by (12). We use approximations to the FEXT transfer functions. For comparison, the lower bound of first order approximation method is also provided in the following Theorem 1.

Theorem 1. *Let $\mathbf{H} = (h_{ij})$ be the channel transform matrix.*

$$\mathbf{\Delta}_{il}(f) = \sum_{k \neq i,l} \frac{h_{ik}(f)h_{kl}(f)}{h_{kk}(f)} \,. \tag{16}$$

The received signal and noise plus residual cross-talk power for channel i are defined respectively as

$$P_{sig}(f,i) = P_{s_i}(f)|\mathbf{\Delta}_{ii}(f)|^2 \,, \tag{17}$$

$$P_{noise}(f,i) = v(f) + \sum_{l \neq i} P_{s_l}(f)\,|\mathbf{\Delta}_{kl}(f)|^2 \,. \tag{18}$$

Then the total achievable rate for channel i when using first order precoding is given by

$$C_{FO_i} = \sum_k C_i(f_k) \ , \tag{19}$$

where

$$C_i(f_k) = \Delta f \log_2(1 + \frac{P_{sig}(f_k, i)}{\Gamma P_{noise}(f_k, i)}). \tag{20}$$

Furthermore, if we define the signal to noise ratio and residual cross-talk to noise ratio respectively by

$$\begin{aligned} SNR(f,i) &= \frac{P_{s_{max}}(f)|h_{k_ik_i}(f)|^2}{v(f)}, \\ INR(f) &= \frac{(p-1)\alpha_1^2(f)|h_{k_0k_0}(f)|^2 P_{s_{max}}(f)}{v(f)}. \end{aligned} \tag{21}$$

The achievable rate is lower bounded by

$$C_i(f_k) \geq \Delta f \log_2\left(\frac{SNR(f_k,i)}{\Gamma}\right)$$
$$-\Delta f \log_2\left(1+\alpha_2^2(f_k)(p-2)^2 INR(f_k)\right)+\Delta f \log_2\left(1-(p-1)\alpha_1(f_k)\alpha_2(f_k)\right)^2 \ ,$$

where

$$|h_{k_0k_0}(f)| = \max_k |h_{kk}(f)| \qquad P_{s_{max}}(f) = \max_k P_{s_k}(f) \tag{22}$$

are respectively the maximal channel transfer function and maximum signal PSD. $\alpha_1(f_k), \alpha_2(f_k)$ are bounds on the FEXT coupling between pair i and other pairs and between other pairs and pair i respectively, i.e.

$$\alpha_1(f) = \max_{\ell \neq k} \frac{|h_{\ell k}(f)|}{|h_{kk}(f)|}, \quad \alpha_2(f) = \max_{\ell \neq k} \frac{|h_{kl}(f)|}{|h_{kk}(f)|}. \tag{23}$$

The proof is given in [7].

We now generalize Theorem 1 to second order precoding. To that end let

$$\Omega_{il}(f) = \sum_{m \neq i} \left(\sum_{k \neq m,l} \frac{h_{mk}(f) h_{kl}(f)}{h_{mm}(f) h_{kk}(f)} h_{im} \right). \tag{24}$$

The performance of the second order precoder is now given by:

Theorem 2. *Let $\Omega_{il}(f)$ be defined by (24), the received signal and noise plus residual cross-talk power for channel i be defined by*

$$P_{sig}(f,i) = P_{s_i}(f)|\Omega_{ii}(f)|^2 \tag{25}$$

and

$$P_{noise}(f,i) = v(f) + \sum_{l \neq i} P_{s_l}(f)\, |\Omega_{kl}(f)|^2 \tag{26}$$

respectively. Then the total capacity for channel i when using second order precoding is given by

$$C_{SO_i} = \sum_k C_i\,(f_k)\ ,$$

where

$$C_i(f_k) = \Delta f \log_2 \left(1 + \frac{P_{sig}(f_k,i)}{\Gamma P_{noise}(f_k,i)}\right). \tag{27}$$

Moreover, the achievable rate at frequency f_k is lower bounded by:

$$\begin{aligned} C(f_k) \geq \Delta f \log_2 \frac{SNR(f_k,i)}{\Gamma} &- \Delta f \log_2 \left(1 + \beta(f_k) INR(f_k)\right) \\ &+ \Delta f \log_2 (1-(p-1)(p-2)\alpha_1^2(f_k)\alpha_2(f_k))^2\ , \end{aligned} \tag{28}$$

where $\beta(f_k) = (p-1)(p-2)^3\alpha_1^2(f_k)\alpha_2^2(f_k)$, $SNR(f_k), INR(f_k)$ are defined by (21) and $\alpha_1(f), \alpha_2(f)$ are defined by (23).

Proof. We derive the lower bound on the performance of the second order precoding. After second order precoding, the received signal is given by

$$\mathbf{x} = \left(\mathbf{D} + \mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^2\right)\mathbf{s} + \boldsymbol{\nu}\,. \tag{29}$$

Therefore for each $1 \leq i \leq p$ the received signal at the i-th receiver is given by

$$x_i(f) = h_{ii}(f)s_i + \sum_{l=1}^{p} \Omega_{il}(f)s_l + \nu_i = h'_{ii}(f)s_i + v'_i(f) \tag{30}$$

where $\mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^2 = (\Omega_{ij}(f))$, $v'_i(f)$ is the total noise and residual crosstalk at the i-th channel

$$v'_i(f) = -\sum_{l \neq i} \Omega_{il}(f)s_l + \nu_i \tag{31}$$

and $h'_{ii}(f) = h_{ii}(f) - \Omega_{ii}(f) = h_{ii}(f) - \sum_{l\neq i}\left(\sum_{k\neq i,l} \frac{h_{lk}(f)h_{ki}(f)}{h_{ll}(f)h_{kk}(f)}\right) h_{il}(f)$.

The received signal and noise power are given respectively by:

$$P_{sig}(f,i) = P_{s_i}(f)|h'_{ii}(f)|^2\,, \tag{32}$$

where

$$P_{noise}(f,i) = v(f) + \sum_{l \neq i} P_{sig}(f,l)\, |\Omega_{kl}(f)|^2\,. \tag{33}$$

Note that

$$\left|h'_{ii}(f)\right| = |h_{ii}(f) + \Omega_{ii}(f)| \geq (|h_{ii}(f)| - |\Omega_{ii}(f)|) \tag{34}$$

and

$$|\Omega_{ii}(f)|^2 = \left|\sum_{l\neq i}\left(\sum_{k\neq i,l}\frac{h_{lk}(f)h_{ki}(f)}{h_{ll}(f)h_{kk}(f)}\right)h_{il}(f)\right|^2 \leq \left(\sum_{l\neq i}|h_{il}(f)|^2\right)$$
$$\times\left(\sum_{l\neq i}\left|\sum_{k\neq i,l}\frac{h_{lk}(f)h_{ki}(f)}{h_{ll}(f)h_{kk}(f)}\right|^2\right) \leq (p-1)(p-2)\alpha_1^2(f)\alpha_2(f)|h_{k_0k_0}(f)|^2\,. \tag{35}$$

For $j \neq i$ we obtain similarly

$$|\Omega_{ij}(f)|^2 = \left|\sum_{l\neq i}\left(\sum_{k\neq l,j}\frac{h_{lk}(f)h_{kj}(f)}{h_{ll}(f)h_{kk}(f)}\right)h_{il}(f)\right|^2$$
$$\leq \left(\sum_{l\neq i}|h_{il}(f)|^2\right)\left(\sum_{l\neq i}\left|\sum_{k\neq l,j}\frac{h_{lk}(f)h_{kj}(f)}{h_{ll}(f)h_{kk}(f)}\right|^2\right)\,. \tag{36}$$

Note that

$$\sum_{l\neq i}|h_{il}(f)|^2 \leq (p-1)\alpha_1^2(f)|h_{k_0k_0}(f)|^2 \tag{37}$$

and

$$\sum_{l\neq i}\left|\sum_{k\neq l,j}\frac{h_{lk}(f)h_{kj}(f)}{h_{ll}(f)h_{kk}(f)}\right|^2 \leq \sum_{l\neq i}\left(\sum_{k\neq l,j}\left|\frac{h_{lk}(f)}{h_{ll}(f)}\right|^2\right)\left(\sum_{k\neq l,j}\left|\frac{h_{kj}(f)}{h_{kk}(f)}\right|^2\right)$$
$$= \left(\sum_{k\neq i}\left|\frac{h_{lk}(f)}{h_{ll}(f)}\right|^2\right)\left(\sum_{k\neq i}\left|\frac{h_{kj}(f)}{h_{kk}(f)}\right|^2\right)$$
$$+\sum_{l\neq i,j}\left(\sum_{k\neq l,j}\left|\frac{h_{lk}(f)}{h_{ll}(f)}\right|^2\right)\left(\sum_{k\neq l,j}\left|\frac{h_{kj}(f)}{h_{kk}(f)}\right|^2\right)$$
$$\leq (p-1)^2\alpha_1^2(f)\alpha_2^2(f) + (p-2)^3\alpha_1^2(f)\alpha_2^2(f)\,. \tag{38}$$

Hence

$$|\Omega_{ij}(f)|^2 \leq (p-1)\left((p-1)^2 + (p-2)^3\right)\alpha_1^4(f)\alpha_2^2(f)|h_{k_0k_0}(f)|^2\,. \tag{39}$$

Therefore the capacity of the k-th frequency bin $f_k \leq f \leq f_k + \Delta f$ with gap Γ

is bounded by

$$C_i\left(f_k\right) = \Delta f \log_2\left(1 + \frac{P_{sig,i}\left(f_k\right)}{\Gamma P_{noise,i}\left(f_k\right)}\right) . \tag{40}$$

Since

$$P_{sig,i} = P_{s_i}\left(f_k\right)\left(\left|h_{ii}\left(f_k\right)\right| - (p-1)(p-2)\alpha_1(f_k)^2\alpha_2(f_k)\left|h_{k_0k_0}\left(f_k\right)\right|\right)^2 \tag{41}$$

and

$$P_{noise,i} = N\left(f_k\right) + (p-1)^2((p-1)^2 + (p-2)^3)\alpha_1(f_k)^4\alpha_2(f_k)^2\left|h_{k_0k_0}\left(f_k\right)\right|^2 P_{s_{max}}\left(f_k\right) , \tag{42}$$

we obtain

$$C_i\left(f_k\right) \geq \Delta f \log_2 \frac{SNR(f_k)}{\Gamma} - \Delta f \log_2\left(1 + \beta(f_k) INR(f_k)\right) + \Delta \log_2(1 - (p-1)(p-2)\alpha_1^2(f_k)\alpha_2(f_k))^2 , \tag{43}$$

where $\beta(f_k) = (p-1)(p-2)^3\alpha_1(f_k)^2\alpha_2(f_k)^2$ and

$$SNR = \frac{P_{s_{max}}(f_k)|h_{k_ik_i}(f_k)|^2}{v(f_k)} , \quad INR = \frac{(p-1)\alpha_1^2(f_k)|h_{k_0k_0}(f_k)|^2 P_{s_{max}}(f_k)}{v(f_k)} \tag{44}$$

are respectively the signal to noise and total FEXT to noise ratio. □

The above analysis demonstrates that performance of first and second-order precoding depends on $\alpha_1(f)$ and $\alpha_2(f)$. In practice for most frequencies the proposed precoding approaches the optimal processing. Since the second order precoding depends on α_i^2 instead of α_i the loss of the second order precoder is much lower than that of the first order precoder. We will show the tightness of the second order bound in simulations based on both theoretical models and measured data.

5. Performance Comparisons

In this section, some computer simulation results are provided to verify our analysis results. In the following experiments, we evaluate the performance of lower bounds through 4 VDSL lines over 4 different distances 75m, 150m, 300m and 600m. The transmit PSD was -60 dBm/Hz. The noise is assumed to be AGWN with PSD of -140 dBm/Hz. Each modem has a coding gain of 3.8 dB, noise margin of 6 dB which results in a gap $\Gamma = 12dB$ for bit error rate (BER) of 10^{-7}. The VDSL frequency bands follow the ITU-T 998 bandplan (see [1])

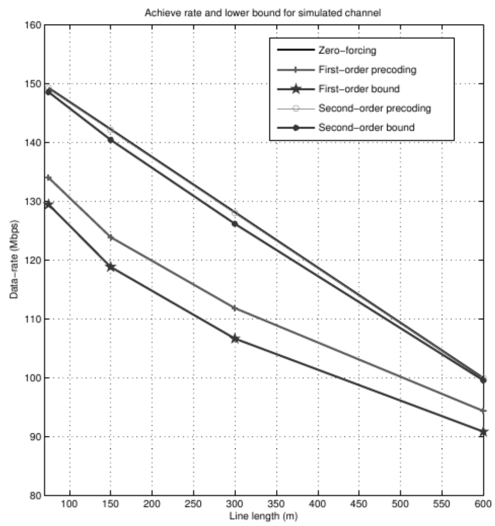

Figure 2: 4 lines, empirical, VDSL bandplan

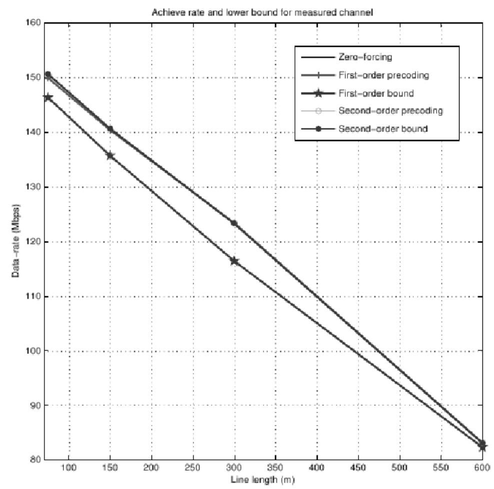

Figure 3: 4 lines, measured, VDSL bandplan

where downstream frequencies are $0.138-3.75$ MHz, and $5.2-8.5$ MHz. In the first experiment, the channel transfer function and FEXT used for simulation using 24-gauge line without bridge taps are based on the model from [2]. Figure 2 depicts estimated capacity and lower bounds of first-order, and second-order precoding, respectively. For comparison purpose, the capacity of zero-forcing method is also added.

Figure 3 shows the results based on the France Telecom measured data.

To demonstrate the tightness of the derived lower bounds, we also consider the special case where the off diagonal elements in each tone is the same. Figure 4 shows the result in this case.

6. Conclusions

In this paper, we derived a lower bound of the achievable rate based on second order linear precoding method. This bound is useful to predict the performance of the second order precoder and allows realtime choice between first and second order precoding. Computer simulation results based on simulated and measured

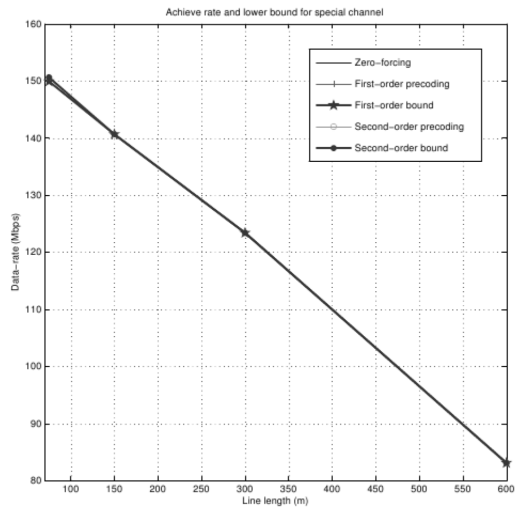

Figure 4: 4 lines, measured, VDSL bandplan

crosstalk channel data are provided to verify the tightness of our analysis.

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