International Journal of Pure and Applied Mathematics

Volume 47 No. 1 2008, 41-51

A STRATEGY FOR PEST MANAGEMENT

Chunjin Wei[1 §], Ruiqing Shi[2], Lansun Chen[3]

[1,2,3]Department of Applied Mathematics
Dalian University of Technology
Dalian, 116024, P.R. CHINA
[1]e-mails: chunjinwei@yahoo.cn; lschen@amss.ac.cn
[1]College of Science
Jimei University
Xiamen, 361021, P.R. CHINA

Abstract: In this paper, we consider a delayed SI model for pest management. The pest population is subdivided into three subgroups: immature pest eggs, susceptible mature pests, infectious pests. By using analysis skills and comparison theorem, we obtain that the solution of the system is uniformly ultimately bounded, and the sufficient condition under which the amount of susceptible pests is less than the given economic threshold value E. Hence we can use the theoretical result to control the pests at desirable low level by impulsively releasing infected pests.

AMS Subject Classification: 90BXX, 90B50
Key Words: impulsive, stage-structure, delay, pest-management

1. Introduction

Pests have heavily increased these years, which cause serious threat to crops. How to control pests effectively has become a critical issue. There are many methods to control pests such as chemical control, biological control, physical control and so on. Pesticides can quickly kill a significant proportion of pest population and they sometimes provide the only feasible way to prevent economic loss. However, pesticide pollution is considered as a major hazard to

Received: March 19, 2008

§Correspondence address: Department of Applied Mathematics, Dalian University of Technology, Dalian, 116024, P.R. CHINA

the health of human beings and natural enemies. For example, it was found that the farmers use chemical methods to kill field mice which resulted in the quantities of hawk decreasing significantly by Ganshu Agriculture Institute (in China). Hence chemical control methods needs to be used cautiously.

Compared to chemical control, biological control including microbial control with pathogens are safer to man and environment and more effective for the longer period of time. For example, Bacillus thuringiensis, which is available in commercial preparations, is used in the control of a large number of pests [7, 8, 3]. Recently the authors in [2, 10, 13, 14, 6, 5, 4] also studied models for pest control and obtained some results.

In the natural world, many pests such as Saltcedar leaf beetle have the stage structure of immature egg stage and mature stage. Pathogens may not be effective against pest egg for its eggshell. That is, the disease only attacks the susceptible mature pest population. In addition, the time to maturity of the immature pest population should not be ignored in reasonable pest control tactics. In this paper, according to the above ecological background, we propose a delayed epidemic model with pulses in order to investigate how epidemics influence the pest control process. In this model the control variable is the release amount of infected pests. Our model is the impulsive differential equation. The theory and application of impulsive differential equations were introduced systematically in [1, 11, 12, 18]. Literatures on combining pest control and infectious disease are seldom [16, 9]. Most of these research literatures studied the condition for pests eradication. But, in practice, eradication for pests is difficult both practically and economically as pests can breed quickly, so we only need to control the susceptible pest population under the economic injury level and it is not necessary to eradicate the susceptible pests totally.

The organization of this paper is as follows: In the next section, we construct our model and give some essential hypotheses. To prove our main result, we also give several definitions, notations and lemmas. In Section 4, a sufficient condition is obtained to assure the amount of the susceptible mature pests below the threshold value of economic injury level. In the final section, we give some simple discussion.

2. Model Formulation

In [17], Zhang and Chen studied the SI model:

$$\begin{cases} S'(t) = -\beta S^q(t)I(t) + S(t)(a - S(t)), \\ I'(t) = \beta S^q(t)I(t) - (\gamma + \mu)I(t), \end{cases} \tag{2.1}$$

where $S(t), I(t)$ denote the amount of susceptible pests and infective pests at time t, $\beta S^q(t) > 0$ is the incidence rate per infective individual, and readers may see [17] for more details. In [15], Aiello and Freedman studied the simple species model with stage structure:

$$\begin{cases} x'(t) = \alpha y(t) - \gamma x(t) - \alpha e^{-\gamma\tau} y(t-\tau), \\ y'(t) = \alpha e^{-\gamma\tau} y(t-\tau) - \beta y^2(t), \end{cases} \tag{2.2}$$

with initial values

$$\begin{cases} x(s) = \Phi(s), y(s) = \Psi(s), s \in [-\tau, 0], \\ \Phi, \Psi \in C([-\tau, 0], R_+^2), \Phi(0) > 0, \Psi(0) > 0. \end{cases} \tag{2.3}$$

Enlightened by the above two models, we construct the following model:

$$\begin{cases} \left.\begin{array}{l} S_i'(t) = B(S(t))S(t) - d_1 S_i(t) - e^{-d_1\tau} B(S(t-\tau))S(t-\tau), \\ S'(t) = e^{-d_1\tau} B(S(t-\tau))S(t-\tau) - d_2 S(t) - \beta S^2(t)I(t), \\ I'(t) = \beta S^2(t)I(t) - d_3 I(t), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} \triangle S_i(t) = 0, \\ \triangle S(t) = 0, \\ \triangle I(t) = \mu, \end{array}\right\} t = nT, n = 1, 2 \cdots. \end{cases} \tag{2.4}$$

The initial data $(\Phi_1(t), \Phi_2(t), \Phi_3(t))$ for system (2.4) are

$$(\Phi_1(t), \Phi_2(t), \Phi_3(t)) \in C_3^+, \qquad \Phi_i(0) > 0, \quad i = 1, 2, 3\,, \tag{2.5}$$

where

$$C_3^+ \doteq C([-\tau, 0], R_+^3 \doteq \{(x_1, x_2, x_3) : x_i \geq 0, i = 1, 2, 3\})\,.$$

Following, we impose the condition

$$\Phi_1(0) = \int_{-\tau}^{0} e^{d_1\theta} B(S(\theta))S(\theta)d\theta \tag{2.6}$$

which is completely reasonable from the point of view of ecology (referring to [15]). Here the pest population is divided into three classes: immature eggs, susceptible mature pests and infective pests which cannot destroy crops. The density of each class at time t is denoted by $S_i(t), S(t), I(t)$, respectively. And $\triangle S_i(t) = S_i(nT^+) - S_i(nT), \triangle S(t) = S(nT^+) - S_i(nT), \triangle I(t) = I(nT^+) - I(nT)$, $\mu > 0$ is the release amount of infected pests which are cultivated in laboratories each time in order to drive the susceptible mature pest population to generate an endemic. The following assumptions are made in our model:

(H_1) The death rate of the immature pest eggs population is proportional to the existing immature pest eggs population with a proportionality constant d_1. The parameters d_2 and d_3 represent the death rate of the susceptible mature pest population and infective pest population, respectively. Only the mature pest population can reproduce. $B(S)S$ is a birth rate function of the susceptible mature pest population with $B(S)$ satisfying the following basic assumptions for $S \in (0, \infty)$:

(a_1) $B(S) > 0$;

(a_2) $B(S) \in C^2((0, \infty), (0, \infty))$ with $B'(S) < 0$;

(a_3) $B(0^+) > d_2 > \sigma > B(\infty), \sigma \doteq \frac{1}{2} \min\{d_1, d_2, d_3\}$.

(H_2) τ represents a constant time to maturity. The product term $e^{-d_1\tau}B(S(t-\tau))S(t-\tau)$ describes the immature pest eggs who were born at time $t-\tau$ and survive at time t, hence represents the transformation from the immature pest egg population to the susceptible mature pest population.

(H_3) $\beta S^2 I$ represents the incidence rate, βS^2 is the incidence rate per infective individual.

3. Preliminaries

In this section, we will give some definitions, notations and lemmas, which will be useful for our main results.

The solution of (2.4), denoted by $x(t) = (S_i(t), S(t), I(t))$, is a piecewise continuous function $x(t)$: $R_+ \to R_+^3$, $x(t)$ is continuous on $(nT, (n+1)T]$, $n \in N$ and $x(nT^+) = \lim_{t \to nT^+} x(t)$ exists. Obviously, the existence and uniqueness of solutions of (2.4) is guaranteed by the smoothness properties of $f = (f_1, f_2, f_3)$, which denotes the mapping defined by right-side of the first three equations in system (2.4) (for more details see [11]).

Let $V : R_+ \times R^3 \to R_+$, then $V \in V_0$ if:

(i) V is continuous in $(nT, (n+1)T] \times R^3$ and $\lim_{(t,z)\to(nT^+,x)} V(t,z) = V(nT^+, x)$ exists, for each $x \in R_+^3, n \in N$.

(ii) V is locally Lipschitzian in x.

Definition 1. $V \in V_0$, then for $(t, x) \in (nT, (n+1)T] \times R^3$, the upper right derivative of $V(t,x)$ with respect to system (2.4) is defined as

$$D^+V(t,x) = \lim_{h\to 0^+}\sup \frac{1}{h}[V(t+h, x+hf(t,x)) - V(t,x)]\,.$$

Lemma 1. *Suppose $x(t) = (S_i(t), S(t), I(t))$ is a solution of (2.4) satisfing the initial (2.5), then $x(t) > 0$ for all $t > 0$.*

Proof. (i) First, we prove $S(t) > 0$ for all $t > 0$.

Suppose $S(t) > 0$. Otherwise, there exists t_0 such that $S(t_0) = 0$. Set $t_0 = \inf\{t > 0 : S(t) = 0\}$. Then we have $S'(t_0 - \varepsilon) \le 0$ for sufficiently small constant ε. But according to system (2.4), we know $S'(t_0) = e^{-d_1\tau}B(S(t_0 - \tau))S(t_0 - \tau) > 0$, then $S'(t_0 - \varepsilon) > 0$, which contradict with $S'(t_0 - \varepsilon) > 0$. So we get $S(t) > 0$ for all $t > 0$.

(ii) Second, we prove $S_i(t) > 0$ for all $t > 0$. Now consider the following equation:

$$u'(t) = -e^{-d_1\tau}B(S(t-\tau))S(t-\tau) - d_1u(t), u(0) = S_i(0)\,.$$

We have $S_i(t) > u(t)$ for $0 < t \le \tau$.

The solution of this equation is:

$$u(t) = e^{-d_1t}[u(0) - \int_0^t e^{d_1(\sigma-\tau)}B(S(\sigma-\tau))S(\sigma-\tau)d\sigma]\,.$$

According to (2.6), we know:

$$u(\tau) = e^{-d_1\tau} \times [\int_{-\tau}^0 e^{d_1\theta}B(S(\theta))S(\theta)d\theta - \int_0^\tau e^{d_1(\sigma-\tau)}B(S(\sigma-\tau))S(\sigma-\tau)d\sigma] = 0\,.$$

Since $u(t)$ is decreasing strictly, then we have $u(t) > u(\tau) = 0$ for $t \in [0,\tau)$. According to comparison theorem, we have $S_i(t) > u(t) > 0$ for $0 < t \le \tau$. Similarly, we can prove $S_i(t) > 0$ for $n\tau < t \le (n+1)\tau, n = 1, 2, \cdots$, then $S_i(t) > 0$ for all $t > 0$.

(iii) It is clear that $I(t) > 0$ for all $t > 0$.

The proof is complete. □

Now, we show that all solutions of (2.4) are uniformly ultimately bounded.

Lemma 2. *There exists a constant $M > 0$ such that $S_i(t) \le M, S(t) \le M, I(t) \le M$ for each solution $(S_i(t), S(t), I(t))$ of (2.4) with all t large enough.*

Proof. Define $V(t) = S_i(t) + S(t) + I(t)$. Since $V(t) \in V_0$, then we have

$$\begin{aligned} D^+V(t) &= B(S(t))S(t) - d_1S_i(t) - d_2S(t) - d_3I(t) \\ &\le B(S(t))S(t) - \delta(S_i(t) + S(t) + I(t)) \end{aligned}$$

for $t \in (nT, (n+1)T]$, in which $\delta \doteq 2\sigma$, then we derive

$$D^{+}V(t) + \sigma V(t) \leq B(S(t))S(t) - \sigma S(t), t \in (nT, (n+1)T]\,.$$

From (H_1), it is easy to know that there exists a constant $K > 0$ such that

$$\begin{cases} D^{+}V(t) \leq -\sigma V(t) + K, & t \neq nT, \\ V(nT^{+}) = V(nT) + u, & t = nT. \end{cases} \tag{3.1}$$

According to Theorem 1.5.2 in [11], we derive

$$\begin{aligned} V(t) &\leq (V(0^{+}) - \frac{K}{\sigma})e^{-\sigma t} + \frac{u(1-e^{-n\sigma T})e^{-\sigma(t-nT)}}{1-e^{-\sigma T}} + \frac{K}{\sigma} \\ &\to \frac{K}{\sigma} + \frac{ue^{\sigma T}}{e^{\sigma T - 1}} \quad \text{as} \quad t \to \infty\,. \end{aligned}$$

Let $M \doteq \frac{K}{\sigma} + \frac{ue^{\sigma T}}{e^{\sigma T-1}}$, then by the definition of $V(t)$ we obtain that each positive solution of system (2.4) is uniformly ultimately bounded. The proof is complete.

Since the second and third equations are independent of S_i, we study the following subsystem:

$$\begin{cases} \left.\begin{aligned} S'(t) &= e^{-d_1\tau}B(S(t-\tau))S(t-\tau) - d_2S(t) - \beta S^2(t)I(t), \\ I'(t) &= \beta S^2(t)I(t) - d_3I(t), \end{aligned}\right\} \quad t \neq nT \\ \left.\begin{aligned} \triangle S(t) &= 0, \\ \triangle I(t) &= u, \end{aligned}\right\} t = nT, n = 1, 2\cdots. \end{cases} \tag{3.2}$$

Lemma 3. *Consider the following sub-system of system (3.2)*

$$\begin{cases} I'(t) = -d_3I(t), & t \neq nT\,, \\ \triangle I(t) = u, & t = nT\,. \end{cases} \tag{3.3}$$

Then system (3.3) has a positive periodic solution $I^*(t)$ and for every solution $I(t)$ of system (3.3), $\mid I(t) - I^*(t) \mid \to 0$ as $t \to \infty$, where

$$I^*(t) = \frac{ue^{-d_3(t-nT)}}{1-e^{-d_3T}} \quad \textit{and} \quad I^*(0^{+}) = \frac{u}{1-e^{-d_3T}}\,.$$

Proof. The lemma is obvious, in fact the solution of system (3.3) is

$$I(t) = (I(0^{+}) - \frac{u}{1-e^{-d_3T}})e^{-d_3t} + I^*(t), \qquad t \in (nT, (n+1)T],\ n \in N\,.$$

4. Main Result

In this section, we give the main result of system (3.2). Suppose the threshold value of economic injury level (EIL) is E.
Let

$$u^* = \frac{(e^{d_3 T} - 1)(B(0)e^{-d_1\tau} - d_2)}{E\beta},$$

and

$$T^* = \frac{1}{d_3} \ln\Big(\frac{u\beta E}{B(0)e^{-d_1\tau} - d_2} + 1\Big).$$

Theorem. *If $u > u^*$ or $T < T^*$, then the amount of the susceptible pests will be less than the threshold E as t large enough.*

Proof. Since $u > u^*$, then we can select a sufficiently small constant $\varepsilon > 0$ such that

$$\frac{B(0)e^{-d_1\tau} - d_2}{\beta\eta} < E,$$

where $\eta = \frac{u}{e^{d_3 T}-1} - \varepsilon > 0$. From the second and fourth equations of system (3.2), we derive

$$\begin{cases} I'(t) \geq -d_3 I(t), & t \neq nT, n \in N, \\ \triangle I(t) = u, & t = nT, n \in N. \end{cases} \tag{4.1}$$

By the comparison theorem and Lemma 3, we derive

$$\lim_{t\to\infty} \inf I(t) \geq \frac{u}{e^{d_3 T} - 1}.$$

Therefore, for the above $\varepsilon > 0$, there exists an integer $n_1 > 0$ such that for any $t > n_1 T$

$$I(t) > \frac{u}{e^{d_3 T} - 1} - \varepsilon \doteq \eta > 0.$$

By the first equation of system (3.2), we have

$$S'(t) < B(0)e^{-d_1\tau} S(t-\tau) - d_2 S(t) - \beta\eta S^2(t) \tag{4.2}$$

for all $t > n_1 T + \tau$.

Consider the following comparison equation:

$$x'(t) = B(0)e^{-d_1\tau} x(t-\tau) - d_2 x(t) - \beta\eta x^2(t). \tag{4.3}$$

Next, we will prove that

$$\lim_{t\to\infty} x(t) = \frac{B(0)e^{-d_1\tau} - d_2}{\beta\eta}$$

It is obvious that the solution of equation (4.3) is positive and bounded for any $t > 0$, and $x^* = \frac{B(0)e^{-d_1\tau}-d_2}{\beta\eta}$ is the unique positive equilibrium of equation (4.3).

Case 1. If $x(t)$ is ultimately monotone, then the limit of $x(t)$ exists for $t \to \infty$. Suppose $l \doteq \lim_{t\to\infty} x(t)$, then we have $l = x^*$. Otherwise, if $l > x^*$, then we have

$$\lim_{t\to\infty} x'(t) = B(0)e^{-d_1\tau} l - d_2 l - \beta\eta l^2 < 0$$

which means $x(t) \to -\infty$, for $t \to +\infty$. It is a contradiction. Similarly, if $l < x^*$, we also have $x(t) \to +\infty$, for $t \to +\infty$, which is a contradiction. Hence $l = x^*$.

Case 2. If $x(t)$ is not ultimately monotone, then we have $\sigma \doteq \sup \mid x(t) - x^* \mid = 0$. Otherwise, if $\sigma > 0$, there exists $x(t_i)(t_{i+1} > t_i, \lim_{i\to+\infty} t_i = +\infty)$ such that $\lim_{i\to+\infty} x(t_i) = x^* + \sigma$ or $\lim_{i\to+\infty} x(t_i) = x^* - \sigma$. Without loss of generality, we only consider $\lim_{i\to+\infty} x(t_i) = x^* + \sigma$. Since

$$B(0)e^{-d_1\tau}(x^* + \sigma) - d_2(x^* + \sigma) - \beta\eta(x^* + \sigma)^2 < 0,$$

we can choose a sufficiently small constant $\varepsilon > 0$ such that

$$B(0)e^{-d_1\tau}(x^* + \sigma + \varepsilon) - d_2(x^* + \sigma - \varepsilon) - \beta\eta(x^* + \sigma - \varepsilon)^2 < 0. \tag{4.4}$$

For this ε, there exists $T = T(\varepsilon) > \tau$ such that

$$x(t_i) < x^* + \sigma + \varepsilon \tag{4.5}$$

for all $t_i > T - \tau$. Notice that there exists $\widetilde{t_i} > T$ such that $x'(\widetilde{t_i}) = 0$, and $x(\widetilde{t_i}) - x^* > \sigma - \varepsilon$, which implies that

$$B(0)e^{-d_1\tau} x(\widetilde{t_i} - \tau) = d_2 x(\widetilde{t_i}) + \beta\eta x^2(\widetilde{t_i})\,.$$

Hence

$$B(0)e^{-d_1\tau} x(\widetilde{t_i} - \tau) > d_2(x^* + \sigma - \varepsilon) + \beta\eta(x^* + \sigma - \varepsilon)^2. \tag{4.6}$$

According to (4.4) and (4.6), we have $x(\widetilde{t_i} - \tau) > x^* + \sigma + \varepsilon$, which contradicts with (4.5). Therefore $\sigma = 0$, i.e., $\lim_{t\to+\infty} x(t) = x^*$. From (4.2) and comparison theorem of differential equation, we have $S(t) < x(t)$ for sufficiently large t.

Notice that $S(\theta) = x(\theta) = \Phi_2(\theta) > 0$ for $t \in [-\tau, 0]$, then our result is obtained, i.e., $S(t) \le \frac{B(0)e^{-d_1\tau}-d_2}{\beta\eta} < E$. This completes the proof. □

Corollary. *If* $\tau > \frac{1}{d_1} \ln \frac{B(0)(e^{d_3 T}-1)}{d_2(e^{d_3 T}-1)+E\beta u} \doteq \tau^*$*, then the amount of the susceptible pest is less than the threshold value* E*.*

5. Discussion

We have studied the delayed epidemic model with stage-structure and impulses. From the aspect of ecology and economics, it is very difficult to eradicate the pests completely and it is also not necessary. Hence, we only consider the amount of susceptible mature pests below the threshold value. According to the above theorem, we can evaluate the maximum impulsive period T^* and the minimum amount of release of infected pests u^*. We notice that u^* and T^* depend on the parameter τ, which implies that the time delay τ plays a very important role in the model. Our theoretical results provide a strategy for pest management. However, In the real world, since the pest is with seasonal damages, should we consider the parameters varying with time t or study the problem in a finite time? We will continue to study these problems in the future.

Acknowledgements

The research is supported by National Natural Science Foundation of P.R. China (10771179).

References

[1] D. Bainov, P. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, Longman Scientific and Technical, Burnt Mill (1993).

[2] H.J. Barcly, Models for pest control using predator release, habitat management and pesticide release in combination, *J. Appl. Ecol.*, **19** (1982), 337-348.

[3] H.D. Burges, N.W. Hussey, *Microbial Control of Insections and Mites*, Academic Press, New York (1971).

[4] A.J. Cherry, C.J. Lomer, D. Djegui, F. Schulthess, Pathogen incidence and their potential as microbial control agents in IPM of maize stemborers in West Africa, *Biocontrol*, **44** (1999), 301-327.

[5] P. Debach, D. Rosen, *Biological Control by Natural Enemies*, Second Edn., Cambridge, Cambridge University Press (1991).

[6] H.I. Freedman, Graphical stability, enrichment, and pest control by a natural enemy, *Math. Biosci.*, **31** (1976), 207-225.

[7] L.A. Falcon, *Use of Bacteria for Microbial Controlof Insects*, Academic Press, New York (1971).

[8] L.A. Falcon, Problems associated with the use of arthropod viruses in pest control, *Annu. Rev. Entomol.*, **21** (1976), 305-324.

[9] B.S. Goh, *Management and Analysis of Biological Populations*, Amsterdam Oxford, New York (1980).

[10] J. Grasman, O.A. Van Herwaarden, L. Hemerik, J.C. Van Lenteren, A two-component model of host parasititoid interactions: determination of the size of inundative releases of parasitoids in biological pest control, *Math. Biosci.*, **169** (2001), 207-216.

[11] V. Lakshmikantham, D.D. Bainov, P.S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore (1989).

[12] P.S. Simeonov, D.D. Bainov, Stability with respect to part of the variables in system with impulsive effect, *J. Math. Anal. Appl.*, **117** (1996), 247-263.

[13] S.Y. Tang, Y.N. Xiao, L.S. Chen, R.A. Cheke, Integrated pest management models and their dynamical behaviour, *Bull. Math. Biol.*, **67** (2005), 115-135.

[14] J.C. Van Lenteren, Integrated pest management in protected crops, In: *Integrated Pest Management* (Ed. D. Dent), Chapman and Hall, London (1995), 311-320.

[15] G.A. Walter, H.I. Freedman, A time-delay model of single species growth with stage structure, *J. Math. Bios.*, **101** (1990), 139-153.

[16] K. Wickwire, Mathematical models for the control of pests and infectious disease: a survey, *Theor. Pop. Biol.*, **11** (1977), 182-238.

[17] X.A. Zhang, L.S. Chen, The periodic solution of a class of epidemic models, *Comput. Math. Appl.*, **38**, No-s: 3, 4 (1999), 61-71.

[18] H. Zhang, R. Zhu, L.S. Chen, A periodic beddington-deangelis type predator-prey system with time delay and pulses, *International Journal of Pure and Applied Mathematics*, **37**, No. 2 (2007), 147-160.