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INTUITIONISTIC FUZZY ALMOST STRONG PRECONTINUITY IN COKER'S SENSE

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Abstract: Intuitionistic fuzzy almost strongly precontinuous mappings and intuitionistic fuzzy almost strongly preopen (preclosed) mappings between Coker's intuitionistic fuzzy topological spaces have been introduced and studied. A characterization of intuitionistic fuzzy open mappings by using those mappings and some weaker forms of intuitionistic fuzzy continuous mappings has been established.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh in his classic paper [10]. Using the concept of fuzzy sets Chang [2] introduced the notion of fuzzy topological spaces. Since Atanassov [1] introduced the notion of intuitionistic fuzzy

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sets, Coker [3] defined the intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of directions is related to the intuitionistic generalized sets introduced by Coker [4]. As a continuation of this work, Jeon [6] introduced the concept of intuitionistic α-continuity and intuitionistic fuzzy precontinuity.

In this paper, as an extension of authors concept presented in [7], [8], [9], we will introduce intuitionistic fuzzy almost strongly precontinuous mappings and intuitionistic fuzzy strongly preopen (preclosed) mappings. We will study some of their properties and establish their relationships with other weaker forms of intuitionistic fuzzy continuous mappings.

2. Preliminaries

We introduce some basic notions and results that are used in the sequel.

Definition 2.1. (see [1]) Let X be a nonempty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\},$$

where the mapping $\mu_A(x) : X \to I$ and $\nu_A(x) : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}.$$

Definition 2.2. (see [1]) Let A and B be IFSs of the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in X\}$. Then:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;

(ii) $A = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$;

(iii) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$;

(iv) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$.

We will use the notation $A =< x, \mu_A, \nu_A >$ instead of $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$. A constant fuzzy set α taking value $\alpha \in [0,1]$ will be denote by $\underline{\alpha}$. The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{< x, \underline{0}, \underline{1} >: x \in X\}$ and $1_\sim = \{< x, \underline{1}, \underline{0} >: x \in X\}$.

An intiutionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is an IFS in X defined by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

Let f be a mapping from an ordinary set X into an ordinary set Y. If $B = \{< y, \mu_B(y), \nu_B(y) >: y \in Y\}$ is an IFS in Y, then the inverse image of B under f is an IFS in X defined by

$$f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) >: x \in X\}.$$

The image of an IFS $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ under f is an IFS defined by

$$f(A) = \{< y, f(\mu_A)(y), f(\nu_A)(y) >: y \in Y\},$$

where

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise}, \end{cases}$$

for each $y \in Y$.

Definition 2.3. (see [3]) An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(T_1) $0_\sim$, $1_\sim \in \tau$;

(T_2) $G_1 \cap G_2 \in \tau$ for any G_1, $G_2 \in \tau$;

(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in I\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS in τ is called intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4. (see [4]) An IFS A in an IFTS X is called:

(i) an intuitionistic fuzzy regular open (IFROS) if and only if $A = int(clA)$;

(ii) an intuitionistic fuzzy α-open set (IFαOS) if and only if $A \subseteq int(cl(intA))$;

(iii) an intuitionistic fuzzy semiopen set (IFSOS) if and only if $A \subseteq cl(intA)$;

(iv) an intuitionistic fuzzy preopen set (IFPOS) if and only if $A \subseteq int(clA)$.

An IFS A is called an intuitionistic fuzzy regular closed set, an intuitionistic fuzzy α-closed set, an intuitionistic fuzzy semiclosed set, an intuitionistic fuzzy preclosed set, respectively (IFRCS, IFαCS, IFSCS and IFPCS) if and only if $\overline{A}$ is an IFROS, IFαOS, IFSOS and IFPOS, respectively.

Definition 2.5. (see [3], [7]) Let A be an IFS in IFTS X. Then:

$intA = \cup\{G : G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;

$clA = \cap\{G : G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A;

$pintA = \cup\{G : G$ is an IFPOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy preinterior of A;

$pclA = \cap\{G : G$ is an IFPCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy preclosure of A.

Definition 2.6. (see [9]) An IFS A in an IFTS X is called an intuitionistic fuzzy strongly preopen set (IFSPOS) if and only if $A \subseteq int(pclA)$.

An IFS A is called an intuitionistic fuzzy strongly preclosed set (IFSPCS) if and only if $\overline{A}$ is an IFSPOS.

Definition 2.7. (see [3], [7]) Let A be an IFS in IFTS X. Then:

$spintA = \cup\{G : G$ is an IFSPOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy strongly preinterior of A;

$spclA = \cap\{G : G$ is an IFSPCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy strongly preclosure of A.

Theorem 2.1. (see [3], [7]) *Let A be an IFS of IFTS X. Then:*

(i) $cl\overline{A} = \overline{intA}$; *(ii)* $int\overline{A} = \overline{clA}$;

(iii) $pcl\overline{A} = \overline{pintA}$; *(iv)* $pint\overline{A} = \overline{pclA}$;

(v) $spcl\overline{A} = \overline{spintA}$; *(vi)* $spint\overline{A} = \overline{spclA}$.

Theorem 2.2. (see [6], [7]) *An IFS A in an IFTS X is an IFαOS if and only if it is both an IFSPOS and IFSOS.*

Definition 2.8. (see [6], [7], [9]) Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) intuitionistic fuzzy continuous if and only if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y.

(ii) intuitionistic fuzzy α-continuous if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFOS B in Y;

(iii) intuitionistic fuzzy precontinuous if and only if $f^{-1}(B)$ is an IFPOS in

X, for each IFOS B in Y.

(iv) intuitionistic fuzzy strongly precontinuous if and only if $f^{-1}(B)$ is an IFSPOS in X, for each IFOS B in Y.

(v) intuitionistic fuzzy irresolute regular continuous if and only if $f^{-1}(B)$ is an IFROS in X, for each IFROS B in Y.

(vi) intuitionistic fuzzy irresolute semicontinuous if and only if $f^{-1}(B)$ is an IFSOS in X, for each IFSOS B in Y.

(vii) intuitionistic fuzzy strongly irresolute precontinuous if and only if $f^{-1}(B)$ is an IFSPOS in X, for each IFSPOS B in Y.

Definition 2.9. (see [6], [8], [9]) Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) intuitionistic fuzzy open (closed) if and only if $f(A)$ is an IFOS (IFCS) in Y, for each IFOS (IFCS) A in X.

(ii) intuitionistic fuzzy α-open (α-closed) if and only if $f(A)$ is an IFαOS (IFαCS) in Y, for each IFOS (IFCS) A in X.

(iii) intuitionistic fuzzy preopen (preclosed) if and only if $f(A)$ is an IFPOS (IFPCS) in Y, for each IFOS (IFCS) A in X.

(iv) intuitionistic fuzzy irresolute semiopen (semiclosed) if and only if $f(A)$ is an IFSOS (IFSCS) in Y, for each IFSOS (IFSCS) A in X.

(v) intuitionistic fuzzy strongly irresolute preopen (preclosed) if and only if $f(A)$ is an IFSPOS (IFSPCS) in Y, for each IFSPOS (IFSPCS) A in X.

Definition 2.10. (see [5]) Let (X,τ) and (Y,σ) are IFTSs and $A \in \tau$, $B \in \sigma$. We say that (X,τ) is product related to (Y,σ) if for any IFSs C in X and D in Y, whenever $\overline{A} \supseteq C$ and $\overline{B} \supseteq D$ implies $\overline{A} \times 1_\sim \cup 1_\sim \times \overline{B} \supseteq C \times D$, there exist $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1} \times 1_\sim \cup 1_\sim \times \overline{B_1} = \overline{A} \times 1_\sim \cup 1_\sim \times \overline{B}$.

Theorem 2.3. (see [5]) *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS. If $g : X \to X \times Y$ is a graph of the mapping f then $g^{-1}(1_\sim \times B) = 1_\sim \cap f^{-1}(B)$ for each IFS B in Y.*

3. Intuitionistic Fuzzy almost Strong Precontinuity

Definition 3.1. An IFS A of an IFTS X is called an intuitionistic fuzzy δ-open set (IFδOS) if and only if there exist IFROSs A_i, $i \in I$ such that $A = \cup_{i\in I} A_i$.

An IFS A is called an intuitionistic fuzzy δ-closed set (IFδCS) if and only if $\overline{A}$ is an IFδOS.

Definition 3.2. Let A be an IFS in IFTS X. Then:

$int_\delta A = \cup\{G : G$ is an IFROS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy δ-interior of A;

$cl_\delta A = \cap\{G : G$ is an IFRCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy δ-closure of A.

Let (X, τ) be an IFTS. Since the intersection of two IFROSs is an IFROS, the family of all IFROSs in (X, τ) forms a base for a smaller topology $S(X)$ on X, called semiregularization of X. An IFTS (X, τ) is called fuzzy semiregular if $S(X) = \tau$.

Definition 3.4. A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called intuitionistic fuzzy almost strongly precontinuous if and only if $f^{-1}(B)$ is an IFSPOS in X, for each IFROS B in Y.

Remark 3.1. Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. If f is intuitionistic fuzzy strongly precontinuous, then f is an intuitionistic fuzzy almost strongly precontinuous mapping. The following example shows that the converse statement may not be true.

Example 3.1. Let $X = \{a, b, c\}$ and A, B, C be IFS's in X defined as follows:

$$A =< x, (\frac{a}{0,5}, \frac{b}{0,2}, \frac{c}{0,6}), (\frac{a}{0,5}, \frac{b}{0,7}, \frac{c}{0,4}) >,$$
$$B =< x, (\frac{a}{0,2}, \frac{b}{0,4}, \frac{c}{0,3}), (\frac{a}{0,7}, \frac{b}{0,6}, \frac{c}{0,6}) > .$$

We put $\tau_1 = \{0_\sim, B, A \cup B, 1_\sim\}$ and $\tau_2 = \{0, A, B, A \cap B, A \cup B, 1\}$. Then the mapping $f = id : (X, \tau_1) \to (Y, \tau_2)$ is intuitionistic fuzzy almost strongly precontinuous, but f is not intuitionistic fuzzy strongly precontinuous.

Theorem 3.1. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent:*

(i) f is an intuitionistic fuzzy almost strongly precontinuous mapping;

(ii) $f^{-1}(B)$ is an IFSPCS in X, for each IFRCS in Y;

(iii) $spcl f^{-1}(cl(intB)) \subseteq f^{-1}(B)$, for each IFCS B in Y;

(iv) $f^{-1}(B) \subseteq spint f^{-1}(int(clB))$, for each IFOS B in Y;

(v) $f^{-1}(B) \subseteq spint f^{-1}(int(cl(intB)))$, for each IF$\alpha$OS B in Y;

(vi) $spcl f^{-1}(cl(int(clB))) \subseteq f^{-1}(B)$, for each IF$\alpha$CS B in Y;

(vii) $spcl f^{-1}(cl(int B)) \subseteq f^{-1}(B)$, *for each IFPCS* B *in* Y;

(viii) $f^{-1}(B) \subseteq spint f^{-1}(int(cl B))$, *for each IFPOS* B *in* Y;

(ix) $spcl f^{-1}(cl(int(cl B))) \subseteq f^{-1}(cl B)$, *for each IFS* B *in* Y;

(x) $f^{-1}(int B) \subseteq spint f^{-1}(int(cl(int B)))$, *for each IFS* B *in* Y.

Proof. $(i) \Rightarrow (ii)$ Let B be any IFRCS in Y. Then $\overline{B}$ is an IFROS in Y. According to the assumption $f^{-1}(\overline{B})$ is an IFSPOS in X. From $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ follows that $f^{-1}(B)$ is an IFSPCS in Y.

$(ii) \Rightarrow (iii)$ Let B be any IFCS in Y. From $cl(int B) \subseteq B$ follows that $f^{-1}(cl(int B)) \subseteq f^{-1}(B)$. Since $cl(int B)$ is an IFRCS we obtain that $f^{-1}(cl(int B))$ is an IFSPCS in X. Hence $spcl f^{-1}(cl(int B)) \subseteq f^{-1}(B)$.

$(iii) \Rightarrow (iv)$ It can be proved by using the complement.

$(iv) \Rightarrow (v)$ Let B be any IFαOS B in Y. Then $B \subseteq int(cl(int B))$, so $f^{-1}(B) \subseteq f^{-1}(int(cl(int B)))$. Since $int(cl(int B))$ is an IFOS in Y, according to the assumption we have that

$$
\begin{aligned}
f^{-1}(int(cl(int B))) &\subseteq spint f^{-1}(int(cl(int(cl(int B))))) \\
&= spint f^{-1}(int(cl(int B))).
\end{aligned}
$$

$(v) \Rightarrow (vi)$ It can be proved by using the complement.

$(vi) \Rightarrow (vii)$ Let B be any IFPCS in Y. Then, from $B \supseteq cl(int B)$ follows that $f^{-1}(B) \supseteq f^{-1}(cl(int B))$. Since $cl(int B)$ is an IFαCS, from the assumption we obtain that $f^{-1}(B) \supseteq f^{-1}(cl(int B)) \supseteq spcl f^{-1}(cl(int(cl(cl(int B))))) = spcl f^{-1}(cl(int B))$.

$(vii) \Rightarrow (viii)$ It can be proved by using the complement.

$(viii) \Rightarrow (ix)$ Let B be any IFS in Y. Then $int\overline{B}$ is an IFPOS. According to the assumption we have $f^{-1}(int\overline{B}) \subseteq spint f^{-1}(int(cl(int\overline{B})))$. By using the complement we obtain $spcl f^{-1}(cl(int(cl B))) \subseteq f^{-1}(cl B)$.

$(ix) \Rightarrow (x)$ It can be proved by using the complement.

$(x) \Rightarrow (i)$ Let B be any IFROS in Y. From the assumption it follows that

$$
f^{-1}(B) = f^{-1}(int B) \subseteq spint f^{-1}(int(cl(int B))) = spint f^{-1}(B).
$$

Hence $f^{-1}(B) = spint f^{-1}(B)$, so f is intuitionistic fuzzy almost strongly precontinuous. ☐

Theorem 3.2. *Let* $f : X \to Y$ *be a mapping from an IFTS* X *into an IFTS* Y. *Then, the following statements are equivalent:*

(i) f *is an intuitionistic fuzzy almost strongly precontinuous mapping;*

(ii) $spcl f^{-1}(B) \subseteq f^{-1}(cl B)$, *for each IFSOS* B *in* Y;

(iii) $f^{-1}(intB) \subseteq spintf^{-1}(B)$, *for each IFSCS* B *in* Y.

Proof. $(i) \Rightarrow (ii)$ Let B be any IFSOS in Y. Then: from $B \subseteq cl(intB)$, it follows that $f^{-1}(B) \subseteq f^{-1}(cl(intB))$. Because $cl(intB)$ is an IFRCS we obtain that $f^{-1}(cl(intB))$ is an IFSPOS. Hence $spclf^{-1}(B) \subseteq spclf^{-1}(cl(intB)) \subseteq f^{-1}(clB)$.

$(ii) \Rightarrow (iii)$ It can be proved by using the complement.

$(iii) \Rightarrow (i)$ Let B be any IFRCS in Y. Then B is an IFSCS. According to the assumption we have $\overline{f^{-1}(B)} = \overline{f^{-1}(clB)} = f^{-1}(int\overline{B}) \subseteq spintf^{-1}(\overline{B}) = \overline{spclf^{-1}(B)}$. Hence $f^{-1}(B) = spclf^{-1}(B)$, so f is an intuitionistic fuzzy almost strongly precontinuous mapping. □

Corollary 3.3. *Let* $f : X \to Y$ *be an intuitionistic fuzzy almost strongly precontinuous mapping from an IFTS* X *into an IFTS* Y*. Then, the following statements hold:*

(i) $spclf^{-1}(B) \subseteq f^{-1}(clB)$, *for each IFOS* B *in* Y;

(ii) $f^{-1}(intB) \subseteq spintf^{-1}(B)$, *for each IFCS* B *in* Y.

The following theorem gives some local characterizations of the intuitionistic fuzzy almost strongly precontinuous mappings.

Theorem 3.4. *Let* $f : X \to Y$ *be a mapping from an IFTS* X *into an IFTS* Y*. Then, the following statements are equivalent:*

(i) f *is an intuitionistic fuzzy almost strongly precontinuous mapping;*

(ii) for each intuitionistic fuzzy point x_α *in* X *and each IFOS* B *in* Y *containing* $f(x_\alpha)$*, there exists IFSPOS* A *in* X *containing* x_α *such that* $f(A) \subseteq int(clB)$;

(iii) for each intuitionistic fuzzy point x_α *in* X *and each IFROS set* B *in* Y *containing* $f(x_\alpha)$*, there exists IFSPOS* A *in* X *containing* x_α *such that* $f(A) \subseteq B$.

Proof. $(i) \Rightarrow (ii)$ Let f be any intuitionistic fuzzy almost strongly precontinuous mapping, x_α be any intuitionistic fuzzy point in X and let B be an IFOS in such that $f(x_\alpha) \subseteq B$. Then $x_\alpha \subseteq f^{-1}(B) \subseteq spintf^{-1}(int(clB))$. We put $A = spintf^{-1}(int(clB))$. Then A is an IFSPOS and

$$f(A) \subseteq f(spintf^{-1}(int(clB))) \subseteq ff^{-1}(int(clB)) \subseteq int(clB).$$

$(ii) \Rightarrow (iii)$ Let x_α be any intuitionistic fuzzy point of X and let B be an IFROS in Y containing $f(x_\alpha)$. Then B is an IFOS. According to the assumption there exists an IFSPOS A in X containing x_α such that $f(A) \subseteq int(clB) = B$.

$(iii) \Rightarrow (i)$ Let B be any IFROS in Y and let x_α be an intuitionistic fuzzy point in X such that $x_\alpha \subseteq f^{-1}(B)$. According to the assumption there exists an IFSPOS A in X such that $x_\alpha \in A$ and $f(A) \subseteq B$. Hence $x_\alpha \in A \subseteq f^{-1}f(A) \subseteq f^{-1}(B)$ and $x_\alpha \in A = spintA \subseteq spintf^{-1}(B)$. Since x_α is arbitrary and $f^{-1}(B)$ is the union of all intuitionistic fuzzy points of $f^{-1}(B)$, $f^{-1}(B) \subseteq spintf^{-1}(B)$. Thus f is an intuitionistic fuzzy almost srong precontinuous mapping. □

Theorem 3.5. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then, the following statements are equivalent:*

(i) f is an intuitionistic fuzzy almost precontinuous mapping;

(ii) $f^{-1}(B)$ is an IFSPOS in X, for each IFδOS B in Y;

(iii) $f^{-1}(B)$ is an IFSPCS in X, for each IFδCS B in Y;

(iv) $f(spclA) \subseteq cl_\delta f(A)$, for each IFS A in X;

(v) $spclf^{-1}(B) \subseteq f^{-1}(cl_\delta B)$, for each IFS B in Y;

(vi) $f^{-1}(int_\delta B) \subseteq spintf^{-1}(B)$, for each IFS B in Y.

Proof. $(i) \Rightarrow (ii)$ Let B be any IFδOS in Y. Then $B = \cup_{\alpha\in I} B_\alpha$, where B_α is an IFROSs in Y, for each $\alpha \in I$. From $f^{-1}(B) = f^{-1}(\cup_{\alpha\in I} B_\alpha) = \cup_{\alpha\in I} f^{-1}(B_\alpha)$ follows that $f^{-1}(B)$ is an IFSPOS as a union of IFSPOSs.

$(ii) \Rightarrow (iii)$ Can be proved by using the complement.

$(iii) \Rightarrow (iv)$ Let A be any IFS in X. Then $cl_\delta f(A)$ is an IFδCS in Y. According to the assumption we have that $f^{-1}(cl_\delta f(A))$ is an IFSPCS in X. Hence $spclA \subseteq spclf^{-1}(f(A)) \subseteq spclf^{-1}(cl_\delta f(A)) = f^{-1}(cl_\delta f(A))$, so $f(spclA) \subseteq cl_\delta f(A)$.

$(iv) \Rightarrow (v)$ Let B be any IFS in Y. From the assumption it follows that $f(spclf^{-1}(B)) \subseteq cl_\delta f(f^{-1}(B)) \subseteq cl_\delta B$. Thus

$$spclf^{-1}(B) \subseteq f^{-1}f(spclf^{-1}(B)) \subseteq f^{-1}(cl_\delta B).$$

$(v) \Rightarrow (vi)$ Can be proved by using the complement.

$(vi) \Rightarrow (i)$ Let B be any IFROS in Y. Then $B = int_\delta B$. According to the assumption we have $f^{-1}(B) = f^{-1}(int_\delta B) \subseteq spintf^{-1}(B) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = spintf^{-1}(B)$, so $f^{-1}(B)$ is an IFSPOS. Thus f is an intuitionistic fuzzy almost precontinuous mapping. □

Corollary 3.6. *A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy almost strongly precontinuous if and only if the mapping $f : X \to (Y, S(X))$ is intuitionistic fuzzy strongly precontinuous.*

Corollary 3.7. *Let $f : X \to Y$ be a mapping from an IFTS X into a semiregular IFTS Y. The mapping f is intuitionistic fuzzy almost strongly precontinuous if and only if f is intuitionistic fuzzy strongly precontinuous.*

Theorem 3.8. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. The mapping f is intuitionistic fuzzy almost strongly precontnuous if and only if $int_\delta f(A) \subseteq f(spintA)$, for each IFS A in X.*

Proof. We suppose that f is intuitionistic fuzzy almost strongly precontinuous. Then $f^{-1}(int_\delta f(A))$ is an IFSPOS in X, for any IFS A in X. Since f is injective, from Theorem 3.5 it follows $f^{-1}(int_\delta f(A)) = spintf^{-1}(int_\delta f(A)) \subseteq spintf^{-1}(f(A)) = spintA$. Again, since f is surjective, we obtain $int_\delta f(A) = ff^{-1}(int_\delta f(A)) \subseteq f(spintA)$.

Conversely, let B be any IFδOS in Y. Then $int_\delta B = B$. According to the assumption we have $f(spintf^{-1}(B)) \supseteq int_\delta ff^{-1}(B) = int_\delta B = B$. Therefore $f^{-1}f(spintf^{-1}(B)) \supseteq f^{-1}(B)$. Since f is injective we obtain $spintf^{-1}(B) = f^{-1}f(spintf^{-1}(B)) \supseteq f^{-1}(B)$. Hence $spintf^{-1}(B) = f^{-1}(B)$, so $f^{-1}(B)$ is an IFSPOS. Thus f is an intuitionistic fuzzy almost precontinuous mapping. □

Definition 3.5. A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called:

(i) intuitionistic fuzzy almost α-continuity if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFROS B in Y.

(ii) fuzzy almost semicontinuity if and only if $f^{-1}(B)$ is an IFSOS in X, for each IFROS B in Y.

Theorem 3.9. *Let X, X_1 and X_2 are IFTSs and $p_i : X_1 \times X_2 \to X_i$, $i = 1, 2$ are the projections of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is an intuitionistic fuzzy almost strongly precontinuous, then $p_i f$ are intuitionistic fuzzy almost precontinuous mappings, as well.*

Proof. Since the projections are intuitionistic fuzzy continuous and intuitionistic fuzzy open mappings we have $clp_i^{-1}(B) \subseteq p_i^{-1}(clB)$ and $intp_i^{-1}(B) \subseteq p_i^{-1}(intB)$, for each IFS B in X_i. Hence

$$(p_i f)^{-1}(B) = f^{-1}({p_i}^{-1}(B)) \subseteq spint(int(cl(p_i^{-1}(B))) \subseteq$$
$$\subseteq spintf^{-1}(p_i^{-1}(int(cl(B))) \subseteq spint(p_i f)^{-1}((int(cl(B))),$$

for each IFOS B in X_i. □

Theorem 3.10. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. If the graph $g : X \to X \times Y$ of f is intuitionistic fuzzy almost strongly precontinuous, then f is intuitionistic fuzzy almost strongly precontinuous, as well.*

Proof. By Theorem 2.3 $f^{-1}(B) = 1_\sim \cap f^{-1}(B) = g^{-1}(1_\sim \times B)$, for each IFROS B in Y. Since g is fuzzy almost precontinuous and $1_\sim \times B$ is an IFROS in $X \times Y$, $f^{-1}(B)$ is an IFSPOS in X, so f is intuitionistic fuzzy almost strongly precontinuous. □

Theorem 3.11. *Let $f : X \to Y$ be an intuitionistic fuzzy strongly irresolute preopen mapping and an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X onto an IFTS Y and let $g : Y \to Z$ be a mapping from an IFTS Y into an IFTS Z. The mapping gf is intuitionistic fuzzy almost strongly precontinuous if and only if g is an intuitionistic fuzzy almost strongly precontinuous.*

Proof. Let gf be an intuitionistic fuzzy almost strongly precontinuous. Then $g^{-1}(C) = f(gf)^{-1}(C)$ is an IFSPOS in Y, for each IFROS C in Z. Hence g is an intuitionistic fuzzy almost strongly precontinuous mapping.

Conversely, let g be an fuzzy almost strongly precontinuous mapping and let C be an IFROS in Z. From $(gf)^{-1}(C) = f^{-1}(g^{-1}(C)$ follows that gf is an intuitionistic fuzzy almost strongly precontinuous mapping. □

Theorem 3.12. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. The mapping f is intuitionistic fuzzy almost α-continuous if and only if it is both intuitionistic fuzzy almost semicontinuous and intuitionistic fuzzy almost strongly precontinuous.*

Proof. It follows from Theorem 3.2. □

Theorem 3.13. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. If the mapping f is both intuitionistic fuzzy almost strongly precontinuous and intuitionistic fuzzy irresolute semicontinuous then f is intuitionistic fuzzy irresolute regular continuous.*

Proof. Let B be any IFROS in Y. Since f is intuitionistic fuzzy almost strongly precontinuous $f^{-1}(B)$ is an IFSPOS in X. Since each IFROS is an IFSCS, from f is intuitionistic fuzzy irresolute semicontinuous it follows that $f^{-1}(B)$ is an IFSCS. Therefore $f^{-1}(B)$ is an IFROS, so f is intuitionistic fuzzy irresolute regular continuous. □

Corollary 3.14. *Let $f : X \to Y$ be a mapping from an IFTS X into a semiregular IFTS Y. If the mapping f is both intuitionistic fuzzy almost strongly precontinuous and intuitionistic fuzzy irresolute semicontinuous then f is an intuitionistic fuzzy continuous mapping.*

4. Intuitionistic Fuzzy Almost Strongly Preopen and Fuzzy Almost Strongly Preclosed Mappings

Definition 4.1. A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called:

(i) intuitionistic fuzzy almost strongly preopen if and only if $f(A)$ is an IFSPOS in Y, for each IFROS A in X.

(ii) intuitionistic fuzzy almost strongly preclosed if and only if $f(A)$ is an IFSPCS in Y, for each IFRCS A in X.

Remark 4.1. Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. If f is an intuitionistic fuzzy strongly preopen (preclosed) mapping, then f is an intuitionistic fuzzy almost strongly preopen (preclosed) mapping. The following example shows that the converse statement may not be true.

Example 4.1. We consider Example 3.1. The mapping $f = id : (X, \tau_2) \to (Y, \tau_1)$ is intuitionistic fuzzy almost strongly preopen (preclosed) but f is not intuitionistic fuzzy strongly preopen (preclosed).

Theorem 4.1. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then f is intuitionistic fuzzy almost strongly preopen (preclosed) if and only if it is intuitionistic fuzzy almost strongly preclosed (preopen).*

Proof. It can be proved by using the complement. □

Theorem 4.2. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy almost strongly preopen (preclosed) if and only if f^{-1} is intuitionistic fuzzy almost strongly precontinuous.*

Proof. It follows from the relation $(f^{-1})^{-1}(A) = f(A)$, for each IFROS (IFRCS) A in X. □

Theorem 4.3. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy almost strongly preopen (preclosed) mapping if and only if $f : (X, S(X)) \to (Y, S(Y))$ is an intuitionistic fuzzy strongly preopen (preclosed) mapping.*

Proof. Let A be any IFOS in $S(X)$. Then $A = \cup_{\alpha \in I} A_\alpha$, where A_α is an IFROSs in X, for each $\alpha \in I$. From $f(A) = f(\cup_{\alpha \in I} A_\alpha) = \cup_{\alpha \in I} f(A_\alpha)$ it follows that $f(A)$ is an IFSPOS as a union of IFSPOSs.

The converse follows from the condition that each IFROS in X is an IFOS in $S(X)$. □

Corollary 4.4. *Let $f : X \to Y$ be a mapping from a semiregular IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy almost strongly preopen* *(preclosed) mapping if and only if it is an intuitionistic fuzzy strongly preopen (preclosed) mapping.*

Theorem 4.5. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy almost strongly preopen mapping if and only if $f(intA) \subseteq spintf(A)$, for each IFSCS A in X.*

Proof. Let f be an intuitionistic fuzzy almost strongly preopen mapping and let A be any fuzzy semiclosed set of X. Then $intA = int(clA)$. According to the assumption we have

$$f(intA) = f(int(clA)) = spintf(int(clA)) = spintf(intA) \subseteq spintf(A).$$

Conversely, let A be any IFROS in X. Then A is an IFSCS in X. According to the assumption we have $f(A) = f(intA) \subseteq spintf(A)$. Thus $f(A)$ is an IFSPOS in X. □

Theorem 4.6. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy almost strongly preclosed mapping if and only if $spcl(f(A)) \subseteq f(clA)$, for each fuzzy semiopen set A in X.*

Proof. It can be proved in a similar manner as the previous theorem. □

Theorem 4.7. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent:*

(i) f is an intuitionistic fuzzy almost strongly preopen (preclosed) mapping.

(ii $f(intA) \subseteq spintf(A)$, for each IFSCS A in X.

(iii) $spclf(A) \subseteq f(clA)$for each IFSOS A in X.

Proof. It follows from Theorem 3.1, Theorem 3.5 and the Theorem 3.6. □

Theorem 4.8. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements holds:*

(1) f is an intuitionistic fuzzy almost strongly preopen mapping if and only if $f(intA) \subseteq int(pclf(A))$, for each IFSCS A in X.

(2) f is an intuitionistic fuzzy almost strongly preclosed mapping if and only if $cl(pintf(A)) \subseteq f(clA)$, for each IFSOS A in X.

Proof. We will prove the statements (1) only. Let f be an intuitionistic fuzzy almost strongly preopen mapping. Then, for any IFSCS A of X we have $intA = int(clA)$, so $f(intA)$ is an IFSPOS in Y. Thus

$$f(intA) = int(pclf(intA)) \subseteq int(pclf(A)).$$

Conversely, let A be an IFROS in X. From $f(A) = f(intA) \subseteq int(pclf(A))$, we conclude that $f(A)$ is an IFSPOS, so f is an intuitionistic fuzzy almost strongly preopen mapping. □

Theorem 4.9. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is intuitionistic fuzzy almost strongly preopen if and only if for each IFS B in Y and each IFRCS A in X such that $f^{-1}(B) \subseteq A$, there exists an IFSPCS C in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. Let B be any IFS in Y and let A be a IFRCS in X such that $f^{-1}(B) \subseteq A$. Then $\overline{A} \subseteq f^{-1}(\overline{B})$, so $f(\overline{A}) \subseteq ff^{-1}(\overline{B}) \subseteq \overline{B}$. Since $\overline{A}$ is an IFROS, $f(\overline{A})$ is an IFSPOS, so $f(\overline{A}) \subseteq spint\overline{B}$. Hence $\overline{A} \subseteq f^{-1}f(\overline{A}) \subseteq f^{-1}(spint\overline{B})$. Therefore $A \supseteq \overline{f^{-1}(spint\overline{B})} = f^{-1}(spclB)$. The result follows for $C = spaclB$.

Conversely, let U be an IFROS in X. We will show that $f(U)$ is an IFSPOS in Y. From $U \subseteq f^{-1}f(U)$ follows that $\overline{U} \supseteq \overline{f^{-1}f(U)} \supseteq f^{-1}\overline{f(U)}$ where $\overline{U}$ is an IFRCS in X. Hence there is an IFSPCS B in Y such that $B \supseteq \overline{f(U)}$ and $f^{-1}(B) \subseteq \overline{U}$. From $B \supseteq \overline{f(U)}$ follows that $B \supseteq spcl\overline{f(U)}$, so $\overline{B} \subseteq \overline{spcl\overline{f(U)}} \subseteq spint f(U)$. From $f^{-1}(B) \subseteq \overline{U}$ we have $B \supseteq f^{-1}(\overline{B}) \supseteq U$, so $\overline{B} \supseteq ff^{-1}(\overline{B}) \supseteq f(U)$. Hence $f(U) = spint f(U)$. Thus $f(U)$ is an IFSPOS, so f is an intuitionistic fuzzy almost strongly preopen mapping. □

Theorem 4.10. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is intuitionistic fuzzy almost strongly preclosed if and only if for each IFS B in Y and each IFROS A in X such that $f^{-1}(B) \subseteq A$ there exists an IFSPOS C in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. It can be proved in a similar manner as Theorem 4.9. □

Theorem 4.11. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. The mapping f is intuitionistic fuzzy almost α-open (α-closed) if and only if it is both intuitionistic fuzzy almost semiopen (semiclosed) and intuitionistic fuzzy almost strongly preopen (preclosed).*

Proof. The proof is similar to the proof of Theorem 3.12. □

Theorem 4.12. *Let $f : X \to Y$ be a mapping from an an IFTS X into an IFTS Y. If the mapping f is both intuitionistic fuzzy almost strongly preopen (preclosed) and intuitionistic fuzzy irresolute semiclosed (semiopen), then f is fuzzy intuitionistic fuzzy irresolute regular (closed) open.*

Proof. The proof is similar to the proof of Theorem 3.13. □

Corollary 4.13. *Let $f : X \to Y$ be a mapping from a semiregular IFTS X into an IFTS Y. If the mapping f is both intuitionistic fuzzy almost strongly preopen (preclosed) and intuitionistic fuzzy irresolute semiclosed (semiopen), then f is an intuitionistic fuzzy (closed) open mapping.*

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