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SWITCHING OF PREDATION ON PREY SPECIES IN THE PRESENCE OF PREDATOR INTERFERENCE

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Abstract: The present paper is an extension of previous work by the authors on a two prey-one predator habitats system where the prey species can move freely within the habitats and the predator can switch to the habitat with most abundant species. In this paper, predation has been studied in the presence of predator interference. The stability analysis has been carried out for non-zero equilibrium values. Conversion rate of the prey to predator has been taken as a bifurcation parameter and for select data sets, bifurcation points have been found. Interestingly, contrary to the earlier work, all the three types of situations, namely no bifurcation, single and multiple bifurcations exist.

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1. Introduction

In the modeling of any predator-prey system, the feeding rate of predators

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is very important. Generally, this information is not completely known for any system and assumptions have to be made in order to construct a usable predatory-rate function. Commonly used predatory-rate functions depend on the prey populations and hence do not take into account any interaction of the predators among themselves.

Recently, Skalkski and Gillian [7], presented statistical evidence that interactions between the predators or predator interference appear to take place and should be considered. It has been suggested that, in a variety of predator-prey systems, the predatory behavior can better be understood if the predator interference is also incorporated in the model. In their models, Beddington [1], De Angelis, Goldstein and O'Neil [5], Crowley and Martin [4] and Hassel and Varley [6] examined several predator-prey systems with predator interference and concluded that the inclusion of predator interference in their models provided better descriptions in many of the systems.

The object of the present work is to consider the systems of Bhatt, Khan and Jaju [2] and Bhatt, Owen and Jaju [3] which deal with environments with prey species in two different habitats and a predator species which can switch to the most abundant prey species and extend their work to include predator interference.

In Section 2, we give the system of equations defining the model. In Section 3, we examine the stability of the equilibrium states, while in Section 4 we give the numerical results together with concluding remarks.

2. The Model

Our model with predatory rate depending upon predator interference is given by

$$\begin{aligned} \frac{dx_1}{dt} &= (\alpha_1 - \epsilon_1)x_1 + \epsilon_2 p_{21} x_2 - \beta_1 x_1 y k_1(x_1, x_2, y)\,, \\ \frac{dx_2}{dt} &= (\alpha_2 - \epsilon_2)x_2 + \epsilon_1 p_{12} x_1 - \beta_2 x_2 y k_2(x_1, x_2, y)\,, \\ \frac{dy}{dt} &= [-\mu + c_1\beta_1 x_1 k_1(x_1, x_2, y) + c_2\beta_2 x_2 k_2(x_1, x_2, y)]\, y\,, \end{aligned} \tag{1}$$

where

$$k_1(x_1, x_2, y) = \frac{1}{1 + \left(\dfrac{x_2 + cy}{x_1}\right)^n}\,, \quad k_2(x_1, x_2, y) = \frac{1}{1 + \left(\dfrac{x_1 + cy}{x_2}\right)^n} \tag{2}$$

for $n = 1, 2, 3, \dots$ and:

x_i: represents the prey population in the two different habitats;

y: represents the abundance of predator species;

β_i: the predator response rates towards the prey x_i;

c_i: the rate of conversion of prey to predator;

ϵ_i: inversion barrier strength in going out of the habitat;

p_{ij}: the probability of successful transition from the i-th habitat to the j-th habitat;

α_i: specific growth rate of the prey in the absence of predation;

μ: per capita death rate of the predator.

The parameter c is obtained from [7] for constructing the functions k_1 and k_2 in equation (3) and together with the other parameters $\alpha_i, \beta_i, \epsilon_i, c_i, p_{ij}, \mu$ are all positive.

It may be noted that the predatory functions used in this work are an extension of those used in the paper by Hassel-Varley [6] and those in [2], [3].

3. Stability of Equilibria

Denoting the equilibrium values of equation (1) by (X_1, X_2, Y), we see that X_1, X_2, Y satisfy the equations

$$u_{12} - \beta_1 X_1 Y k_1(X_1, X_2, Y) = 0\,, \tag{3}$$

$$v_{12} - \beta_2 X_2 Y k_2(X_1, X_2, Y) = 0\,, \tag{4}$$

and

$$Y = \frac{1}{\mu}\left(c_1 u_{12} + c_2 v_{12}\right)\,, \tag{5}$$

where

$$u_{12} = (\alpha_1 - \epsilon_1)X_1 + \epsilon_2 p_{21} X_2\,, \quad v_{12} = (\alpha_2 - \epsilon_2)X_2 + \epsilon_1 p_{12} X_1\,.$$

Setting $X_1 = \bar{X} X_2$, $u_{12} = U_{12}(\bar{X})X_2$ and $v_{12} = V_{12}(\bar{X})X_2$, where $U_{12}(\bar{X}) = (\alpha_1 - \epsilon_1)\bar{X} + \epsilon_2 p_{21}$ and $V_{12}(\bar{X}) = (\alpha_2 - \epsilon_2) + \epsilon_1 p_{12}\bar{X}$, we solve equations (3) and (5) for X_2 to get

$$X_2 = \frac{(\alpha_1 - \epsilon_1)\bar{X} + \epsilon_2 p_{21}}{\beta_1 \bar{X}\left[\dfrac{c_1 U_{12}(\bar{X}) + c_2 V_{12}(\bar{X})}{\mu}\right] k_1(X_1, X_2, Y)}. \tag{6}$$

Now using equations (4) and (5) we see that we can get another expression for X_2, namely

$$X_2 = \frac{\alpha_2 - \epsilon_2 + \epsilon_1 p_{12}\bar{X}}{\beta_2 \left[\dfrac{c_1 U_{12}(\bar{X}) + c_2 V_{12}(\bar{X})}{\mu}\right] k_2(X_1, X_2, Y)}, \tag{7}$$

X_1 can be obtained from $X_1 = \bar{X} X_2$, say

$$X_1 = \frac{(\alpha_1 - \epsilon_1)\bar{X} + \epsilon_2 p_{21}}{\beta_1 \left[\dfrac{c_1 U_{12}(\bar{X}) + c_2 V_{12}(\bar{X})}{\mu}\right] k_1(X_1, X_2, Y)} \tag{8}$$

and Y can be obtained from equation (5).

We note that with the predatory functions as defined in equation (2) we can write $k_1(X_1, X_2, Y)$, and $k_2(X_1, X_2, Y)$ as functions of $\bar{X}$ only. As it stands, since we have two expressions for X_2, the equilibrium point (X_1, X_2, Y) may not exist and even if it exists it may not represent real populations. To ensure that it exists we must first equate the two expressions, equations (6) and (7), for X_2. That is, we must choose $\bar{X}$ to satisfy

$$\frac{(\alpha_1 - \epsilon_1)\bar{X} + \epsilon_2 p_{21}}{\alpha_2 - \epsilon_2 + \epsilon_1 p_{12}\bar{X}} = \frac{\beta_1 \bar{X} k_1(X_1, X_2, Y)}{\beta_2 k_2(X_1, X_2, Y)}, \tag{9}$$

where

$$k_1(X_1, X_2, Y) = \frac{1}{1 + \left[\dfrac{X_2 + \dfrac{c}{\mu}(c_1 u_{12} + c_2 v_{12})}{X_1}\right]^n},$$

$$k_2(X_1, X_2, Y) = \frac{1}{1 + \left[\dfrac{X_1 + \dfrac{c}{\mu}(c_1 u_{12} + c_2 v_{12})}{X_2}\right]^n}$$

and

$$\alpha_2 - \epsilon_2 + \epsilon_1 p_1 2\bar{X} \neq 0.$$

In order to ensure that X_1, X_2, Y represent real populations we need to have $X_1 > 0, X_2 > 0, Y > 0$. Indeed, if we choose

$$(\alpha_1 - \epsilon_1)\bar{X} + \epsilon_2 p_{21} > 0, \quad \alpha_2 - \epsilon_2 + \epsilon_2 p_{12}\bar{X} > 0, \tag{10}$$

then it is seen that X_1, X_2, Y are all positive. From the above we may write the conditions for the equilibrium point, (X_1, X_2, Y), of equations (1) to exist and represent real populations, as follows:

The equilibrium point, (X_1, X_2, Y), of equations (1) exists and represents real populations if $\bar{X}$ satisfies

$$\frac{(\alpha_1-\epsilon_1)\bar{X}+\epsilon_2 p_{21}}{\alpha_2-\epsilon_2+\epsilon_2 p_{12}\bar{X}} = \frac{\beta_1 \bar{X} k_1}{\beta_2 k_2} \text{ and } \frac{\epsilon_2-\alpha_2}{\epsilon_1 p_{12}} < \bar{X} < \frac{\epsilon_2 p_{21}}{\epsilon_1-\alpha_1}\,, \tag{11}$$

where

$$k_1 = \frac{1}{1+\left[\dfrac{1+\dfrac{c}{\mu}(c_1((\alpha_1-\epsilon_1)\bar{X}+\epsilon_2 p_{21})+c_2(\alpha_2-\epsilon_2+\epsilon_1 p_{12}\bar{X}))}{\bar{X}}\right]^n},$$

$$k_2 = \frac{1}{1+\left[\bar{X}+\dfrac{c}{\mu}(c_1((\alpha_1-\epsilon_1)\bar{X}+\epsilon_2 p_{21})+c_2(\alpha_2-\epsilon_2+\epsilon_1 p_{12}\bar{X}))\right]^n}. \tag{12}$$

3.1. Stability Analysis

We shall now examine the stability of the equilibrium point (X_1, X_2, Y), where X_1, X_2, Y are all positive. We shall first linearize equations (1) by admitting a small perturbation about the equilibrium point, that is, by substituting $x_1 = X_1+u, x_2 = X_2+v, y = Y+w$ into the functions k_i and expanding them using Taylor's Theorem while neglecting higher order terms in u, v and w. Defining $A, B, C_0, \tilde{C}_0, D, \tilde{D}, \tilde{E}, w_1, w_2$ by

$$\begin{aligned}
A &= c_1\beta_1(K_1(S)-SK_{1p}(S))+c_2\beta_2 K_{2p}(\tilde{S}),\\
B &= c_1\beta_1 K_{1p}(S)+c_2\beta_2(K_2(\tilde{S})-\tilde{S}K_{2p}(\tilde{S})),\\
C_0 &= \epsilon_1 p_{12}-\beta_2 Y K_{2p}(\tilde{S}),\\
\tilde{C}_0 &= -\bar{X}C_0+c\beta_2\frac{Y^2}{X_2}K_{2p}(\tilde{S}),\\
D &= \epsilon_2 p_{21}-\beta_1 Y K_{1p}(S),\\
\tilde{D} &= -\frac{D}{\bar{X}}+c\beta_1\frac{Y^2}{X_1}K_{1p}(S),\\
\tilde{E} &= cY(c_1\beta_1 K_{1p}(S)+c_2\beta_2 K_{2p}(\tilde{S})),\\
w_1 &= X_1(c\frac{Y}{X_1}K_{1p}(S)+K_1(S)) \text{ and}\\
w_2 &= X_2(c\frac{Y}{X_2}K_{2p}(\tilde{S})+K_2(\tilde{S})),
\end{aligned} \tag{13}$$

where

$$K_1(S) = \frac{1}{(1+S^n)}, \quad S = \frac{X_2+cY}{X_1},$$

$$K_2(\tilde{S}) = \frac{1}{(1+\tilde{S}^n)}, \quad \tilde{S} = \frac{X_1 + cY}{X_2}$$

and $K_{1p}(S)$, $K_{2p}(\tilde{S})$ are the derivatives of $K_1(S)$, $K_2(\tilde{S})$ with respect to $S, \tilde{S}$ respectively, we are able to write the linearized form of equations (1) as

$$\frac{dV}{dt} = JV\,,$$

where $V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ and with the aid of equation (13), the characteristic equation can be written as

$$|J| = \begin{vmatrix} \tilde{D}-\lambda & D & -\beta_1 w_1 \\ C_0 & \tilde{C}_0 - \lambda & -\beta_2 w_2 \\ AY & BY & \tilde{E}-\lambda \end{vmatrix} = 0. \tag{14}$$

This equation can be written as

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0\,, \tag{15}$$

where

$$\begin{aligned} b_1 &= -(\tilde{C}_0 + \tilde{D} + \tilde{E}), \\ b_2 &= \tilde{D}\tilde{E} + \tilde{D}\tilde{C}_0 + \beta_1 AYw_1 + \beta_2 BYw_2 + \tilde{C}_0\tilde{E} - C_0 D, \\ b_3 &= -\tilde{C}_0\tilde{D}\tilde{E} + \beta_2 ADYw_2 + \beta_1 BC_0Yw_1 - \beta_1 A\tilde{C}_0Yw_1 - \beta_2 B\tilde{D}Yw_2 + C_0D\tilde{E}. \end{aligned} \tag{16}$$

Now the eigenvalue solutions, λ, must have negative real parts in order that the equilibrium points be stable. Conditions for this to hold are provided by the Routh-Hurwitz criteria. These criteria say that the eigenvalues will have negative real parts if and only if

$$b_1 > 0, \quad b_2 > 0, \quad b_1b_2 - b_3 > 0.$$

However, from equations (13) we can show that $b_1 > 0$, hence we have stability of the equilibrium point if and only if

$$b_2 > 0, \quad b_1b_2 - b_3 > 0\,.$$

With the help of the above we can write the following:

If (X_1, X_2, Y) is an equilibrium point of equations (1) and the conditions of (11) and (12) hold and $A, B, C, \tilde{C}, D, \tilde{D}, \tilde{E}, w_1, w_2$ are defined by equation (13), then the stability of the equilibrium point (X_1, X_2, Y) is assured if and only if

$$b_2 > 0, \quad b_1b_2 - b_3 > 0\,,$$

where b_1, b_2, b_3 are given by equations (16).

4. Applications and Numerical Results

In one of our applications we looked at the stability behavior for $n = 1$ and 2, $\beta_1 < \beta_2$ and $\beta_2 < \beta_1$ for varying values of c_1 and c_2. We give the results for only two values of β_1, β_2. These results are shown in Table 1. The parameter c was obtained from [1] and three values for c, namely, $0.0316, 0.2927$ and 0.7296 were used. The values of the other parameters used are $\mu = 0.01, \alpha_1 = 0.015, \alpha_2 = 0.025, \beta_1 = 0.01, \beta_2 = 0.02$ (and also $\beta_1 = 0.02, \beta_2 = 0.01), \epsilon_1 = 0.02, \epsilon_2 = 0.03, p_{12} = 0.3$ and $p_{21} = 0.2$.

The upper-part of Table 1 has $\beta_1 < \beta_2$ while the lower-part has $\beta_1 > \beta_2$, actually $\beta_1 \leftrightarrow \beta_2$. We observe that the stability behavior corresponds to some kind of "translation" of the upper-part unto the lower-part.

In further investigation we considered $p_{12} < p_{21}$ and $p_{21} < p_{12}$. We give the results for only two values of $p_{12},\ p_{21}$. Again we took $n = 1$ and 2 and $c = 0.0316, 0.2927$ and 0.7296 while $\mu = 0.01, \beta_1 = 0.01, \beta_2 = 0.02$ and the following three data sets of values of the other variables:

	ϵ_1	ϵ_2	p_{12}	p_{21}	α_1	α_2
SET(1)	0.04	0.03	0.5	0.7	0.015	0.025
SET(2)	0.1	0.3	0.2	0.7	0.05	0.25
SET(3)	0.1	0.3	0.5	0.3	0.05	0.25

The results are shown in Table 2.

Contrary to our earlier work in [2] and [3], it is interesting to note that, in Table 1, multiple bifurcations, marked as (i), (ii) occur, at the same point in each case, when c_1 (or c_2) are varying. Similarly in Table 2, SET(1) and SET(3), with $c_2 = 0.03$, have multiple bifurcation points, marked as (iii) and (iv), when c_1 is varying. Also non-existence of the bifurcation can be observed in several cases.

To support our theory we produce seven figures, Figure 1 to Figure 4 correspond to single bifurcation cases while Fig 5. and Fig 6. correspond to multiple bifurcation point cases, shown as (iii) and (iv) in Table 2. Lastly Figure 7 corresponds to (ii) in Table 1.

For Figure 1 to Figure 4, we chose parameter values, $\mu = 0.01, \alpha_1 = 0.015, \alpha_2 = 0.025, \epsilon_1 = 0.03, \epsilon_2 = 0.02,\ p_{12} = 0.3, p_{21} = 0.2$ and $c = 0.0316$ together with those in Table 3.

For Figure 5 we chose the parameter values, $\mu = 0.01, \alpha_1 = 0.015, \alpha_2 = 0.025, \beta_1 = 0.01, \beta_2 = 0.02, p_{12} = 0.5, p_{21} = 0.7, \epsilon_1 = 0.04, \epsilon_2 = 0.03, c_2 =$

n	c	STABLE	UNSTABLE	Bifurcation Point
		$\beta_1 = 0.01, \beta 2 = 0.02,$	$c_2 = .03$	
1	0.0316	$0 \leq c_1 \leq 0.060132$	$c_1 >.060132$	0.060132
1	0.2927	$0 \leq c_1 \leq 0.063438$	$c_1 >.063438$	0.063438
1	0.7296	$0 \leq c_1 \leq 0.069986$	$c_1 >.069986$	0.069986
2	0.0316	$0 \leq c_1 \leq 0.059322$	$c_1 >.059322$	0.059322
2	0.2927	$0 \leq c_1 \leq 0.064483$	$c_1 >.064483$	0.064483
2	0.7296	$0 \leq c_1 \leq 0.076037$	$.076038 \leq c_1$ &	0.076037 &
		$c_1 \geq 0.749111$	$c_1 <.749111$	0.749111 **(i)**
		$\beta_1 = 0.01, \beta 2 = 0.02,$	$c_1 = .03$	
1	0.0316	$c_2 \geq 0.015015$	$0 \leq c_2 <.015015$	0.015015
1	0.2927	$c_2 \geq 0.014636$	$0 \leq c_2 <.014636$	0.014636
1	0.7296	$c_2 \geq 0.014041$	$0 \leq c_2 <.014041$	0.014041
2	0.0316	$c_2 \geq 0.015243$	$0 \leq c_2 <.015243$	0.015243
2	0.2927	$c_2 \geq 0.014657$	$0 \leq c_2 <.014657$	0.014657
2	0.7296	$c_2 \geq 0.013760$	$0 \leq c_2 <.013760$	0.013760
		$\beta_1 = 0.02, \beta 2 = 0.01,$	$c_2 = .03$	
1	0.0316	$c_1 \geq 0.015015$	$0 \leq c_1 <.015015$	0.015015
1	0.2927	$c_1 \geq 0.014636$	$0 \leq c_1 <.014636$	0.014636
1	0.7296	$c_1 \geq 0.014041$	$0 \leq c_1 <.014041$	0.014041
2	0.0316	$c_1 \geq 0.015243$	$0 \leq c_1 <.015243$	0.015243
2	0.2927	$c_1 \geq 0.014657$	$0 \leq c_1 <.014657$	0.014657
2	0.7296	$c_1 \geq 0.013760$	$0 \leq c_1 <.013760$	0.013760
		$\beta_1 = 0.02, \beta 2 = 0.01,$	$c_1 = .03$	
1	0.0316	$0 \leq c_2 \leq 0.060132$	$c_2 >.060132$	0.060132
1	0.2927	$0 \leq c_2 \leq 0.063438$	$c_2 >.063438$	0.063438
1	0.7296	$0 \leq c_2 \leq 0.069986$	$c_2 >.069986$	0.069986
2	0.0316	$0 \leq c_2 \leq 0.059322$	$c_2 >.059322$	0.059322
2	0.2927	$0 \leq c_2 \leq 0.064483$	$c_2 >.064483$	0.064483
2	0.7296	$0 \leq c_2 \leq 0.076037$	$.076038 \leq c_2$ &	0.076037 &
		$c_2 \geq 0.749111$	$c_2 <.749111$	0.749111 **(ii)**

Table 1:

$0.03, c = 0.7296$ and $n = 1$. In this case the parameter, c_1, had the values $0.1, 0.3$ and 0.5, so that $0 < c_1 < 0.217864$, $0.217865 \leq c_1 \leq 0.315678$ and $0.315679 < c_1 \leq 1.0$ which come from the entries in Table 2 containing two equilibria in SET(1), with $c_2 = 0.03$. These come from the entry in Table 2 containing two equilibria, marked as (iii).

For Figure 6 we chose the parameter values, $\mu = 0.01, \alpha_1 = 0.05, \alpha_2 = 0.25, \beta_1 = 0.01, \beta_2 = 0.02, p_{12} = 0.5, p_{21} = 0.3, \epsilon_1 = 0.1, \epsilon_2 = 0.3, c_2 = 0.03, c =$

n	c	STABLE	UNSTABLE	Bifurcation Point
		SET(1), $c_1 = .03$		
1	.0316	$.011926 \leq c_2 \leq 1$	$c_2 < .011926$	.011926
1	.2927	$.010986 \leq c_2 \leq 1$	$c_2 < .010986$	.010986
1	.7296	$.009735 \leq c_2 \leq 1$	$c_2 < .009735$	.009735
2	.0316	$.010371 \leq c_2 \leq 1$	$c_2 < .010371$	.010371
2	.2927	$.009534 \leq c_2 \leq 1$	$c_2 < .009534$	.009534
2	.7296	$.008421 \leq c_2 \leq 1$	$c_2 < .008421$	.008421
		SET(2), $c_1 = .03$		
1	.0316	$0 \leq c_2 \leq .112872$	$c_2 > .112872$	.112872
1	.2927	$0 \leq c_2 \leq 1$	NOWHERE	NONE
1	.7296	$0 \leq c_2 \leq 1$	NOWHERE	NONE
2	.0316	$0 \leq c_2 \leq .128027$	$c_2 > .128027$	.128027
2	.2927	$0 \leq c_2 \leq 1$	NOWHERE	NONE
2	.7296	$0 \leq c_2 \leq 1$	NOWHERE	NONE
		SET(3), $c_1 = .03$		
1	.0316	$.019223 \leq c_2 \leq 1$	$c_2 < 0.019223$	.019223
1	.2927	$.005110 \leq c_2 \leq 1$	$c_2 < 0.005110$	.005110
1	.7296	$0 \leq c_2 \leq 1$	NOWHERE	NONE
2	.0316	$.007677 \leq c_2 \leq 1$	$c_2 < .007677$	.007677
2	.2927	$0 \leq c_2 \leq 1$	NOWHERE	NONE
2	.7296	$0 \leq c_2 \leq 1$	NOWHERE	NONE
		SET(1), $c_2 = .03$		
1	.0316	$0 \leq c_1 \leq .076694$	$c_1 > .076694$	.076694
1	.2927	$0 \leq c_1 \leq .097849$	$c_1 > .097849$	.097849
1	.7296	$0 \leq c_1 \leq .217864$	$.217865 \leq c_1$ &	.217864 &
		$c_1 \geq .315679$	$c_1 \leq .315678$	.315679 **(iii)**
2	.0316	$0 \leq c_1 \leq .088602$	$c_1 > .088602$	.088602
2	.2927	$0 \leq c_1 \leq .119191$	$c_1 > .119191$	.119191
2	.7296	$0 \leq c_1 \leq 1$	NOWHERE	NONE
1	.0316	$.009645 \leq c_1 \leq 1$	$c_1 < .009645$	.009645
1	.2927	$.005954 \leq c_1 \leq 1$	$c_1 < .005954$	.005954
1	.7296	$.003405 \leq c_1 \leq 1$	$c_1 < .003405$	.003405
2	.0316	$.007627 \leq c_1 \leq 1$	$c_1 < .007627$	.007627
2	.2927	$.006314 \leq c_1 \leq 1$	$c_1 < .006314$	.006314
2	.7296	$.004997 \leq c_1 \leq 1$	$c_1 < .004997$	.004997
		SET(3), $c_2 = .03$		
1	.0316	$0 \leq c_1 \leq .053325$	$.053326 \leq c_1$ &	.053325 &
		$c_1 \geq .376864$	$c_1 \leq .376863$	.376864 **(iv)**
1	.2927	$0 \leq c_1 \leq 1$	NOWHERE	NONE
1	.7296	$0 \leq c_1 \leq 1$	NOWHERE	NONE
2	.0316	$0 \leq c_1 \leq 1$	NOWHERE	NONE
2	.2927	$0 \leq c_1 \leq 1$	NOWHERE	NONE
2	.7296	$0 \leq c_1 \leq 1$	NOWHERE	NONE

Table 2:

β_1	$\beta 2$	c_1	c_2	n	Behaviour
0.01	0.02	0.02	0.03	1	Stable
0.01	0.02	0.08	0.03	2	Unstable
0.02	0.01	0.01	0.03	1	Unstable
0.02	0.01	0.02	0.03	2	Stable

Table 3:

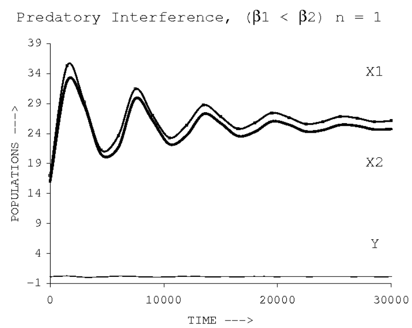

Figure 1:

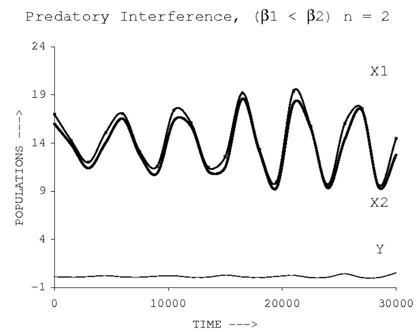

Figure 2:

0.0316 and $n = 1$. Here the parameter, c_1, had the values $0.02, 0.3$ and 0.5, so that $0 < c_1 < 0.053325$, $0.053326 \leq c_1 \leq 0.376863$ and $0.376864 < c_1 \leq 1.0$ which come from the entries in Table 2 containing two equilibria in SET (3), with $c_2 = 0.03$. These come from the entry in Table 2 containing two equilibria, marked as (iv).

For Figure 7 we chose the parameter values, $\mu = 0.01, \alpha_1 = 0.015$, $\alpha_2 = 0.025, \beta_1 = 0.01, \beta_2 = 0.02, p_{12} = 0.3, p_{21} = 0.2, \epsilon_1 = 0.02, \epsilon_2 = 0.03, c_2 = 0.03, c = 0.7296$ and $n = 2$. Here the parameter, c_1, had the values $0.02, 0.5$ and 0.08, so that $0 < c_1 < 0.076037$, $0.076038 \leq c_1 \leq 0.749110$ and $0.749111 < c_1 \leq 1.0$. These come from the entry in Table 1 containing two equilibria, marked as (ii).

As expected, these numerical results, as seen from the figures, confirm our analytical results.

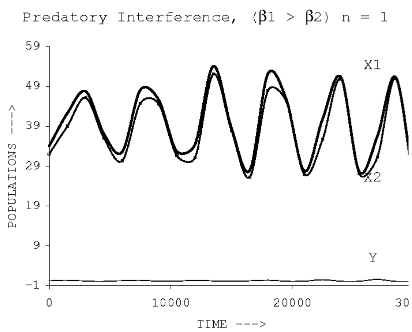

Figure 3:

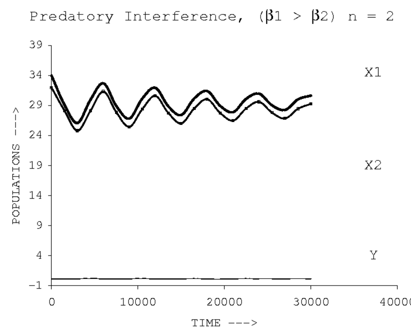

Figure 4:

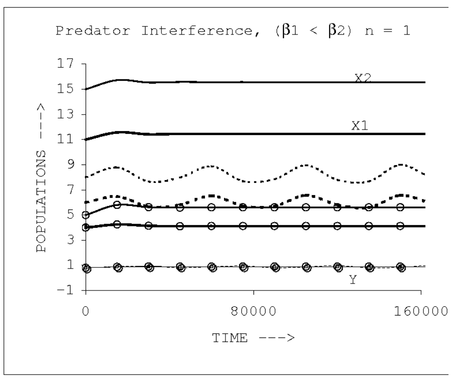

Figure 5:

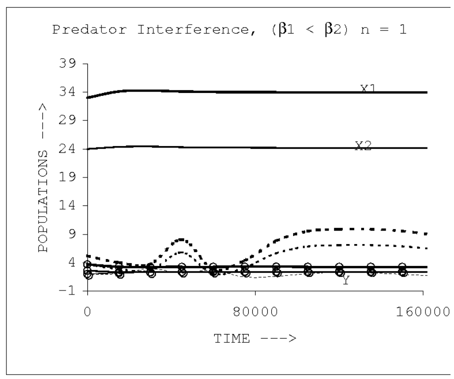

Figure 6:

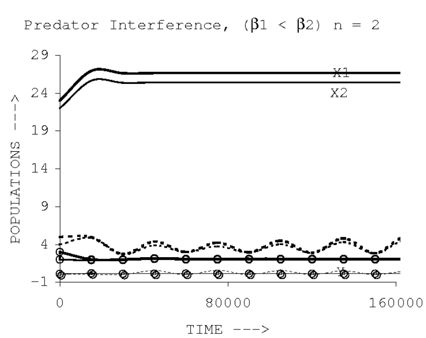

Figure 7:

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