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COINCIDENCE POINT THEOREMS FOR FOUR CLASSES OF CONTRACTIVE MAPPINGS IN PSEUDOCOMPACT TICHONOV SPACES

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Abstract: In this paper four classes of contractive mappings are introduced in pseudocompact Tichonov spaces and coincidence point theorems for these contractive mappings are proved.

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1. Introduction and Preliminaries

Coincidence point theorems have been studied by many scholars under vari-

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ous conditions [1-8]. Harinath [2] established the first existence result of fixed points for a class of contractive mappings in a pseudocompact Tichonov space X. Afterwards, Fisher [1], Jain and Dixit [3], Liu [4-6], Naidu and Rao [7], Rao [8] and others extended Harinath's results in various directions. Liu [4] proved coincidence point theorems for self mappings f, g, h in a pseudocompact Tichonov space X which satisfy the following condition:

$$F(fx, gy) < \max\Big\{F(hy, hx), F(fx, hx), F(hy, gy), \frac{F(fx, hx)F(hy, gy)}{F(hy, hx)}, \frac{F(fx, hy)F(hx, gy)}{F(hy, hx)}\Big\}$$

for all $x, y \in X$ with $hx \neq hy$. Further, Liu [6] established coincidence point theorems for the following self mappings f, g, s, t in a pseudocompact Tichonov space X

$$F(fx, gy) < \max\Big\{F(ty, sx), F(fx, sx), F(ty, gy), \frac{F(fx, sx)F(ty, gy)}{F(ty, sx)}, \frac{F(fx, ty)F(sx, gy)}{F(ty, sx)}, \frac{F(fx, ty)F(sx, gy)}{F(fx, gy)}, \frac{F^2(ty, sx)}{F(fx, gy)}, \frac{F^2(fx, sx)}{F(fx, gy)}, \frac{F^2(ty, gy)}{F(fx, gy)}\Big\},$$

for all $x, y \in X$ with $fx \neq gy$, $ty \neq sx$, $fx \neq sx$, $ty \neq gy$.

Inspired by the above mentioned results, we introduce four classes of contractive mappings in a pseudocompact Tichonov space X as follows:

$$\begin{aligned} &F(hy, hx)F(fx, gy) \\ &\quad < \max\Big\{F^2(hy, hx), F(fx, hx)F(hy, gy), F(hx, gy)F(fx, hy), \\ &\qquad F(hy, fx)F(hx, gy), F(gy, hx)F(fx, hy), \\ &\qquad \frac{F(hy, fx)F(fx, fy)F(gy, hx)}{F(hy, hx)}, \frac{F(hy, fx)F(gy, hx)F(gx, gy)}{F(hy, hx)}, \\ &\qquad \frac{F(hy, hx)A^2(x, y)}{F(fx, gy)+1}, \frac{F^2(fx, hy)F(gy, hx)}{F(x, y)+1}, \\ &\qquad \frac{F^2(gy, hx)F(hy, fx)}{F(fx, gx)+1}, \frac{F^2(hx, gy)F(fx, hy)}{F(fy, gy)+1}\Big\} \end{aligned} \tag{1.1}$$

for all $x, y \in X$ with $hx \neq hy$;

$$\begin{aligned} &\max\{F^2(hy, hx), F^2(fx, gy)\} \\ &\quad < \max\Big\{A^2(x, y), F(hy, hx)A(x, y), F(hx, gy)F(fx, hy), \\ &\qquad F(hy, fx)F(hx, gy), F(gy, hx)F(fx, hy), \end{aligned}$$

$$\frac{F(hy,fx)F(fx,fy)F(gy,hx)}{F(hy,hx)}, \frac{F(fx,hy)F(gy,hx)F(gx,gy)}{F(hy,hx)},$$
$$\frac{F(hy,hx)A^2(x,y)}{F(fx,gy)+1}, \frac{F(x,y)F(fx,hy)F(gy,hx)}{F(x,fy)+1},$$
$$\left.\frac{F(x,y)F(hy,fx)F(gy,hx)}{F(y,gx)+1}, \frac{F^2(hx,gy)F^2(fx,hy)}{[F(x,y)+1]^2}\right\} \quad (1.2)$$

for all $x, y \in X$ with $hx \neq hy$;

$$\begin{aligned}
&\min\{F(hy,hx)+F(fx,gy), F(fx,hx)+F(hy,gy)\} \\
&< \max\Big\{\min\{2F(fx,hx), 2F(hy,gy), \\
&\quad F(hy,hx)+F(fx,hx), F(fx,gy)+F(hy,gy)\}, \\
&\quad \min\{F(fx,hy), F(hx,gy)\} + \frac{F(hy,hx)F(fx,gy)}{B(x,y)+1}, \\
&\quad \min\Big\{\frac{F^2(hy,fx)}{F(hy,hx)}, \frac{F^2(gy,hx)}{F(hy,hx)}\Big\} + \frac{F(hy,hx)F(fx,gy)}{A(x,y)+1}, \\
&\quad \frac{F(fx,hy)F(gy,hx)}{F(x,y)+1} + F(fx,gy), \\
&\quad \frac{F(hy,fx)F(hx,gy)}{F(fx,gy)+1} + F(hy,hx), \\
&\quad \frac{F(hy,hx)A(x,y)}{F(fx,hx)+1} + \frac{F(fx,gy)B(x,y)}{F(hy,gy)+1}, \frac{2F(hy,hx)F(fx,gy)}{A(x,y)+1}\Big\}
\end{aligned} \quad (1.3)$$

for all $x, y \in X$ with $hx \neq hy$;

$$\begin{aligned}
&F(fx,gy)F(ty,sx) \\
&< \max\Big\{F^2(ty,sx), F(fx,sx)F(ty,gy), F(fx,ty)F(sx,gy), \\
&\quad \frac{F^2(ty,gy)F(ty,sx)}{F(fx,gy)}, \frac{F^2(ty,gy)F(fx,sx)}{F(fx,gy)}, \\
&\quad \frac{F^2(ty,fx)F(gy,sx)}{F(gy,fx)}, \frac{F^2(ty,sx)F(ty,gy)}{F(fx,sx)}, \\
&\quad \frac{F^2(gy,sx)F(fx,ty)}{F(ty,sx)}, \frac{F^2(sx,gy)F(fx,ty)}{F(sx,ty)}\Big\}
\end{aligned} \quad (1.4)$$

for all $x, y \in X$ with $fx \neq gy$, $ty \neq sx$, $fx \neq sx$, $ty \neq gy$, where

$$A(x,y) = \max\{F(fx,hx), F(hy,gy)\},$$
$$B(x,y) = \min\{F(fx,hx), F(hy,gy)\}.$$

Under certain conditions we prove the existence of coincidence points for the contractive mappings (1.1), (1.2), (1.3) and (1.4), respectively, in pseudocompact Tichonov spaces.

Recall that a topological space X is said to be pseudocompact if and only if every real valued continuous function in X is bounded. By Tichonov space we mean a completely regular Hausdorff space. It may be noted that each compact space is pseudocompact. If X is an arbitrary Tichonov space, then X is pseudocompact if and only if every real valued continuous function over X is bounded and assumes its bounds [9].

Throughout this paper, we assume that $\mathbb{R}^+ = [0, +\infty)$, X is a pseudocompact Tichonov space and $F : X \times X \to \mathbb{R}^+$ satisfies that $F(x, y) = 0$ if and only if $x = y$.

2. Coincidence Point Theorems for Contractive Mappings

Now we give sufficient conditions of the existence of coincidence points for the four classes of contractive mappings (1.1), (1.2), (1.3) and (1.4), respectively, in pseudocompact Tichonov spaces.

Theorem 2.1. *Let f, g, h be three mappings from a pseudocompact Tichonov space X into itself satisfying $f(X) \cup g(X) \subseteq h(X)$ and (1.1). If $a, b : X \to \mathbb{R}^+$ defined by $a(x) = F(fx, hx)$ and $b(x) = F(hx, gx)$ are continuous in X, then f and h or g and h have a coincidence point in X.*

Proof. Since X is a pseudocompact Tichonov space and a, b are continuous, it follows that there exist two points $u, v \in X$ such that

$$a(u) = \inf\{a(x) : x \in X\} \quad \text{and} \quad b(v) = \inf\{b(x) : x \in X\}. \tag{2.1}$$

We now consider two cases.

Case 1. Assume that $a(u) \leq b(v)$. Since $f(X) \subseteq h(X)$, there exists a point $w \in X$ such that $fu = hw$. We claim that u is a coincidence point of f and h. If not, $fu \neq hu$, which leads to $hu \neq hw$. Using (1.1), we obtain that

$$\begin{aligned} &F(hw, hu)F(fu, gw) \\ &\quad < \max\Big\{F^2(hw, hu), F(fu, hu)F(hw, gw), F(hu, gw)F(fu, hw), \\ &\qquad F(hw, fu)F(hu, gw), F(gw, hu)F(fu, hw), \\ &\qquad \frac{F(hw, fu)F(fu, fw)F(gw, hu)}{F(hw, hu)}, \frac{F(hw, fu)F(gw, hu)F(gu, gw)}{F(hw, hu)}, \\ &\qquad \frac{F(hw, hu)A^2(u, w)}{F(fu, gw) + 1}, \frac{F^2(fu, hw)F(gw, hu)}{F(u, w) + 1}, \end{aligned}$$

$$\frac{F^2(gw,hu)F(hw,fu)}{F(fu,gu)+1}, \frac{F^2(hu,gw)F(fu,hw)}{F(fw,gw)+1}\Big\},$$

which together with (2.1) gives that

$$a^2(u) \le a(u)b(w)$$
$$< \max\Big\{a^2(u), a(u)b(w), 0,0,0,0,0, \frac{a(u)b^2(w)}{b(w)+1}, 0,0,0\Big\} = a^2(u),$$

which is a contradiction and hence $fu = hu$.

Case 2. Assume that $b(v) < a(u)$. In light of $g(X) \subseteq h(X)$, there exists a point $c \in X$ such that $gv = hc$. We are convinced that v is a coincidence point of g and h. Otherwise, $gv \ne hv$, thus $hv \ne hc$. By virtue of (1.1), we get that

$$F(hv,hc)F(fc,gv)$$
$$< \max\Big\{F^2(hv,hc), F(fc,hc)F(hv,gv), F(hc,gv)F(fc,hv),$$
$$F(hv,fc)F(hc,gv), F(gv,hc)F(fc,hv),$$
$$\frac{F(hv,fc)F(fc,fv)F(gv,hc)}{F(hv,hc)}, \frac{F(hv,fc)F(gv,hc)F(gc,gv)}{F(hv,hc)},$$
$$\frac{F(hv,hc)A^2(c,v)}{F(fc,gv)+1}, \frac{F^2(fc,hv)F(gv,hc)}{F(c,v)+1},$$
$$\frac{F^2(gv,hc)F(hv,fc)}{F(fc,gc)+1}, \frac{F^2(hc,gv)F(fc,hv)}{F(fv,gv)+1}\Big\},$$

which implies that

$$b(v)a(c) < \max\Big\{b^2(v), a(c)b(v), 0,0,0,0,0, \frac{b(v)a^2(c)}{a(c)+1}, 0,0,0\Big\} = b^2(v). \quad (2.2)$$

In view of (2.1) and (2.2), we find that

$$b^2(v) \le b(v)a(c) < b^2(v),$$

which is impossible. Therefore $gv = hv$. This completes the proof. □

Theorem 2.2. *Let f,g,h be three mappings from a pseudocompact Tichonov space X into itself satisfying $f(X) \cup g(X) \subseteq h(X)$ and (1.2). If $a,b : X \to \mathbb{R}^+$ defined by $a(x) = F(fx,hx)$ and $b(x) = F(hx,gx)$ are continuous in X, then f and h or g and h have a coincidence point in X.*

Proof. Due to the fact that X is a pseudocompact Tichonov space and a,b are continuous, there exist two points $u,v \in X$ satisfying (2.1). We now take two cases into account:

Case 1. Suppose that $a(u) \le b(v)$. Note that $f(X) \subseteq h(X)$. Thus there exists a point $w \in X$ such that $fu = hw$. We now affirm that u is a coincidence point of f and h. If not, $fu \neq hu$, which infers that $hu \neq hw$. In terms of (1.2), we have that

$$\begin{aligned}
&\max\{F^2(hw,hu), F^2(fu,gw)\} \\
&< \max\Big\{A^2(u,w), F(hw,hu)A(u,w), F(hu,gw)F(fu,hw), \\
&\quad F(hw,fu)F(hu,gw), F(gw,hu)F(fu,hw), \\
&\quad \frac{F(hw,fu)F(fu,fw)F(gw,hu)}{F(hw,hu)}, \frac{F(fu,hw)F(gw,hu)F(gu,gw)}{F(hw,hu)}, \\
&\quad \frac{F(hw,hu)A^2(u,w)}{F(fu,gw)+1}, \frac{F(u,w)F(fu,hw)F(gw,hu)}{F(u,fw)+1}, \\
&\quad \frac{F(u,w)F(hw,fu)F(gw,hu)}{F(w,gu)+1}, \frac{F^2(hu,gw)F^2(fu,hw)}{[F(u,w)+1]^2}\Big\},
\end{aligned}$$

which together with (2.1) implies that

$$\begin{aligned}
b^2(w) &= \max\{a^2(u), b^2(w)\} \\
&< \max\Big\{b^2(w), a(u)b(w), 0, 0, 0, 0, 0, \frac{a(u)b^2(w)}{b(w)+1}, 0, 0, 0\Big\} \\
&= b^2(w),
\end{aligned}$$

which is contradictory. Hence $fu = hu$.

Case 2. Suppose that $b(v) < a(u)$. By reason of $g(X) \subseteq h(X)$, there exists a point $c \in X$ such that $gv = hc$. We claim that v is a coincidence point of g and h. Otherwise, $gv \neq hv$, clearly, $hv \neq hc$. Applying (1.2), we get that

$$\begin{aligned}
&\max\{F^2(hv,hc), F^2(fc,gv)\} \\
&< \max\Big\{A^2(c,v), F(hv,hc)A(c,v), F(hc,gv)F(fc,hv), \\
&\quad F(hv,fc)F(hc,gv), F(gv,hc)F(fc,hv), \\
&\quad \frac{F(hv,fc)F(fc,fv)F(gv,hc)}{F(hv,hc)}, \frac{F(fc,hv)F(gv,hc)F(gc,gv)}{F(hv,hc)}, \\
&\quad \frac{F(hv,hc)A^2(c,v)}{F(fc,gv)+1}, \frac{F(c,v)F(fc,hv)F(gv,hc)}{F(c,fv)+1}, \\
&\quad \frac{F(c,v)F(hv,fc)F(gv,hc)}{F(v,gc)+1}, \frac{F^2(hc,gv)F^2(fc,hv)}{[F(c,v)+1]^2}\Big\},
\end{aligned}$$

which together with (2.1) yields that

$$\begin{aligned}a^2(c) &= \max\{b^2(v), a^2(c)\}\\ &< \max\left\{a^2(c), b(v)a(c), 0, 0, 0, 0, 0, \frac{b(v)a^2(c)}{a(c)+1}, 0, 0, 0\right\}\\ &= a^2(c),\end{aligned}$$

which is a contradiction and hence $gv = hv$. This completes the proof. □

Theorem 2.3. *Let f, g, h be three mappings from a pseudocompact Tichonov space X into itself satisfying $f(X) \cup g(X) \subseteq h(X)$ and (1.3). If $a, b : X \to \mathbb{R}^+$ defined by $a(x) = F(fx, hx)$ and $b(x) = F(hx, gx)$ are continuous in X, then f and h or g and h have a coincidence point in X.*

Proof. Using the fact that X is a pseudocompact Tichonov space and a, b are continuous, then there exist two points $u, v \in X$ satisfying (2.1). We need to consider two cases.

Case 1. Suppose that $a(u) \le b(v)$. In view of $f(X) \subseteq h(X)$, there exists a point $w \in X$ such that $fu = hw$. We now affirm that u is a coincidence point of f and h. If not, $fu \ne hu$, thus $hu \ne hw$. Using (1.3), we gain that

$$\begin{aligned}&\min\{F(hw, hu) + F(fu, gw), F(fu, hu) + F(hw, gw)\}\\ &< \max\Big\{\min\{2F(fu, hu), 2F(hw, gw),\\ &\quad F(hw, hu) + F(fu, hu), F(fu, gw) + F(hw, gw)\},\\ &\quad \min\{F(fu, hw), F(hu, gw)\} + \frac{F(hw, hu)F(fu, gw)}{B(u, w)+1},\\ &\quad \min\left\{\frac{F^2(hw, fu)}{F(hw, hu)}, \frac{F^2(gw, hu)}{F(hw, hu)}\right\} + \frac{F(hw, hu)F(fu, gw)}{A(u, w)+1},\\ &\quad \frac{F(fu, hw)F(gw, hu)}{F(u, w)+1} + F(fu, gw), \frac{F(hw, fu)F(hu, gw)}{F(fu, gw)+1} + F(hw, hu),\\ &\quad \frac{F(hw, hu)A(u, w)}{F(fu, hu)+1} + \frac{F(fu, gw)B(u, w)}{F(hw, gw)+1}, \frac{2F(hw, hu)F(fu, gw)}{A(u, w)+1}\Big\},\end{aligned}$$

which gives that

$$\begin{aligned}&\min\{a(u) + b(w), a(u) + b(w)\}\\ &< \max\Big\{2a(u), \frac{a(u)b(w)}{a(u)+1}, \frac{a(u)b(w)}{b(w)+1}, b(w), a(u),\\ &\quad \frac{a(u)b(w)}{a(u)+1} + \frac{b(w)a(u)}{b(w)+1}, \frac{2a(u)b(w)}{b(w)+1}\Big\},\end{aligned}$$

which implies that

$$a(u) + b(w) < \max\{2a(u), b(w)\},$$

which together with (2.1) guarantees that either

$$b(w) < a(u) \le b(v) \le b(w)$$

or

$$a(u) < 0 \le a(u),$$

each of which is impossible. Consequently, $fu = hu$.

Case 2. Suppose that $b(v) < a(u)$. In light of $g(X) \subseteq h(X)$, there exists a point $c \in X$ such that $gv = hc$. We claim that v is a coincidence point of g and h. Otherwise, $gv \ne hv$, thus $hv \ne hc$. Applying (1.3), we get that

$$\begin{aligned}
&\min\{F(hv,hc) + F(fc,gv), F(fc,hc) + F(hv,gv)\}\\
&< \max\Big\{\min\{2F(fc,hc), 2F(hv,gv),\\
&\quad F(hv,hc) + F(fc,hc), F(fc,gv) + F(hv,gv)\},\\
&\quad \min\{F(fc,hv), F(hc,gv)\} + \frac{F(hv,hc)F(fc,gv)}{B(c,v)+1},\\
&\quad \min\Big\{\frac{F^2(hv,fc)}{F(hv,hc)}, \frac{F^2(gv,hc)}{F(hv,hc)}\Big\} + \frac{F(hv,hc)F(fc,gv)}{A(c,v)+1},\\
&\quad \frac{F(fc,hv)F(gv,hc)}{F(c,v)+1} + F(fc,gv), \frac{F(hv,fc)F(hc,gv)}{F(fc,gv)+1} + F(hv,hc),\\
&\quad \frac{F(hv,hc)A(c,v)}{F(fc,hc)+1} + \frac{F(fc,gv)B(c,v)}{F(hv,gv)+1}, \frac{2F(hv,hc)F(fc,gv)}{A(c,v)+1}\Big\},
\end{aligned}$$

that is,

$$\begin{aligned}
&\min\{b(v) + a(c), a(c) + b(v)\}\\
&< \max\Big\{2b(v), \frac{b(v)a(c)}{b(v)+1}, \frac{b(v)a(c)}{a(c)+1}, a(c), b(v),\\
&\quad \frac{b(v)a(c)}{a(c)+1} + \frac{a(c)b(v)}{b(v)+1}, \frac{2b(v)a(c)}{a(c)+1}\Big\},
\end{aligned}$$

which means that

$$a(c) + b(v) < \max\{2b(v), a(c)\},$$

which together with (2.1) infers that either

$$a(c) < b(v) < a(u) \le a(c),$$

or

$$b(v) < 0 \le b(v),$$

each of which is absurd and hence $gv = hv$. This completes the proof. □

Theorem 2.4. *Let f, g, s, t be four mappings from a pseudocompact Tichonov space X into itself satisfying $f(X) \subseteq t(X)$, $g(X) \subseteq s(X)$ and (1.4). Assume that $a, b : X \to \mathbb{R}^+$ are defined by $a(x) = F(fx, sx)$ and $b(x) = F(tx, gx)$ in X. If one of a, b is continuous in X, then f and s or g and t have a coincidence point in X.*

Proof. Assume that b is continuous in X. We are convinced that there exists $x_0 \in X$ such that $b(x_0) = \inf\{b(x) : x \in X\}$. From the assumption that $f(X) \subseteq t(X)$ and $g(X) \subseteq s(X)$, which guarantees that there exist $x_1, x_2 \in X$ such that $gx_0 = sx_1$ and $fx_1 = tx_2$. Suppose that neither f and s nor g and t have a coincidence point in X. In view of (1.4), we have that

$$\begin{aligned}
&F(fx_1, gx_2)F(tx_2, sx_1)\\
&< \max\Big\{F^2(tx_2, sx_1), F(fx_1, sx_1)F(tx_2, gx_2), F(fx_1, tx_2)F(sx_1, gx_2),\\
&\quad \frac{F^2(tx_2, gx_2)F(tx_2, sx_1)}{F(fx_1, gx_2)}, \frac{F^2(tx_2, gx_2)F(fx_1, sx_1)}{F(fx_1, gx_2)},\\
&\quad \frac{F^2(tx_2, fx_1)F(gx_2, sx_1)}{F(gx_2, fx_1)}, \frac{F^2(tx_2, sx_1)F(tx_2, gx_2)}{F(fx_1, sx_1)},\\
&\quad \frac{F^2(gx_2, sx_1)F(fx_1, tx_2)}{F(tx_2, sx_1)}, \frac{F^2(sx_1, gx_2)F(fx_1, tx_2)}{F(sx_1, tx_2)}\Big\}\\
&= \max\Big\{a^2(x_1), a(x_1)b(x_2), 0, \frac{b^2(x_2)a(x_1)}{b(x_2)}, \frac{b^2(x_2)a(x_1)}{b(x_2)}, 0,\\
&\quad \frac{a^2(x_1)b(x_2)}{a(x_1)}, 0, 0\Big\},
\end{aligned}$$

that is,

$$b(x_2)a(x_1) < \max\{a^2(x_1), a(x_1)b(x_2)\},$$

which implies that

$$b(x_2) < a(x_1). \tag{2.3}$$

By virtue of (1.4), we obtain that

$$\begin{aligned}
&F(fx_1, gx_0)F(tx_0, sx_1)\\
&< \max\Big\{F^2(tx_0, sx_1), F(fx_1, sx_1)F(tx_0, gx_0), F(fx_1, tx_0)F(sx_1, gx_0),\\
&\quad \frac{F^2(tx_0, gx_0)F(tx_0, sx_1)}{F(fx_1, gx_0)}, \frac{F^2(tx_0, gx_0)F(fx_1, sx_1)}{F(fx_1, gx_0)},
\end{aligned}$$

$$\frac{F^2(tx_0, fx_1)F(gx_0, sx_1)}{F(gx_0, fx_1)}, \frac{F^2(tx_0, sx_1)F(tx_0, gx_0)}{F(fx_1, sx_1)},$$
$$\frac{F^2(gx_0, sx_1)F(fx_1, tx_0)}{F(tx_0, sx_1)}, \frac{F^2(sx_1, gx_0)F(fx_1, tx_0)}{F(sx_1, tx_0)}\Big\}$$
$$= \max\Big\{b^2(x_0), a(x_1)b(x_0), 0, \frac{b^3(x_0)}{a(x_1)}, \frac{b^2(x_0)a(x_1)}{a(x_1)}, 0, \frac{b^3(x_0)}{a(x_1)}, 0, 0\Big\},$$

that is,

$$a(x_1)b(x_0) < \max\Big\{b^2(x_0), a(x_1)b(x_0), \frac{b^3(x_0)}{a(x_1)}\Big\},$$

which leads to

$$a(x_1) < b(x_0). \tag{2.4}$$

It follows from (2.3) and (2.4) that

$$b(x_2) < a(x_1) < b(x_0) \leq b(x_2),$$

which is a contradiction. So either f and s or g and t have a coincidence point.

Assume that a is continuous in X. Obviously, there exists $x_0 \in X$ such that $a(x_0) = \inf\{a(x) : x \in X\}$. On the grounds that $f(X) \subseteq t(X)$ and $g(X) \subseteq s(X)$, there exist $x_1, x_2 \in X$ such that $fx_0 = tx_1$ and $gx_1 = sx_2$. Suppose that neither f and s nor g and t have a coincidence point in X. In terms of (1.4), we get that

$$F(fx_0, gx_1)F(tx_1, sx_0)$$
$$< \max\Big\{F^2(tx_1, sx_0), F(fx_0, sx_0)F(tx_1, gx_1), F(fx_0, tx_1)F(sx_0, gx_1),$$
$$\frac{F^2(tx_1, gx_1)F(tx_1, sx_0)}{F(fx_0, gx_1)}, \frac{F^2(tx_1, gx_1)F(fx_0, sx_0)}{F(fx_0, gx_1)},$$
$$\frac{F^2(tx_1, fx_0)F(gx_1, sx_0)}{F(gx_1, fx_0)}, \frac{F^2(tx_1, sx_0)F(tx_1, gx_1)}{F(fx_0, sx_0)},$$
$$\frac{F^2(gx_1, sx_0)F(fx_0, tx_1)}{F(tx_1, sx_0)}, \frac{F^2(sx_0, gx_1)F(fx_0, tx_1)}{F(sx_0, tx_1)}\Big\}$$
$$= \max\Big\{a^2(x_0), a(x_0)b(x_1), 0, \frac{b^2(x_1)a(x_0)}{b(x_1)}, \frac{b^2(x_1)a(x_0)}{b(x_1)}, 0,$$
$$\frac{a^2(x_0)b(x_1)}{a(x_0)}, 0, 0\Big\},$$

that is,

$$b(x_1)a(x_0) < \max\{a^2(x_0), a(x_0)b(x_1)\},$$

which yields that

$$b(x_1) < a(x_0). \tag{2.5}$$

Using (1.4), we gain that

$$\begin{aligned}
&F(fx_2, gx_1)F(tx_1, sx_2)\\
&< \max\Big\{F^2(tx_1, sx_2), F(fx_2, sx_2)F(tx_1, gx_1), F(fx_2, tx_1)F(sx_2, gx_1),\\
&\quad \frac{F^2(tx_1, gx_1)F(tx_1, sx_2)}{F(fx_2, gx_1)}, \frac{F^2(tx_1, gx_1)F(fx_2, sx_2)}{F(fx_2, gx_1)},\\
&\quad \frac{F^2(tx_1, fx_2)F(gx_1, sx_2)}{F(gx_1, fx_2)}, \frac{F^2(tx_1, sx_2)F(tx_1, gx_1)}{F(fx_2, sx_2)},\\
&\quad \frac{F^2(gx_1, sx_2)F(fx_2, tx_1)}{F(tx_1, sx_2)}, \frac{F^2(sx_2, gx_1)F(fx_2, tx_1)}{F(sx_2, tx_1)}\Big\}\\
&= \max\Big\{b^2(x_1), a(x_2)b(x_1), 0, \frac{b^3(x_1)}{a(x_2)}, \frac{b^2(x_1)a(x_2)}{a(x_2)}, 0, \frac{b^3(x_1)}{a(x_2)}, 0, 0\Big\},
\end{aligned}$$

that is,

$$a(x_2)b(x_1) < \max\Big\{b^2(x_1), a(x_2)b(x_1), \frac{b^3(x_1)}{a(x_2)}\Big\},$$

which infers that

$$a(x_2) < b(x_1). \tag{2.6}$$

It follows from (2.5) and (2.6) that $a(x_2) < b(x_1) < a(x_0)$, which is a contradiction. Therefore, either f and s or g and t have a coincidence point. This completes the proof. □

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