International Journal of Pure and Applied Mathematics

Volume 46 No. 4 2008, 415-433

THE CROSS-DIFFUSION DRIVEN INSTABILITY IN A SPRUCE BUDWORM-APHID INTERACTION MODEL

Shihua Zhu[1], Zhigui Lin[2 §], Jiahong Liu[3]

[1,2,3]School of Mathematical Science
Yangzhou University
Yangzhou, 225002, P.R. CHINA
[2]e-mail: zglin68@hotmail.com

Abstract: In this paper the instability of the uniform equilibrium of a spruce budworm-aphid interaction model is discussed. In bounded domain and unbounded domain, the sufficient conditions for the instability are obtained respectively. We show that under certain conditions cross-diffusion can induce the instability of the uniform equilibrium, which is stable for the kinetic system and for the self-diffusion reaction system, on the other hand, cross-diffusion can also stabilize the uniform equilibrium, which is stable for the kinetic system but unstable for the self-diffusion reaction system. Numerical simulations are presented to illustrate the main results.

AMS Subject Classification: 35K57, 35E99, 92D40
Key Words: instability, self-diffusion, cross-diffusion

1. Introduction

In 1978, Ludwig et al proposed a practical model for the spruce budworm in Canada and considered in [7] the spruce budworm population dynamics to be governed by the equation

$$\frac{dN}{dt} = Nr(1 - \frac{N}{k}) - p(N). \tag{1.1}$$

Here N is the population of the spruce budworm, r is the linear birth rate of the budworm, k is the carry capacity which is related to the density of foliage

Received: August 1, 2008

§Correspondence author

available on the trees, the $p(N)$-term represents predation generally by birds. Predation usually saturates for large enough N. For small population densities N, the birds tend to seek food elsewhere and so the predation term $p(N)$ drops more rapidly as $N \to 0$, than a linear rate proportional to N. To be specific, $p(N) = \frac{AN^2}{N^2+B^2}$, where A, B both are positive constants, and the dynamics of $N(t)$ is then governed by

$$\frac{dN}{dt} = N[r(1-\frac{N}{k}) - \frac{AN}{N^2+B^2}. \tag{1.2}$$

They showed that the model (1.2) has three uniform equilibriums and studied the stability of these equilibriums. After some years, Murray [8] extended the equation (1.2) to the following system

$$\begin{cases} \frac{dN}{dt} = N[r(1-\frac{N}{k}) - P\frac{AN}{N^2+B^2}], \\ \frac{dP}{dt} = Ps(1-\frac{hP}{N}), \end{cases} \tag{1.3}$$

where N, r, k are defined as above, P is the population of the budworm-aphid and A, B, h are all positive constants. Wollking [15] studied how the positive parameters r, s, h, A, B affect the dynamic characteristic of the system (1.3). When the injurious insect is too much, a effective way is to use the insecticide. Li, Jing and Fu [5] modified the system

$$\begin{cases} \frac{dN}{dt} = N[r(1-\frac{N}{k}) - P\frac{AN}{N^2+B^2}] - N_0, \\ \frac{dP}{dt} = Ps(1-\frac{hP}{N}), \end{cases} \tag{1.4}$$

where N_0 is the linear death rate by using the insecticide. Here we introduce dimensionless variables:

$$u = \frac{N}{k}, \quad v = \frac{hP}{k}, \quad \tau = rt, \quad a = \frac{A}{rh}, \quad b = \frac{s}{r}, \quad d = \frac{B}{k}, \quad F = \frac{N_0}{rk},$$

which on substituting into (1.4) becomes

$$\begin{cases} \frac{du}{d\tau} = u[(1-u) - \frac{auv}{u^2+d^2}] - F := f(u,v), \\ \frac{dv}{d\tau} = bv(1-\frac{v}{u}) := g(u,v). \end{cases} \tag{1.5}$$

Li, Jing and Fu have showed how the F size affect the dynamic characteristic of the systems (1.5), including the number of the positive constant equilibrium solutions, stability and bifurcations as well.

However, a large amount of research has been devoted to the study of symmetry breaking instabilities leading to steady-state solutions in models for chemical, physical and biological pattern formation employing reaction-diffusion systems [8]. One of the most intensively studied of such models is that of Alan. M. Turing, who was one of the greatest scientists in the 20-th century. In his paper " The Chemical basis of morphogenesis" [12], he suggested that under certain conditions, a constant equilibrium solution can be asymptotically stable with the kinetic equation, but it is unstable with its corresponding reaction-diffusion system which only have self-diffusion coefficients. That interesting phenomenon is called *diffusion driven instability* or *Turing instability*. Over the years, Turing's ideas have attracted the attention of a great number of investigators and successfully developed on the theoretical backgrounds [14, 9, 2].

In recent years, more attention has been given to study the effect of cross-diffusion in reaction-diffusion systems. For example, Kareiva and Odell [4] proposed cross-diffusion model for predator-prey interaction. Farkas [3] studied the cross-diffusion effect on the stabilities of capital and labor force in a closed economy model. Abdusalam and Fahmy [1] investigated the cross-diffusion effect in a telegraph reaction diffusion Lotka-Volterra tow competitive system, and Shi [10] also studied a more general cross-diffusion model. In these models, it was noticed that when the cross-diffusion is nonlinear the difficulties in mathematical analysis are increased. Some of these self-organized patterns have been attributed to the cross-diffusion and advection in the systems. Under certain conditions, cross-diffusion can change the stability of a constant equilibrium solution whether it is stable with the corresponding self-diffusion reaction system or not, which is also *diffusion driven instability,* but is not classical *Turing instability*.

In this paper, we consider the reaction-diffusion system

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta[(d_1 + \alpha_{11}u + \alpha_{12}v)u] + \lambda f(u,v), \\ \frac{\partial v}{\partial t} = \Delta[(d_2 + \frac{\alpha_{21}}{\gamma+u} + \alpha_{22}v)v] + \lambda g(u,v), \\ \frac{\partial u}{\partial \nu}(x,t) = \frac{\partial v}{\partial \nu}(x,t) = 0, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), \end{cases} \tag{1.6}$$

where $f(u,v)$ and $g(u,v)$ are defined in the system (1.5), the initial values $\beta_1(x)$ and $\beta_2(x)$ are non-negative smooth functions which are not identically zero, λ is a positive parameter and d_1, d_2, γ are positive constants except for α_{ij} $(i,j=1,2)$ which maybe nonnegative constants. The diffusion terms can be written as

$$\operatorname{div}\{(d_1 + 2\alpha_{11}u + \alpha_{12}v)\nabla u + \alpha_{12}u\nabla v\},$$

$$\operatorname{div}\{\frac{-\alpha_{21}v}{(\gamma+u)^2}\nabla u + (d_2 + \frac{\alpha_{21}}{\gamma+u} + 2\alpha_{22}v)\nabla v\}.$$

The terms

$$d_1 + 2\alpha_{11}u + \alpha_{12}v, \quad d_2 + \frac{\alpha_{21}}{\gamma+u} + 2\alpha_{22}v$$

represent the "self-diffusion" and the terms

$$\alpha_{12}u, \quad \frac{-\alpha_{21}v}{(\gamma+u)^2}$$

represent the "cross-diffusion". Here $\alpha_{12}u > 0$, $\frac{-\alpha_{21}v}{(\gamma+u)^2} < 0$ imply that the flux of u and v in x-direction are toward decreasing population of v and toward increasing population of u respectively. The above model means that, in addition to the dispersive force, the diffusion also depends on population pressure from other species. If $\alpha_{12} = 0$ and $\alpha_{21} = 0$, the cross-diffusion is absent in the above system. In the case when $\alpha_{ij} = 0$ for $i, j = 1, 2$, the system (1.6) is the classic predator-prey model.

The purpose of this article is to explore cross-diffusion-induced instability for the reaction-diffusion systems (1.6). Section 2 deals with the case on the bounded domain Ω in $\mathbb{R}^n$ $(n \geq 1)$ with no-flux boundary condition using the linearizing method similarly as in [6] and [13]. The instability in the whole space $\mathbb{R}^n$ is discussed in Section 3. Section 4 is devoted to the instability on a bounded spatial domain $(0, \pi)$ with no-flux boundary condition, and a more detailed conclusion about the impact of the cross-diffusion is given. Finally numerical simulations are presented in Section 5 to illustrate our results and some discussions are present in the last section.

2. Cross-Diffusion Systems in Bounded Domain

In this sections, we discuss the instability of the system (1.6) on the bounded domain Ω in $\mathbb{R}^n$ with smooth boundary. Assume that there is no-flux on the boundary, then we have the homogeneous Neumann boundary condition and the problem becomes

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta[(d_1 + \alpha_{11}u + \alpha_{12}v)u] + \lambda f(u,v), & t > 0,\ x \in \Omega, \\ \frac{\partial v}{\partial t} = \Delta[(d_2 + \frac{\alpha_{21}}{\gamma+u} + \alpha_{22}v)v] + \lambda g(u,v), & t > 0,\ x \in \Omega, \\ \frac{\partial u}{\partial \nu}(x,t) = \frac{\partial v}{\partial \nu}(x,t) = 0, & t > 0,\ x \in \partial\Omega, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), & x \in \overline{\Omega}, \end{cases} \tag{2.1}$$

where ν is the outward unit normal vector on $\partial\Omega$. The corresponding system of ordinary differential equations of the system (2.1) is

$$\begin{cases} \frac{du}{dt} = \lambda f(u,v) = \lambda\{u[(1-u) - \frac{auv}{u^2+d^2}] - F\}, & t > 0, \\ \frac{dv}{dt} = \lambda g(u,v) = \lambda bv(1 - \frac{v}{u}), & t > 0, \\ u(0) = \beta_1(0),\ v(0) = \beta_2(0). \end{cases} \tag{2.2}$$

Form [5], we know that the system (2.2) admits some positive constant equilibrium solutions when $0 < F < \frac{1}{4}$. Suppose (u_0, v_0) is a positive constant equilibrium solution, i.e.

$$f(u_0, v_0) = 0, \quad \text{and} \quad g(u_0, v_0) = 0.$$

Clearly (u_0, v_0) is also an equilibrium solution of the system (2.1) and $u_0 = v_0$. In the following, we discuss the stability of (u_0, v_0) about the systems (2.1) and (2.2). One isocline is given by:

$$l\text{:} \quad v = \frac{1}{a}[-u^2 + u - (F + d^2) + \frac{d^2}{u} - \frac{d^2 F}{u^2}] = v(u).$$

Let $\omega = \frac{dv}{du}(u_0, v_0)$, we obtain

$$\frac{2d^2 F}{{u_0}^3} = a\omega - (1 - 2u_0 - \frac{d^2}{{u_0}^2}) \tag{2.3}$$

and the Jacobian of the system (2.2) at (u_0, v_0) is λJ, where

$$J = \begin{pmatrix} f_u & f_v \\ g_u & g_v \end{pmatrix} = \begin{pmatrix} a\omega\frac{{u_0}^2}{{u_0}^2+d^2} & -\frac{a{u_0}^2}{{u_0}^2+d^2} \\ b & -b \end{pmatrix}. \tag{2.4}$$

Thus we have

$$\text{Trace}\,(J) = a\omega\frac{{u_0}^2}{{u_0}^2 + d^2} - b = \frac{{u_0}^2}{{u_0}^2 + d^2}(a\omega - b - \frac{bd^2}{{u_0}^2}), \tag{2.5}$$

$$\text{Det}\,(J) = -ba\omega\frac{{u_0}^2}{{u_0}^2 + d^2} + b\frac{a{u_0}^2}{{u_0}^2 + d^2} = \frac{(1-\omega)ab{u_0}^2}{{u_0}^2 + d^2}. \tag{2.6}$$

Now, we look for the conditions, under which the positive constant equilibrium solution (u_0, v_0) is stable for the ODE system (2.2) and is unstable for the PDE system (2.1). We always assume that (u_0, v_0) is linearly stable with respect to the system (2.2), then we have the following result according to [5]:

Lemma 2.1. *Assume that $0 < F < \frac{1}{4}$. If*

$$0 < \omega < 1, \quad h(\omega) < 0, \tag{2.7}$$

or $\omega \le 0$, where $h(\omega) = a\omega - b - \frac{bd^2}{{u_0}^2}$ and the matrix J and ω are defined as above, then the positive constant equilibrium solution (u_0, v_0) is a stable equilibrium solution with respect to (2.2).

In the following, we shall discuss that the positive constant equilibrium solution (u_0, v_0) with respect to (2.1) is unstable, basing on the conditions of Lemma 2.1. To make the writing clearly, let

$$\begin{cases} w(u,v) = (d_1 + \alpha_{11}u + \alpha_{12}v)u, \\ z(u,v) = (d_2 + \frac{\alpha_{21}}{\gamma+u} + \alpha_{22}v)v, \end{cases} \tag{2.8}$$

then the system (2.1) becomes the following system

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta w(u,v) + \lambda f(u,v), & x \in \Omega,\, t>0, \\ \frac{\partial v}{\partial t} = \Delta z(u,v) + \lambda g(u,v), & x \in \Omega,\, t>0, \\ w(u,v) = (d_1 + \alpha_{11} u + \alpha_{12} v)u, & x \in \Omega,\, t>0, \\ z(u,v) = (d_2 + \frac{\alpha_{21}}{\gamma+u} + \alpha_{22} v)v, & x \in \Omega,\, t>0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = \frac{\partial z}{\partial \nu} = 0, & x \in \partial\Omega,\, t>0, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), & x \in \overline{\Omega}. \end{cases} \tag{2.9}$$

It is easy to see that the instability of (u_0, v_0) in the system (2.1) is equivalent to that of (u_0, v_0, w_0, z_0) in the system (2.9), where

$$w_0 = w(u_0, v_0), \quad \text{and} \quad z_0 = z(u_0, v_0).$$

First we linearize the reaction-diffusion system (2.9) about the constant equilibrium solution (u_0, v_0, w_0, z_0), let

$$U(x,t) = u - u_0, \quad V(x,t) = v - v_0, \quad W(x,t) = w - w_0, \quad Z(x,t) = z - z_0$$

be a spatially perturbation at the solution (u_0, v_0, w_0, z_0). For our convenience, we still denote $U(x,t), V(x,t), W(x,t), Z(x,t)$ by $u(x,t), v(x,t), w(x,t), z(x,t)$ respectively, then linearizing the reaction-diffusion system (2.8) at (u_0, v_0, w_0, z_0) gives

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta w + \lambda f_u u + \lambda f_v v, & x \in \Omega,\, t>0, \\ \frac{\partial v}{\partial t} = \Delta z + \lambda g_u u + \lambda g_v v, & x \in \Omega,\, t>0, \\ w = w_u u + w_v v, & x \in \Omega,\, t>0, \\ z = z_u u + z_v v, & x \in \Omega,\, t>0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = \frac{\partial z}{\partial \nu} = 0, & x \in \partial\Omega,\, t>0, \\ u(x,0) = {\beta_1}'(x),\ v(x,0) = {\beta_2}'(x), & x \in \overline{\Omega}. \end{cases} \tag{2.10}$$

Now, we want to look for conditions under which the solution $(0,0,0,0)$ of the linear system (2.10), resp. the solution (u_0, v_0) of the nonlinear system (2.1) loses its stability. In fact the stability of (2.9) depends on the asymptotic property of the perturbation which satisfies (2.10). We also denote the Jacobian of w, z with respect u, v by

$$D = \begin{pmatrix} w_u & w_v \\ z_u & z_v \end{pmatrix} = \begin{pmatrix} d_1 + 2\alpha_{11} u_0 + \alpha_{12} u_0 & \alpha_{12} u_0 \\ \frac{-\alpha_{21}}{(\gamma+u_0)^2} u_0 & d_2 + 2\alpha_{22} u_0 + \frac{\alpha_{21}}{\gamma+u_0} \end{pmatrix}. \tag{2.11}$$

It is easy to find that Trace (D) and Det(D) are always positive, where

$$\text{Trace}\,(D) = (d_1 + 2\alpha_{11} u_0) + (d_2 + 2\alpha_{22} u_0) + \alpha_{12} u_0 + \frac{\alpha_{21}}{\gamma + u_0}, \tag{2.12}$$

$$\begin{aligned} \text{Det(D)} = {} & (d_1 + 2\alpha_{11} u_0)(d_2 + 2\alpha_{22} u_0) + \alpha_{12} u_0 (d_2 + 2\alpha_{22} u_0) \\ & + \frac{\alpha_{21}(d_1 + 2\alpha_{11} u_0 + \alpha_{12} u_0)}{\gamma + u_0} + \frac{\alpha_{12}\alpha_{21} {u_0}^2}{(\gamma + u_0)^2}. \end{aligned} \tag{2.13}$$

Considering the instability of (2.10), let $0 = \mu_0 < \mu_1 < \mu_2 < \cdots\cdots$ be the eigenvalues of the operator $-\Delta$ on the bounded domain Ω with the no-flux boundary condition, and the linearization of (2.10) is equivalent to

$$\begin{cases} \frac{\partial u}{\partial t} + \mu_i w = \lambda f_u u + \lambda f_v v, & x \in \Omega,\ t > 0, \\ \frac{\partial v}{\partial t} + \mu_i z = \lambda g_u u + \lambda g_v v, & x \in \Omega,\ t > 0, \\ w = w_u u + w_v v, & x \in \Omega,\ t > 0, \\ z = z_u u + z_v v, & x \in \Omega,\ t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = \frac{\partial z}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = {\beta_1}'(x),\ v(x,0) = {\beta_2}'(x), & x \in \overline{\Omega}. \end{cases} \tag{2.14}$$

To examine the linear stability of $(0,0,0,0)$ in the system (2.14), let

$$(u, v, w, z)^T = (c_1, c_2, c_3, c_4)^T \exp(\delta t) X_i(x),$$

where $\delta \in \mathbb{C}$ and $X_i(x)$ is the corresponding eigenfunction of μ_i. Substituting $(u, v, w, z)^T$ by $(c_1, c_2, c_3, c_4)^T \exp(\delta t) X_i(x)$ in to (2.14) yields

$$\begin{cases} (\delta c_1 + \mu_i c_3) \exp(\delta t) X_i(x) = \lambda(f_u c_1 + f_v c_2) \exp(\delta t) X_i(x), \\ (\delta c_2 + \mu_i c_4) \exp(\delta t) X_i(x) = \lambda(g_u c_1 + g_v c_2) \exp(\delta t) X_i(x), \\ c_3 \exp(\delta t) X_i(x) = (w_u c_1 + w_v c_2) \exp(\delta t) X_i(x), \\ c_4 \exp(\delta t) X_i(x) = (z_u c_1 + z_v c_2) \exp(\delta t) X_i(x)\,, \end{cases} \tag{2.15}$$

for $t > 0$ and $x \in \Omega$. Since $\exp(\delta t) X_i(x) \neq 0$ for any $t > 0$ and $X_i(x)$, (2.15) is equivalent to the following linear algebraical equations

$$\begin{pmatrix} \delta - \lambda f_u & -\lambda f_v & \mu_i & 0 \\ -\lambda g_u & \delta - \lambda g_v & 0 & \mu_i \\ -w_u & -w_v & 1 & 0 \\ -z_u & -z_v & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{2.16}$$

Nontrivial solutions to (2.16) exists if

$$\mathrm{Det} \begin{pmatrix} \delta - \lambda f_u & -\lambda f_v & \mu_i & 0 \\ -\lambda g_u & \delta - \lambda g_v & 0 & \mu_i \\ -w_u & -w_v & 1 & 0 \\ -z_u & -z_v & 0 & 1 \end{pmatrix} = 0. \tag{2.17}$$

A direct calculation shows

$$\mathrm{Det} \begin{pmatrix} \delta - \lambda f_u + \mu_i w_u & -\lambda f_v + \mu_i w_v \\ -\lambda g_u + \mu_i z_u & \delta - \lambda g_v + \mu_i z_v \end{pmatrix} = 0. \tag{2.18}$$

We define the matrix M_i by the following

$$M_i = \begin{pmatrix} \lambda f_u - \mu_i w_u & \lambda f_v - \mu_i w_v \\ \lambda g_u - \mu_i z_u & \lambda g_v - \mu_i z_v \end{pmatrix} = \lambda J - \mu_i D, \tag{2.19}$$

where the matrix J and D are defined in (2.3) and (2.11). If

$$(u, v, w, z)^T = (c_1, c_2, c_3, c_4)^T \exp(\delta t) X_i(x),$$

would go to infinity as $t \to +\infty$ for some $\mu_i > 0$, then (2.9) is a linear unstable system, so we decide the sign of δ which satisfies (2.18). In fact, we can see those δ are the eigenvalues of the matrix M_i. According to the properties for the eigenvalues of the matrix, we only need discuss the signs of its trace and determinant of M_i:

$$\mathrm{Trace}\,(M_i) = \lambda \mathrm{Trace}\,(J) - \mu_i \mathrm{Trace}\,(D), \tag{2.20}$$

$$\mathrm{Det}(M_i) = {\mu_i}^2 \mathrm{Det(D)} + \mu_i \lambda F(J,D) + \lambda^2 \mathrm{Det(J)}, \tag{2.21}$$

where

$$F(J,D) = -\mathrm{Det}\begin{pmatrix} f_u & f_v \\ z_u & z_v \end{pmatrix} - \mathrm{Det}\begin{pmatrix} w_u & w_v \\ g_u & g_v \end{pmatrix}.$$

Trace(J), Det(J), Trace(D) and Det(D) are defined in (2.5), (2.6), (2.12) and (2.13) respectively. Since that Trace(J) < 0, Trace(D) > 0 and $\lambda > 0$, then Trace$(M_i) < 0$ is always true. Hence if M_i has an eigenvalue with positive real part, then it must be a real value one and the other eigenvalue must be a negative real one. A necessary condition for the instability of (2.9) is $F(J,D) < 0$, i.e.

$$a(\omega z_v + z_u) - b(w_u + w_v)(1 + \frac{d^2}{{u_0}^2}) > 0, \tag{2.22}$$

otherwise Det$(M_i) > 0$ for all $\mu_i > 0$, since Det(D) > 0 and Det(J) > 0. According to the matrix D, if (2.22) is true, we must have $\omega > 0$. Thus in the following, we discuss the problem based on the condition (2.7). For instability we must have Det$(M_i) < 0$ for some $\mu_i > 0$, and we notice that Det(M_i) achieves its minimum

$$\min_{\mu_i \in \mathbb{R}^+} \mathrm{Det}(M_i) = [\mathrm{Det}(J) - \frac{F^2(J,D)}{4\mathrm{Det}(D)}]\lambda^2, \tag{2.23}$$

at the critical point $\mu_* > 0$, where

$$\mu_* = -\frac{F(J,D)\lambda}{2\mathrm{Det}(D)}. \tag{2.24}$$

However, the condition (2.22) holds and the inequality $\min_{\mu_i \in \mathbb{R}^+} \mathrm{Det}(M_i) < 0$, which imply that Det$(M_i) = 0$ has two positive solutions for μ_i, are necessary, but not sufficient for instability in the bounded domain Ω. Because it cannot guarantee there is μ_i such that Det$(M_i) < 0$. Now we discuss Det$(M_i) = 0$

directly. Let $0 < k_1 < k_2$ be the zeros of $\mathrm{Det}(M_i) = 0$, i.e.

$$\begin{aligned} k_1 &= \tfrac{-F(J,D)-\sqrt{F^2(J,D)-4\mathrm{Det}(D)\mathrm{Det}(J))}}{2\mathrm{Det}(D)}\lambda \\ &= \tfrac{\lambda {u_0}^2}{2\mathrm{Det}(D)({u_0}^2+d^2)}[H(\omega) - \sqrt{H^2(\omega) - 4ab(1-\omega)(1+\tfrac{d^2}{{u_0}^2})\mathrm{Det}(D)}] \\ &< \mu_* < k_2 = \tfrac{-F(J,D)+\sqrt{F^2(J,D)-4Det(D)\mathrm{Det}(J))}}{2\mathrm{Det}(D)}\lambda \\ &= \tfrac{\lambda {u_0}^2}{2\mathrm{Det}(D)({u_0}^2+d^2)}[H(\omega) + \sqrt{H^2(\omega) - 4ab(1-\omega)(1+\tfrac{d^2}{{u_0}^2})\mathrm{Det}(D)}], \end{aligned} \tag{2.25}$$

where

$$H(\omega) = a(\omega z_v + z_u) - b(w_u + w_v)(1 + \frac{d^2}{{u_0}^2}). \tag{2.26}$$

When $0 < k_1 < \mu_i < k_2$ for some $i \in \mathbb{N}^+$, the matrix $\lambda J - \mu_i D$ has an eigenvalue which is positive for these i. Summarizing the above calculation yields

Theorem 2.2. *Suppose that (2.7) holds, (u_0, v_0) is a positive constant equilibrium solution of (2.1) and (2.2) and μ_i, the matrixes J, D are defined as above. If $\min\limits_{\mu_i \in \mathbb{R}^+} \mathrm{Det}(M_i) < 0$ and $H(\omega) > 0$ are satisfied, and*

$$0 < k_1 \le \mu_i \le k_2, \tag{2.27}$$

for some μ_i, where k_1 and k_2 are defined by (2.25), then (u_0, v_0) is an unstable equilibrium solution with respect to the reaction-diffusion system (2.1), but a stable equilibrium solution with respect to the ordinary differential equation system (2.2).

Corollary 2.3. *Suppose that (2.7) holds, (u_0, v_0) is a positive constant equilibrium solution of (2.1) and (2.2), if $\min\limits_{\mu_i \in \mathbb{R}^+} \mathrm{Det}(M_i) > 0$ or $H(\omega) \le 0$, then (u_0, v_0) is also a stable equilibrium solution with respect to the system (2.1).*

Remark 2.1. We call some properties for $\mu_i (i \in N)$ which is the eigenvalue of $-\Delta$ in the bounded domain Ω with no-flux boundary condition. μ_i are discrete. $\mu_0 = 0$ and $\mu \to +\infty$ when $i \to +\infty$. If the domain Ω is changed, the corresponding eigenvalues are changed continuously. From the condition (2.25), we can see that the instability of (2.1) depends on the size of domain Ω. So when the size of Ω tends to ∞, the eigenvalues $\{\mu_i\}_{i=0}^{+\infty}$ tend to density which implies that the relation (2.25) is naturally satisfied. On the other hand, when the size of Ω is sufficiently small, the eigenvalues $\{\mu_i\}_{i=0}^{+\infty}$ are more discrete so that the relation (2.25) is naturally unsatisfied, in other words the phenomenon in Theorem 2.2 does not appear. This just verify the well-known fact, which was first stated by Shigesada et al in [11].

Remark 2.2. From the condition (2.25), we can find that the size of the positive parameter λ has effect on the unstable condition and has not effect on the stable condition from Corollary 2.3.

Remark 2.3. Theorem 2.2 gives a general criterion for the instability when self-diffusion and/or cross-diffusion is introduced into the system (2.1). When the cross-diffusion is absent in the system (2.1), i.e. $\alpha_{12} = \alpha_{21} = 0$, Theorem 2.2 is a result about self-diffusion driven instability. When $\alpha_{ij} = 0$ for $i, j = 1, 2$, Theorem 2.2 is a result about classical *Turing instability*.

3. Cross-Diffusion Systems in $\mathbb{R}^n$

In this section, we discuss the system (1.6) in the whole space $\mathbb{R}^n$:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta w(u,v) + \lambda f(u,v), & t > 0,\ x \in \mathbb{R}^n, \\ \frac{\partial v}{\partial t} = \Delta z(u,v) + \lambda g(u,v), & t > 0,\ x \in \mathbb{R}^n, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), & x \in \mathbb{R}^n, \end{cases} \tag{3.1}$$

where the function $w(u,v)$, $z(u,v)$, $f(u,v)$ and $g(u,v)$ are defined in the system (1.5) and (2.8). Suppose that (u_0, v_0) is a constant equilibrium of the corresponding kinetic equation of (3.1) which is given by the system (2.2). The sufficient condition for (u_0, v_0) being linearly stable with respect to the system (2.2) is the condition (2.7). As in Section 2, the system (3.1) becomes the following system with four equations:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta w(u,v) + \lambda f(u,v), \\ \frac{\partial v}{\partial t} = \Delta z(u,v) + \lambda g(u,v), \\ w(u,v) = (d_1 + \alpha_{11} u + \alpha_{12} v) u \\ z(u,v) = (d_2 + \frac{\alpha_{21}}{\gamma + u} + \alpha_{22} v) v, \end{cases} \tag{3.2}$$

for $t > 0$ and $x \in \mathbb{R}^n$. It is easy to see that the instability of (3.1) at (u_0, v_0) is equivalent to that of (3.2) at (u_0, v_0, w_0, z_0), where $w_0 = w(u_0, v_0)$, $z_0 = z(u_0, v_0)$. The linearization of (3.2) at (u_0, v_0, w_0, z_0) is

$$\begin{cases} \frac{\partial u}{\partial t} + \Delta w = \lambda f_u u + \lambda f_v v, \\ \frac{\partial v}{\partial t} + \Delta z = \lambda g_u u + \lambda g_v v, \\ w = w_u u + w_v v, \\ z = z_u u + z_v v \end{cases} \tag{3.3}$$

for $t > 0$ and $x \in \mathbb{R}^n$.

Now we want to look for condition under which the solution $(0, 0, 0, 0)$ of the linear system (3.3), resp. the solution (u_0, v_0) of the nonlinear system (3.1) loses its stability. To examine the linear stability of (u_0, v_0, w_0, z_0), let

$$(u, v, w, z)^T = (c_1, c_2, c_3, c_4)^T \exp(\delta t + i\mathbf{k}x),$$

where $\lambda \in \mathbb{R}$ and $\mathbf{k} = (k_1, k_2, \cdots k_n),\ k_i \geq 0,\ |\mathbf{k}| > 0$. Substituting $(u, v, w, z)^T$ by $(c_1, c_2, c_3, c_4)^T \exp(\delta t + i\mathbf{k}x)$ in to (3.3), then nontrivial solutions to (3.3) are possible provide

$$\text{Det}\begin{pmatrix} \delta - \lambda f_u & -\lambda f_v & |\mathbf{k}|^2 & 0 \\ -\lambda g_u & \delta - \lambda g_v & 0 & |\mathbf{k}|^2 \\ -w_u & -w_v & 1 & 0 \\ -z_u & -z_v & 0 & 1 \end{pmatrix} = 0, \tag{3.4}$$

where $|\mathbf{k}|^2 = \sum_{i=1}^{n} {k_i}^2$. A direct calculation shows

$$\text{Det}\begin{pmatrix} \delta - \lambda f_u + |\mathbf{k}|^2 w_u & -\lambda f_v + |\mathbf{k}|^2 w_v \\ -\lambda g_u + |\mathbf{k}|^2 z_u & \delta - \lambda g_v + |\mathbf{k}|^2 z_v \end{pmatrix} = 0. \tag{3.5}$$

We define the matrix M_i by the following

$$M_i = \begin{pmatrix} \lambda f_u - |\mathbf{k}|^2 w_u & \lambda f_v - |\mathbf{k}|^2 w_v \\ \lambda g_u - |\mathbf{k}|^2 z_u & \lambda g_v - |\mathbf{k}|^2 z_v \end{pmatrix} = \lambda J - |\mathbf{k}|^2 D, \tag{3.6}$$

where the matrix J and D are defined in (2.3) and (2.11). As in Section 2, M_i has at least an eigenvalue with positive real part if $\text{Det}(M_i) < 0$ for some $|\mathbf{k}| > 0$, where

$$\text{Det}(M_i) = |\mathbf{k}|^4 \text{Det(D)} + |\mathbf{k}|^2 \lambda F(J, D) + \lambda^2 \text{Det(J)}, \tag{3.7}$$

which achieve its minimum

$$\min_{|\mathbf{k}| \in \mathbb{R}^+} \text{Det}(M_i) = [\text{Det}(J) - \frac{F^2(J, D)}{4\text{Det}(D)}]\lambda^2, \tag{3.8}$$

at the critical value $|\mathbf{k}_*| > 0$, where

$$|\mathbf{k}_*|^2 = -\frac{F(J, D)\lambda}{2\text{Det}(D)}. \tag{3.9}$$

If $F(J, D) < 0$ holds and $\min_{|\mathbf{k}| \in \mathbb{R}^+} \text{Det}(M_i) < 0$, then (u_0, v_0, w_0, z_0) is an unstable equilibrium with respect to the problem (3.3). A direct calculation shows that

$$F(J, D) = -\frac{{u_0}^2}{{u_0}^2 + d^2} H(\omega),$$

$$\min_{|\mathbf{k}| \in \mathbb{R}^+} \text{Det}(M_i) = \frac{\lambda^2 {u_0}^4}{4\text{Det}(D)({u_0}^2 + d^2)^2}[H^2(\omega) - 4ab(1 - \omega)\text{Det}(D)(1 + \frac{d^2}{{u_0}^2})],$$

where $H(\omega)$ defined as (2.26). Therefore we have

Theorem 3.1. *Suppose that (2.7) holds, (u_0, v_0) is a positive constant equilibrium solution of (3.1) and (2.2), and the matrixes J, D are defined as*

(2.4) and (2.8). If

$$0 < H(\omega) < 2\sqrt{ab(1-\omega)\mathrm{Det}(D)(1+\frac{d^2}{{u_0}^2})} \quad \text{and} \quad |\boldsymbol{k}_*|^2 = \frac{{u_0}^2 H(\omega)}{({u_0}^2+d^2)2\mathrm{Det}(D)}\lambda$$

are satisfied, then (u_0, v_0) *is an unstable equilibrium solution with respect to the reaction-diffusion system (3.1), but a stable equilibrium solution with respect to the ordinary differential equation system (2.2). Furthermore, if*

$$H(\omega) > 2\sqrt{ab(1-\omega)\mathrm{Det}(D)(1+\frac{d^2}{{u_0}^2})} \quad \text{or} \quad H(\omega) \le 0,$$

then (u_0, v_0) *is a stable equilibrium solution with respect to the reaction-diffusion system (3.1).*

4. Cross-Diffusion Systems in an Interval $(0, \pi)$

The analysis in Section 2 can be applied to the situation, where the bounded domain is Ω, but we do not know the size of μ_i which is the eigenvalue of $-\Delta$ in the bounded domain Ω with no-flux boundary condition on the boundary $\partial\Omega$. Here we focus our problem on an interval $(0, \pi)$ and consider the following system

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta w(u,v) + \lambda f(u,v), & 0 < x < \pi,\ t > 0, \\ \frac{\partial v}{\partial t} = \Delta z(u,v) + \lambda g(u,v), & 0 < x < \pi,\ t > 0, \\ \frac{\partial u}{\partial \nu}(t,0) = \frac{\partial v}{\partial \nu}(t,\pi) = 0, & t > 0, \\ \frac{\partial w}{\partial \nu}(t,0) = \frac{\partial z}{\partial \nu}(t,\pi) = 0, & t > 0, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), & 0 \le x \le \pi, \end{cases} \tag{4.1}$$

where $f(u,v)$, $g(u,v)$, $w(u,v)$ and $z(u,v)$ are defined in the system (1.5) and (2.8). $\beta_1(x)$ and $\beta_2(x)$ are the same as in the system (1.6). Again we discuss (4.1) as above and start with the linearized system (2.14), where the eigenvalue of $-\Delta$ with homogeneous Neumann boundary condition is

$$\mu_i = i^2, \quad \text{i.e.} \quad \sqrt{\mu_i} = i, \quad i = 0, 1, 2, \cdots, \tag{4.2}$$

so

$$M_i = \lambda J - i^2 D.$$

According to Theorem 2.2 and Corollary 2.3, we have the following conclusion.

Theorem 4.1. *Suppose that (2.7) holds,* (u_0, v_0) *is a positive constant equilibrium solution of (4.1) and (2.2) and the matrixes* J, D *are defined as*

(2.4) and (2.11). If $\min_{i\in\mathbb{N}^*}\mathrm{Det}(M_i) < 0$ *and* $H(\omega) > 0$ *are satisfied, and*

$$0 < k_1 \leq i^2 \leq k_2, \tag{4.3}$$

for some i*, where* k_1 *and* k_2 *are defined by (2.25), then* (u_0, v_0) *is an unstable equilibrium solution with respect to the reaction-diffusion system (4.1), but a stable equilibrium solution with respect to the ordinary differential equation system (2.2).*

Corollary 4.2. *The conditions are the same as in Theorem 4.1, if*

$$\min_{i\in\mathbb{N}^*}\mathrm{Det}(M_i) > 0$$

or $H(\omega) \leq 0$*, then the positive constant equilibrium solution* (u_0, v_0) *is also a stable equilibrium solution with respect to (4.1).*

Since that analysis and the proof of Theorem 3.1 are similar to that of Theorem 2.2, we omit its proof. Because the eigenvalue of $-\Delta$ with homogenous Neumann boundary condition in $(0, +\infty) \times (0, \pi)$ is $i^2 (i \in \mathbb{N}^*)$, the interval $[k_1, k_2]$ includes at least one $i^2 (i \in \mathbb{N}^*)$, which is a sufficient condition, so the length of the interval $[k_1, k_2]$ is larger than 1^2, i.e.

$$\sqrt{k_2} - \sqrt{k_1} > 1$$

which is equivalent to

$$u_0{}^2 H(\omega) - \sqrt{ab(1-\omega)u_0{}^2(u_0{}^2 + d^2)\mathrm{Det}(D)} > \frac{(u_0{}^2 + d^2)\mathrm{Det}(D)}{\lambda}, \tag{4.4}$$

then we have

Theorem 4.3. *The conditions are the same as in Theorem 3.1, if (4.4) is satisfied for some* $\lambda > 0$*, then the positive constant equilibrium solution* (u_0, v_0) *is also an unstable equilibrium solution with respect to (4.1).*

Corollary 4.4. *The conditions are the same as in Corollary 3.1, if*

$$\min_{i\in\mathbb{N}^*}\mathrm{Det}(M_i) > 0, \quad \text{or} \quad H(\omega) \leq 0, \tag{4.5}$$

then the positive constant equilibrium solution (u_0, v_0) *is also a stable equilibrium solution with respect to (4.1).*

Theorem 4.1 gives a general criterion for the instability when self-diffusion and/or cross-diffusion is added to the system (4.1). In order to introduce the effect of cross-diffusion more clearly, we first assume $\alpha_{11} = \alpha_{22} = 0$, we will analyze the three cases respectively about the condition (4.4), which implies $H(\omega) > 0$.

Case 1. $\alpha_{12} = \alpha_{21} = 0$. The condition (4.4) becomes

$$u_0{}^2 H(\omega) - \sqrt{ab(1-\omega)u_0{}^2(u_0{}^2+d^2)d_1d_2} > \frac{(u_0{}^2+d^2)d_1d_2}{\lambda}, \tag{4.6}$$

where $H(\omega) = ad_2\omega - bd_1(1+\frac{d^2}{u_0{}^2})$. If d_2 is sufficient large, then (4.6) hold for some $\lambda > 0$, but if d_1 is sufficiently large, then (4.6) always does not hold for any $\lambda > 0$.

Case 2. $\alpha_{21} = 0$. The condition (4.4) is

$$u_0{}^2 H_{21}(\omega) - \sqrt{ab(1-\omega)u_0{}^2(u_0{}^2+d^2)d_2(d_1+\alpha_{12}u_0)} > \frac{d_2(u_0{}^2+d^2)(d_1+\alpha_{12}u_0)}{\lambda}, \tag{4.7}$$

where $H_{21}(\omega) = ad_2\omega - b(d_1+2\alpha_{12}u_0)(1+\frac{d^2}{u_0{}^2})$. So if α_{12} is large enough, the above condition (4.7) does not hold.

Case 3. $\alpha_{12} = 0$. The condition (4.4) is

$$u_0{}^2 H_{12}(\omega) - \sqrt{ab(1-\omega)u_0{}^2(u_0{}^2+d^2)d_1(d_2+\frac{\alpha_{21}}{\gamma+u_0})} > \frac{d_1(u_0{}^2+d^2)(d_2+\frac{\alpha_{21}}{\gamma+u_0})}{\lambda}, \tag{4.8}$$

where $H_{12}(\omega) = a[d_2\omega + \frac{\alpha_{21}\omega}{\gamma+u_0} - \frac{\alpha_{21}u_0}{(\gamma+u_0)^2}] - bd_1(1+\frac{d^2}{u_0{}^2})$. When $\omega\gamma + (\omega-1)u_0 > 0$, if d_2 or α_{21} is sufficiently large, then (4.8) is true for some $\lambda > 0$.

Summarizing the above analysis, we have

Theorem 4.5. *Suppose that (2.7) holds, (u_0, v_0) is a positive constant equilibrium solution of (4.1) and (2.2),*

(I) When $\alpha_{12} = \alpha_{21} = 0$, the cross-diffusion are not respecting in system (4.1). If d_2 is large such that (4.6) holds, then the equilibrium solution (u_0, v_0) is unstable with respect to (4.1) for some $\lambda > 0$, but if d_1 is too large such that $H(\omega) \leq 0$, then (u_0, v_0) is stable for (4.1).

(II) When $\alpha_{21} = 0$, there is only one cross-diffusion and u moves to v. If α_{12} is large such that (4.7) holds, then (u_0, v_0) is stable for (4.1).

(III) When $\alpha_{12} = 0$, and u moves to v. When $\omega\gamma + (\omega-1)u_0 > 0$, if α_{21} is large such that (4.8) holds, then the equilibrium solution (u_0, v_0) is unstable with respect to (4.1) for some $\lambda > 0$; but when $\omega\gamma + (\omega-1)u_0 \leq 0$, then the equilibrium solution (u_0, v_0) is stable with respect to (4.1).

Remark 4.1. The above analysis shows that when the condition (4.6) holds, if $H_{21}(\omega) \leq 0$ or $H_{12}(\omega) \leq 0$, then cross-diffusion makes (u_0, v_0) unstable for the system (4.1); but when (4.6) does not hold, cross-diffusion makes (u_0, v_0) stable for the system (4.1), if (4.8) holds. In other words, under certain conditions, cross-diffusion can change the stability of a constant equilibrium solution whether it is stable with the corresponding self-diffusion reaction system or not.

5. Numerical Illustrations

In this section, we present some numerical simulations to illustrate our theoretical analysis.

In the system (4.1), we select

$$a = \frac{49}{114}, \quad b = 1, \quad d = \frac{\sqrt{19}}{19}, \quad F = \frac{1}{8}, \quad \alpha_{11} = \alpha_{22} = 0,$$

thus the PDE model (4.1) becomes

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta[(d_1 + \alpha_{12}v)u] + \lambda\{u[(1-u) - \frac{49uv}{114u^2+6}] - \frac{1}{8}\}, & 0 < x < \pi,\ t > 0, \\ \frac{\partial v}{\partial t} = \Delta[(d_2 + \frac{\alpha_{21}}{\gamma+u})v] + \lambda v(1 - \frac{v}{u}), & 0 < x < \pi,\ t > 0, \\ \frac{\partial u}{\partial \nu}(0,t) = \frac{\partial u}{\partial \nu}(\pi,t) = \frac{\partial v}{\partial \nu}(0,t) = \frac{\partial v}{\partial \nu}(\pi,t) = 0, & t > 0, \\ u(x,0) = \beta_1(x),\ v(x,0) = \beta_2(x), & 0 \le x \le \pi, \end{cases} \tag{5.1}$$

and the corresponding ODE model of (5.1) is

$$\begin{cases} \frac{du}{dt} = \lambda f(u,v) = \lambda\{u[(1-u) - \frac{49uv}{114u^2+6}] - \frac{1}{8}\}, & t > 0, \\ \frac{dv}{dt} = \lambda g(u,v) = \lambda v(1 - \frac{v}{u}), & t > 0, \\ u(0) = \beta_1(0),\ v(0) = \beta_2(0). \end{cases} \tag{5.2}$$

A direct computation shows that there are two positive constant equilibrium solutions $(0.21149, 0.21149)$ and $(\frac{1}{3}, \frac{1}{3})$. According to Lemma 2.1, we know the equilibrium solution $(0.21149, 0.21149)$ is unstable with respect to (5.2), but the equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ is stable with respect to (5.2). There are shown in Figure 1, where the initial condition is taken at $(\frac{1}{4}, \frac{1}{2})$.

In the following, we turn to discuss the instability of the positive constant equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ with respect to the system (5.1). Here $\omega = \frac{1}{2}$. First, we choose two sets of parameters:

$$d_1 = 1, \quad d_2 = 6, \quad \alpha_{12} = \alpha_{21} = 0, \tag{5.3}$$

$$d_1 = 1, \quad d_2 = 6, \quad \alpha_{12} = 0, \quad \alpha_{21} = 300, \quad \gamma = 2, \quad \lambda = 100. \tag{5.4}$$

By Theorem 4.3 (I), (III), we know that, under the set of parameters in (5.3), the equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ of system (5.1) is stable. But under the set of parameters in (5.4), the solution $(\frac{1}{3}, \frac{1}{3})$ of system (5.1) is unstable. These are shown in Figure 2, where the initial condition is taken at $(\frac{1}{3} + \sin x, \frac{1}{3} + \cos x)$.

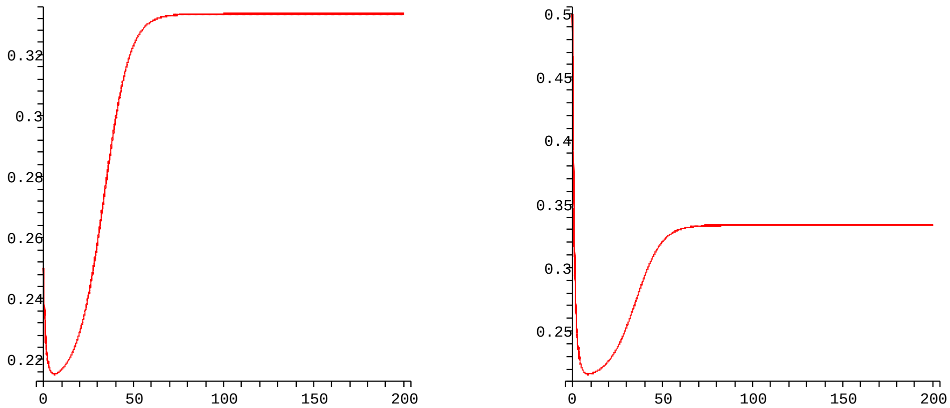


Figure 1: Numerical simulation of an asymptotically stable homogeneous equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ of the system (5.2). Left: component u; right: component v.

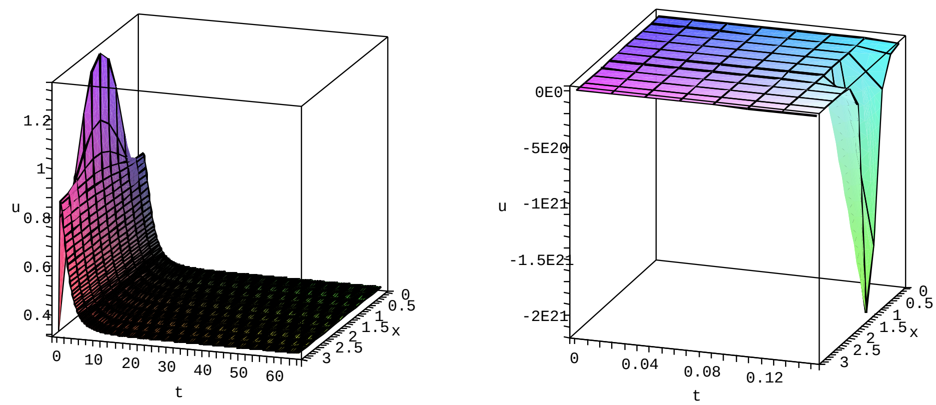
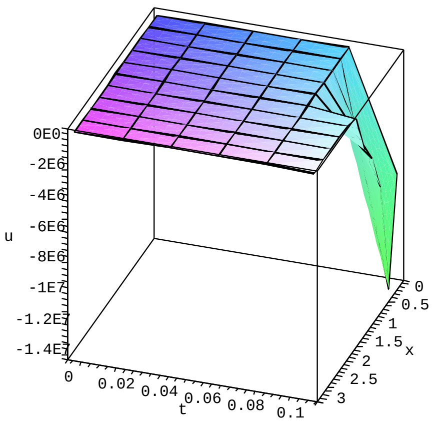


Figure 2: Numerical simulation of a homogeneous equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ of the system (5.1). Left: component u (stable under (5.3)); right: component u (unstable under (5.4)).

Second, we choose two sets of parameters:

$$d_1 = \frac{1}{2}, \quad d_2 = 12, \quad \alpha_{12} = \alpha_{21} = 0, \quad \lambda = 100, \tag{5.5}$$

$$d_1 = \frac{1}{2}, \quad d_2 = 12, \quad \alpha_{12} = 4, \quad \alpha_{21} = 0. \tag{5.6}$$



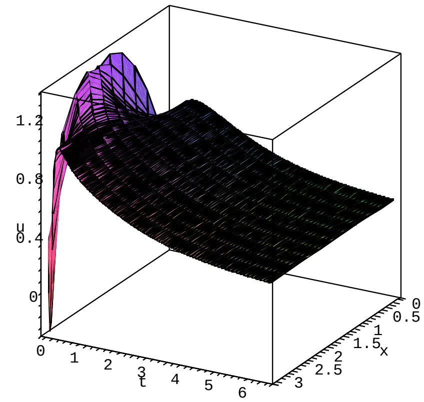


Figure 3: Numerical simulation of a homogeneous equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ of the system (5.1). Left: component u (unstable under (5.5)); right: component u (stable under (5.6)).

By Theorem 4.2 (I), (II), we know that, the equilibrium solution $(\frac{1}{3}, \frac{1}{3})$ of system (5.1) is unstable under the set of parameters in (5.5). But under the set of parameters in (5.6), the solution $(\frac{1}{3}, \frac{1}{3})$ of system (5.1) is stable. These are shown in Figure 3, where the initial condition is also taken at $(\frac{1}{3} + \sin x, \frac{1}{3} + \cos x)$.

6. Discussions

In this paper, we follow the ideas of Turing about diffusion driven instability and consider the impact of cross-diffusion on the stability of a positive constant equilibrium solution in a spruce budworm-aphid interaction model. Classical *Turing instability* suggests that stable in a kinetic system without diffusion does not implies stable in the corresponding system with self-diffusion but without cross-diffusion. Our results get another two ways: one is stable in a system with self-diffusion without cross-diffusion, but unstable in the system with self-diffusion and cross-diffusion; and the other is unstable in a system with self-diffusion without cross-diffusion, but stable in the system with both self-diffusion and cross-diffusion, when the kinetic system without diffusion is stable. Therefore not only diffusion but also cross-diffusion can establish the pattern formation in the biological systems. The pattern formation induced by diffusion and cross-diffusion proves that biological systems are extremely complex.

Acknowledgements

The work is partially supported by PRC grant NSFC 10671172, the NSF of Jiangsu Province (BK2006064) and also by "Blue Project" of Jiangsu Province.

References

[1] H.A. Abdusalam, E.S. Fahmy, Cross-diffusional effect in a telegraph reaction diffusion Lotaka-Volterra tow competitive system, *Chaos. Solitons. Fractals*, **18** (2003), 259-266.

[2] M. Alber, T. Glimm, H.G. E. Hentschel, B. Kazmierczak, A. Stuart Newman, Stability of n-dimensional patterns in a generalized Turing system: implications for biological pattern formation, *Nonlinearity*, **18** (2005), 125-138.

[3] M. Farkas, On the distribution of capital and labor in a closed economy, *Southeast Asian Bull. Math.*, **19** (1995), 27-36.

[4] P. Kareiva, G. Odell, Swarms of preadatora exhibit "preytaxis" if individual predators use area-restricted, *Search. Amer. Natu.*, **130** (1987), 233-270.

[5] A. Li, H. Jing, J. Fu, Dynamic characteristic of spruce budworm-its natural enemy-pesticide interaction modle, *Jour. Theor. Biol.*, **21** (2006), 377-383. (in chinese),

[6] Z.G. Lin, M. Pedersen, Stability in diffusive food-chain model with Michaelis-Menten functional response, *Nonlinear. Anal.*, **57** (2004), 421-433.

[7] D. Ludwig, D.D. Jones, C.S. Holling, Qualitative analysis of insect outbreak systems: the spruce budworm and forest, *J. Amim. Ecol.*, **47** (1978), 315-332.

[8] J.D. Murray, *Mathematical Biology*, Springer-Verlag, New York (2002).

[9] R.A. Satnoianu, P. van den Driessche, Some remarks on matrix stability with application to Turing instability, *Linear. Algebra. Appl.*, **398** (2005), 69-74.

[10] J. Shi, Z. Xie, K. Little, Cross-diffusion induced instability and stability in reaction-diffusion systems, To Appear.

[11] N. Shigesada, K. Kawasaki, E. Teramoto, Spatial segregation of interacting species, *Jour. Theor. Biol.*, **79** (1979), 83-99.

[12] A.M. Turing, The chemical basis of morphogenesis, *Phil. Tran.R. Soc. Lond., Series B*, **237** (1952), 37-72.

[13] M.X. Wang, Stationary patterns for a prey-predator model with prey-dependent and ratio-dependent functional responses and diffusion, *Phys. D*, **196** (2004), 172-192.

[14] H. Werner, L. Kwan, K.M. Peter, Network topology and Turing instabilities in small arrays of diffusively coupled reactors, *Phys. Rev. Lett. A*, **328** (2004), 444-451.

[15] J.D. Wollkind, J.B. Colling, J.A. Logan, Metastability in a temperature-dependent model system for predatorprey mite outbreak interactions on fruit trees, *B. Math. Biol.*, **50** (1988), 379-409.