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FROBENIUS PUSH-FORWARDS ON SMOOTH QUADRICS

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Abstract: Here we study Frobenius push-forwards of line bundles and spinor bundles on smooth quadric hypersurfaces and their indecomposable factors.

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Let X be an integral and smooth n-dimensional projective variety defined over an algebraically closed field $\mathbb{K}$ such that $\mathrm{char}(\mathbb{K}) = p > 0$. Let $F_X : X \to X$ denote the absolute Frobenius morphism. F_X is just set-theoretically the identity, while $F_X^*(f) = f^p$ for all $P \in X$ and all $f \in \mathcal{O}_{X,P}$. F_X is not a $\mathbb{K}$-morphism, but only an $\mathbb{F}_p$ morphism. $\deg(F_X) = p^n$ (see [3], p. 128 and [2]). For every integer $r \geq 1$ let $F_X^r : X \to X$ denote the iteration r times of F_X. The smoothness of X implies that F_X is flat (see [4], Example III.10.9). Hence for every vector bundle E on X the coherent sheaf $F_{X*}^r(E)$ is a vector bundle on X with rank $p^{nr} \cdot \mathrm{rank}(E)$. From now on we assume $n \geq 2$. We will say that X is ACM (i.e. arithmetically Cohen- Macaulay) if $h^i(X, M) = 0$ for all $1 \leq i \leq n-1$ and all $M \in \mathrm{Pic}(X)$. A smooth quadric hypersurface $Q_n \subset \mathbf{P}^{n+1}$ is ACM if and only if $n \geq 3$. From now on we will assume that X is ACM. A vector bundle E on X will be said to be ACM if $h^i(X, E \otimes M) = 0$ for all $1 \leq i \leq n-1$ and all $M \in \mathrm{Pic}(X)$. A vector bundle on $\mathbf{P}^n$ is ACM if and only if it is isomorphic to a direct sum of line bundles (Horrock's criterion or see [3], Lemma 6.3). Here we take as X a smoooth quadric hypersurface $Q_n \subset \mathbf{P}^{n+1}$ and see how to

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compute the indecomposable factors of any ACM vector bundle on Q_n in terms of the Hilbert functions $h^0_E, h^n_E : \mathbb{Z} \to \mathbb{N}$ defined by $h^0_E(t) := h^0(Q_n, E(t))$ and $h^n_E(t) := h^n(Q_n, E(t))$ (see Remarks 8 and 9). In particular we will implicitely determine for each $x \in \mathbb{Z}$ the indecomposable factors (with their multiplicities) of the vector bundle $F^r_{Q_n*}(\mathcal{O}_{Q_n}(x))$ (see Remark 10 and Corollary 1). We have no explicit formula for these multiplicities.

Proposition 1. *Fix an integer $r \geq 1$ and an ACM vector bundle E on X. Then $F^r_{X*}(E)$ is ACM.*

Proof. Fix $M \in \mathrm{Pic}(X)$ and an integer i such that $1 \leq i \leq n-1$. Since F^r_X is finite, $h^i(X, E \otimes F^{r*}_X(M)) = h^i(X, F^r_{X*}(E \otimes F^{r*}_X(M)))$. The projection formula gives $F^r_{X*}(E \otimes F^{r*}_X(M)) \cong F^r_{X*}(E) \otimes M$. Since $M \in \mathrm{Pic}(X)$, we have $F^{r*}_X(M)) \cong M^{\otimes p^r}$. Since E is ACM, $h^i(X, E \otimes M^{\otimes p^r}) = 0$. □

Remark 1. Since $F^{r*}_X(M) \cong M^{\otimes p^r}$ for all $M \in \mathrm{Pic}(X)$, the projection formula gives $F^r_{X*}(E \otimes M^{\otimes p^r}) \cong F^r_{X*}(E) \otimes M$ for every vector bundle E and every $M \in \mathrm{Pic}(X)$.

Remark 2. For any $L \in \mathrm{Pic}(\mathbf{P}^n)$, $F^r_{\mathbf{P}^n*}(L)$ is a direct sum of line bundles ([3], p.128). For given $n, p, r, \deg(L)$ the decomposition of $F^r_{\mathbf{P}^n*}(L)$ may be computed in the following way, generalizing [2], Lemma 3.7. Set $x := \deg(L)$ and write $F^r_{\mathbf{P}^n*}(L) \cong \oplus_{i=1}^b \mathcal{O}_{\mathbf{P}^n}(a_i)^{\oplus m_i}$ with $b \geq 1$ and $a_i > a_j$ for all $i < j$. By Remark 1 we have $h^i(\mathbf{P}^r, F^r_{\mathbf{P}^n*}(L)(t)) = h^i(\mathbf{P}^n, \mathcal{O}_{\mathbf{P}^r}(x + p^r t))$ for all $t \in \mathbb{Z}$ and all $0 \leq i \leq n$. Hence the Euler's function $\chi_{F^r_{\mathbf{P}^n*}(\mathcal{O}_{\mathbf{P}^n}(x))}$ satisfies the formula $\chi_{F^r_{\mathbf{P}^n*}(\mathcal{O}_{\mathbf{P}^n}(x))}(t) = \chi_{\mathcal{O}_{\mathbf{P}^n}(x)}(t)$ for all t. Hence it is uniquely determined by the integers n, p, r, x. Fix a line $L \subset \mathbf{P}^n$. Set $E := \oplus_{i=1}^b \mathcal{O}_{\mathbf{P}^n}(a_i)^{\oplus m_i}$. The integers $b, a_1, m_1, \dots, a_b, m_b$ are uniquely determined by $E|L$ and they may be compute from χ_E in ithe folowing way. Let $\Delta^{(n-1)}(\chi_E)$ denote the $(n-1)$-th difference function of χ_E. $n-1$ exact sequences give $\chi_{E|L} = \Delta^{(n-1)}(\chi_{E|L})$. a_1 is the maximal integer z such that $\chi_{E|L}(-z) > 0$. We have $m_1 = \chi_{E|L}(-a_1)$ and $\chi_{E|L}(-t) \geq m_1(a_1 - t + 1)$ for all $t \leq a_1$. If $b = 1$, we are done. If $b > 1$, then $\chi_{E|L}(-t) > m_1(a_1 - t + 1)$ for $t \ll 0$ and a_2 is the maximal integer $t < a_1$ such that $\chi_{E|L}(-t) > m_1(a_1 - t + 1)$. $m_2 = \chi_{(E|L}(-a_2) - m_1(a_1 - a_2 + 1)$. And so on. The formula $\chi_{F^r_{\mathbf{P}^n*}(\mathcal{O}_{\mathbf{P}^n}(x)))}(t + c) = \chi_{F^r_{\mathbf{P}^n*}(\mathcal{O}_{\mathbf{P}^n}(x + cp^r))}(t)$ for all $c \in \mathbb{Z}$ gives very strong restrictions on the integers $b, a_1, m_1, \dots, a_b, m_b$ and shows that to compute them it is sufficient to compute them when $0 \leq x < p^r$. The proof of [2], Lemma 3.7, shows that if $0 \leq x < p^r$, then $a_1 = 0$ and $m_1 = \binom{n+x}{n}$.

Example 1. Here we take $X := \mathbf{P}^1$, i.e. we look at the case $n = 1$ of Remark 2. Fix integers x, r such that $r \geq 1$. We want to compute $F^r_{\mathbf{P}^1*}(\mathcal{O}_{\mathbf{P}^1}(x))$. Remark 1 shows that $F^r_{\mathbf{P}^1*}(\mathcal{O}_{\mathbf{P}^1}(x + cp^r)) \cong F^r_{\mathbf{P}^1*}(\mathcal{O}_{\mathbf{P}^1}(x))(c)$

for all $c \in \mathbb{Z}$. Hence it is sufficient to prove the case $0 \leq x < p^r$. In this case the finiteness of the Frobenius map gives $h^0(\mathbf{P}^1, F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x))) = x + 1$ and $h^1(\mathbf{P}^1, F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x))) = 0$. The last equality gives that every indecomposable factor of $F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x))$ has degree ≥ -1. Since $x < p^r$, $h^0(\mathbf{P}^1, F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x))(-1)) = 0$. Hence every indecomposable factors of

$$F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x))$$

has degree ≤ 0. Thus $F^r_{\mathbf{P}^1 *}(\mathcal{O}_{\mathbf{P}^1}(x)) \cong \mathcal{O}_{\mathbf{P}^1}^{\oplus (x+1)} \oplus \mathcal{O}_{\mathbf{P}^1}(-1)^{(p^r - x - 1)}$.

Remark 3. Let $Q_n \subset \mathbf{P}^{n+1}$, $n \geq 3$, be a smooth quadric. There is a complete classification of all ACM vector bundles on Q_n ([5], [1]). From their description we see that in odd positive characteristic we may use [7], Section 3, and the cohomological properties of the spinor bundles proved in [6]. The indecomposable ones are the line bundles and the twists by a line bundle of the following ones. If n, is odd, say $n = 2m+1$, there is an indecomposable rank 2^m ACM vector bundle S_n on Q_n with $c_1(S_n) = -2^{m-1}$ and $S_n(t)$, $t \in \mathbb{Z}$, are the only indecomposable vector bundles with rank at least 2. $S_n^* \cong S_n(1)$ and we have an exact sequence on Q_{2m+1}:

$$0 \to S_{2m+1} \to \mathcal{O}_{Q_{2m+1}}^{\oplus 2^{m+1}} \to S_{2m+1}(1) \to 0 \tag{1}$$

$f^*(S_{2m+1}) \cong S_{2m+1}$ for all $f \in \mathrm{Aut}(Q_{2m+1})$. If $n = 2m+1$ is odd, then there are two indecomposable rank 2^{m-1} vector bundles S'_{2m} and S''_{2m} with $c_1(S_{2m}) = c_1(S''_{2m}) = -2^{m-2}$ such that $S'_{2m}(t)$ and $S''_{2m}(t)$ are the only indecomposable ACM vector bundles on Q_{2m} with rank at least 2. We have $S'^*_{2m} \cong S''^*_{2m}$ and $S''^*_{2m} \cong S'^*_{2m}$. $f^*(S'_{2m+1}) \cong S'_{2m+1}$ and $f^*(S''_{2m+1}) \cong S''_{2m+1}$for all $f \in \mathrm{Aut}^0(Q_{2m+1})$, while any $f \in \mathrm{Aut}(Q_{2m+1} \backslash \mathrm{Aut}^0(Q_{2m+1}$ interchanges S'_{2m} and S''_{2m}. We have two exact sequences on Q_{2m}:

$$0 \to S'_{2m+1} \to \mathcal{O}_{Q_{2m+1}}^{\oplus 2^m} \to S''_{2m+1}(1) \to 0\,, \tag{2}$$

$$0 \to S''_{2m+1} \to \mathcal{O}_{Q_{2m+1}}^{\oplus 2^m} \to S'_{2m+1}(1) \to 0\,. \tag{3}$$

If we see Q_{2m} as a hyperplane section of Q_{2m+1} and Q_{2m+1} as a hyperplane section of Q_{2m+2}, then

$$S_{2m+1}|Q_{2m} \cong S'_{2m} \oplus S''_{2m} \text{ and } S'_{2m+2}|Q_{2m+1} \cong S''_{2m+2}|Q_{2m+1} \cong S_{2m+1}.$$

If n is even we will often write S_n for any of the 2 spinor bundles S'_n and S''_n. Take $n = 3$. Fix a smooth conic $D \subset Q_3$. Hence there is a smooth hyperplane section $Q_2 \subset Q_3$ such that D is a plane section of Q_2. Since $D \cong \mathbf{P}^1$, S_3 is a direct sum of 2 lines bundles. Call a, b with $a \geq b$ their degree. We have $a + b = -2$. Since Q_3 is ACM, from (1) for $m = 1$ we get $h^0(D, S_3|D) = 0$ and $h^0(D, S_3(1)|D) = 4$. The first equality gives $a \leq -1$. Hence the second equality gives $a = b = -1$. Now fix any integer $n \geq 4$ and take any smooth quadric $D \subset Q_n$ such that the plane $\langle D\rangle$ is not contained in Q_n. Taking a general hyperplane section of Q_n containing D and using the formulas for $S_n|Q_{n-1}$ we get that $S_n|D$ is a direct sum of line bundles of degree -1. Hence for all integers $b > 0$ and t_i every rank 1 factor of $\oplus_{i=1}^{b} S_n(t_i)|D$ has odd degree.

Remark 4. Fix the integers $n \geq 3$, $r \geq 1$ and x and set $E := F^r_{Q_n *}(\mathcal{O}_{Q_n}(x))$. Proposition 1 gives that E is ACM. Hence its indecomposable factors are either line bundles or twists of a spinor bundles. If n is even and $S'_n(t)$ appears in a decomposition of E with multiplicity $b \geq 0$, then $S''_n(t)$ appears with the same multiplicity, because $f^*(E) \cong E$ for all $f \in \mathrm{Aut}(Q_n)$. Fix a smooth conic $D \subset Q_n$ such that the plane spanned by D is not contained in D. There is a chain $Q_1 := D \subset Q_2 \subset \cdots Q_{n-1} \subset Q_n$ in which each Q_i is a smooth hyperplane section of Q_{i+1}. As in Remark 2 taking $n-1$ standard exact sequences we get $\chi_{E|D} = \Delta^{(n-1)}(\chi_E)$. Remark 1 gives $\chi_E(t) = \chi(\mathcal{O}_{Q_n}(x + tp^r))$ for all $t \in \mathbb{Z}$. Thus $\chi_E(t) = \binom{n+1+x+tp^r}{n+1} - \binom{n-1+x+tp^r}{n+1}$ for all t such that $x + tp^r \geq 0$ (and for all t if we give the right definition of the binomial coefficients).

Proposition 2. *Fix integers $r \geq 1$ and $n \geq 3$. If n is odd, then $F^{r*}_{Q_n}(S_n)$ is not ACM. If n is even, then neither $F^{r*}_{Q_n}(S'_n)$ nor $F^{r*}_{Q_n}(S''_n)$ are ACM.*

Proof. First assume n odd, say $n = 2m + 1$, and that $F^{r*}_{Q_n}(S_n)$ is ACM. Hence either $F^{r*}_{Q_n}(S_n) \cong S_n(t)$ for some integer t or $F^{r*}_{Q_n}(S_n)$ is isomorphic a direct sum of line bundles. Fix any line $R \subset Q_n$. $F^{r*}_{Q_n}(S_n)$ is a direct sum of 2^{m-1} line bundles of degree 0 and 2^{m-1} line bundles of degree $-p^r$, while $S_n(t)|R$ is a direct sum of 2^{m-1} line bundles of degree t and 2^{m-1} line bundles of degree $t-1$. Hence $F^{r*}_{Q_n}(S_n) \ncong S_n(t)$ for any t. Restricting to R shows that if $F^{r*}_{Q_n}(S_n)$ is a direct sum of line bundles, then $F^{r*}_{Q_n}(S_n) \cong \mathcal{O}_{Q_n}^{\oplus 2^{m-1}} \oplus \mathcal{O}_{Q_n}(-p^r)^{\oplus 2^{m-1}}$. Take a smooth conic $D \subset Q_n$ such that Q_n does not contained the plane spanned by D. We saw in Remark 2 that $S_n|D$ is a direct sum of line bundles with the same degree. Hence $F^{r*}_{Q_n}(S_n)|D$ has the same property, contradiction. The proof for n even is similar and omitted. □

Remark 5. Fix an integer $n \geq 1$ and a general $P \in \mathbf{P}^{n+1} \backslash Q_n$. Let $u : Q_n \to \mathbf{P}^n$ be the degree 2 morphism induced by the linear projection from P. Since $p \neq 2$, u is separable and $u_*(\mathcal{O}_{Q_n}) \cong \mathcal{O}_{\mathbf{P}^n} \oplus \mathcal{O}_{\mathbf{P}^n}(-2)$. Since $\mathbf{P}^n$ and Q_n are smooth and u is finite, u is flat ([4], Example III.10.9). Hence $u_*(E)$ is locally free for every vector bundle E on Q_n. Since $u^*(\mathcal{O}_{\mathbf{P}^n}(t)) \cong \mathcal{O}_{Q_n}(t)$ for all $t \in \mathbb{Z}$, the projection formula gives $u_*(\mathcal{O}_{Q_n}(t)) \cong \mathcal{O}_{\mathbf{P}^n}(t) \oplus \mathcal{O}_{\mathbf{P}^n}(t-2)$ for all $t \in \mathbb{Z}$. Fix an integer x. We have $u \circ F^r_{Q_n} = F^r_{\mathbf{P}^n} \circ u$. Hence $u_*(F^r_{Q_n *}(\mathcal{O}_{Q_n}(x)) \cong F^r_{\mathbf{P}^n *}(\mathcal{O}_{\mathbf{P}^n}(x)) \oplus F^r_{\mathbf{P}^n *}(\mathcal{O}_{\mathbf{P}^n}(x-2))$. Hence $u_*(F^r_{Q_n *}(\mathcal{O}_{Q_n}(x))$ is a direct sum of line bundles (Remark 2). Now assume $n \geq 3$ and fix an integer t. We want to check that $u_*(S_n(t))$ (case n odd), $u_*(S'_n(t))$ and $S''_n(t)$ and isomorphic to a direct sum of $2^{\lceil n/2 \rceil + 1}$ line bundles. By Horrock's criterion or [3], Lemma 6.3, it is sufficient to show that $u_*(S_n(t))$ is ACM. Just use the proof of Proposition 1 using u instead of F^r_X. Now we will check that all rank 1 factors of $u_*(S_n(t))$, $u_*(S'_n(t))$ and $S''_n(t)$ have degree $t-1$. Since $u^*(\mathcal{O}_{\mathbf{P}^n}(t-1)) \cong \mathcal{O}_{Q_n}(t-1)$, the projection formula shows that it is sufficient to do the case $t = 1$. We will write S_n when n is even to denote any of the two spinor bundles. Since $h^0(Q_n, S_n) = 0$ and $h^0(Q_n, S_n(1)) = 2 \cdot \mathrm{rank}(S_n(t))$, then $h^0(\mathbf{P}^n, u_*(S_n(1)(-1)) = 0$ and $h^0(\mathbf{P}^n, u_*(S_n(1))) = 2 \cdot \mathrm{rank}(S_n(t))$. Since $u_*(S_n(1))$ is a direct sum of $2 \cdot \mathrm{rank}(S_n(t))$ line bundles, the last two equalities implies that $u_*(S_n(1))$ is trivial.

Remark 6. As in Remark 5 let $u : Q_n \to \mathbf{P}^n$, $n \geq 3$, denote the degree 2 morphism induced by a linear projection from a general point of $\mathbf{P}^{n+1}$. Let E be an ACM vector bundle on Q_n. Hence E is a direct sum of line bundles and of twists of spinor bundles (Remark 3). Remark 5 gives that $u_*(E)$ is a direct sum of line bundles. Since u is finite and $u^*(\mathcal{O}_{\mathbf{P}^n}(1)) \cong \mathcal{O}_{Q_n}(1)$, the Euler's function χ_E of E determines the Euler's function of $u_*(E)$. We saw in Remark 2 that the Euler's function of $u_*(E)$ uniquely determines the splitting type of $u_*(E)$.

Remark 7. Fix integers $n \geq 3$ and t. First assume n odd. Let z (resp. z') be the the minimal integer y such that $h^0(Q_n, S_n(t+y)) > 0$ (resp. $h^0(Q_n, \mathcal{O}_{Q_n}(t+y)) > 0$). Let w (resp. w') be the maximal integer y such that $h^n(Q_n, S_n(t+y)) > 0$ (resp. $h^n(Q_n, \mathcal{O}_{Q_n}(t+y)) > 0$). Since $\omega_{Q_n} \cong \mathcal{O}_{Q_n}(-n)$ and $S^*_n \cong S_n(1))$, $z = 1-t$, $z' = -t$, $w = -t-n$ and $w' = -t-n$. Hence $z - w = n+1$, while $z' - w' = n$. Notice that $h^0(Q_n, \mathcal{O}_{Q_n}(-n)) = 1$ and $h^0(Q_n, S_n(1)) = 2^{(n+1)/2}$. If n is odd, a similar computation is true, because $S'^*_n \cong S''_n$ and $S''^*_n \cong S'_n$.

Remark 8. Fix an odd integer $n \geq 3$ and an ACM vector bundle E on Q_n. Here we will show how the Hilbert function of E determines the indecomposable factors of E with there multiplicities. Let z be the minimal integer t such that $h^0(Q_n, E(t)) > 0$. Set $b := h^0(Q_n, E(z))$, $c := h^E(Q_n, E(z-n))$ and $a := (b-c)2^{-(n+1)/2}$. By Remark 7 a is a non-negative integer and $E \cong A \oplus E'$ with $A \cong \mathcal{O}_{Q_n}(-z) \oplus S_n(1-z)^{\oplus a}$. If $E' = 0$, then we are done. If $E' \neq 0$, we know the functions $h^0_{E'}$ and $h^n_{E'}$, because we know the functions h^0_A and h^n_A and $h^0_{E'} = h^0_E - h^n_A$. $h^0_{E'} = h^n_E - h^n_A$. Hence we may apply the same procedure to

E' and conclude by induction on the rank of E.

Remark 9. Fix an even integer $n \geq 4$. We may use the construction of Remark 8, but we cannot distinguish between $S'_n(t)$ and $S'_n(t)$ because $S'_n(t)$ and $S''_n(t)$ have the same cohomological properties, since $f^*(S'_n) \cong S''_n$ and $f^*(S''_n) \cong S'_n$ for all $f \in \mathrm{Aut}(Q_{2m+1} \backslash \mathrm{Aut}^0(Q_{2m+1})$.

Remark 10. Fix integers $n \geq 3$, $r \geq 1$, x and set $E := F^r_{Q_n *}(\mathcal{O}_{Q_n}(x))$. By Proposition 1 we may apply Remark 5 to the vector bundle E. If n is even, we have the additional property that in the decomposition of E for any t the vector bundles $S'_n(t)$ and $S''_n(t)$ appear with the same multiplicity.

Without doing any computation we may at least get the following result.

Corollary 1. *Fix an integer* ≥ 1. *There is an integer* x *such that* $0 \leq x < p^r$ *and* $F^r_{Q_n *}(\mathcal{O}_{Q_n}(x))$ *is not a direct sum of line bundles.*

Proof. Assume that the result is false. Since $F^r_{Q_n *}(\mathcal{O}_{Q_n}(x + cp^r)) \cong F^r_{Q_n *}(\mathcal{O}_{Q_n}(x))(c)$ for all $c \in \mathbb{Z}$, we get that $F^r_{Q_n *}(\mathcal{O}_{Q_n}(t))$ is a direct sum of line bundles for all $t \in \mathbb{Z}$. Proposition 2 says that if n is odd, then $F^{r*}_{Q_n}(S_n)$ is not ACM, while if n is even, then neither $F^{r*}_{Q_n}(S'_n)$ nor $F^{r*}_{Q_n}(S''_n)$ are ACM. If n is even we will write S_n instead of S'_n. Since $F^{r*}_{Q_n}(S_n)$ is not ACM, there are integers i, t such that $1 \leq i \leq n-1$ and $h^i(Q_n, F^{r*}_{Q_n}(S_n)(t)) > 0$. Since $F^r_{Q_n}$ is finite, $h^i(Q_n, F^{r*}_{Q_n}(S_n)(t)) = h^i(Q_n, F^r_{Q_n *}(F^{r*}_{Q_n}(S_n)(t)))$. We have $F^r_{Q_n *}(F^{r*}_{Q_n}(S_n)(t)) \cong S_n \otimes F^r_{Q_n *}(\mathcal{O}_{Q_n}(t))$ (projection formula). Since we assumed that $F^r_{Q_n *}(\mathcal{O}_{Q_n}(t))$ is a direct sum of line bundles and S_n is ACM, then $S_n \otimes F^r_{Q_n *}(\mathcal{O}_{Q_n}(t))$ is ACM. Thus $h^i(Q_n, S_n \otimes F^r_{Q_n *}(\mathcal{O}_{Q_n}(t))) = 0$, contradiction. □

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References

[1] R.-O. Buchweitz, D. Eisenbud, J. Herzog, Cohen-Macaulay modules on quadrics, Singularities, Representation of Algebras, and Vector bundles, In: *Proceedings, Lambrecht, 1985*, 58-95, Lecture Notes in Math., **1273**, Springer, Berlin (1987).

[2] L. Ein, Stable vector bundles on projective spaces in char $p > 0$, *Math. Ann.*, **254**, No. 1 (1980), 53-72.

[3] R. Hartshorne, *Ample Subvarieties of Algebraic Varieties*, Lect. Notes in Math., **156**, Springer, Berlin (1970).

[4] R. Hartshorne, *Algebraic Geometry*, Springer, Berlin (1977).

[5] H. Knörrer, Cohen-Macaulay modules on hypersurface singularities I, *Invent. Math.*, **88**, No. 2 (1987), 153-164.

[6] G. Ottaviani, Spinor bundles on quadrics, *Trans. Amer. Math. Soc.*, **307**, No. 1 (1988), 301-336.

[7] G. Ottaviani, Some extensions of Horrocks criterion to vector bundles on Grassmannians and quadrics, *Annali Mat. Pura Appl., IV*, **155** (1989), 317-341.