

ORGANIC MATHEMATICS

Moshe Klein¹ §, Doron Shadmi²

^{1,2}Gan Adam Ltd.

P.O. Box 5024, Tivon 36087, ISRAEL

¹e-mail: gan_adam@netvision.net.il

²e-mail: shadmi_doron@yahoo.com

Abstract: David Hilbert, after introducing 23 Open Problems [7], finished his lecture at the ICM 1900 in Paris by explaining how Mathematics is an "Organism" which needs to maintain the connection between its branches and keep them united in order to stay "vital". Unfortunately, he predicted that Mathematics might indeed break apart. This is already starting to happen. We offer a representation of this "Unity Problem" of Mathematics, as stated by David Hilbert. According to our understanding, the solution for this problematic situation is connected to the 6-th Problem Hilbert suggested during that same lecture - involving the relationship between Mathematics and Physics. We shall test the Non-locality Principle in nature in light of the experiment that Alain Aspect conducted (in 1982) [1] as an answer to the EPR Thought Experiment [6]. We believe that establishing a new Mathematical language will possibly unite locality and non-locality General Relativity Theory [5] and Quantum Mechanics.

Proposing a way to solve Hilbert's 6-th Problem.

AMS Subject Classification: 03A05

Key Words: mathematics and physics, Klein bottle, locality, non-locality, education

1. The Unity Problem in Mathematics

We wish to share the Mathematical Research that we have been conducting for

Received: August 14, 2008

© 2008, Academic Publications Ltd.

§Correspondence author

many years, after reading David Hilbert's Lecture from the ICM 1900 in Paris [7]. In his lecture, Hilbert introduced 23 open Mathematical Problems whose solutions, by his understanding, would lead future Mathematical research and indicate the main directions of Mathematical Progression. Many efforts were made to solve these problems and 20 of the problems have already been solved. There are three remaining problems not yet solved: the 6-th, the 8-th and the 16-th.

Although introducing those 23 problems is important, we were impressed mostly by the words in which Hilbert concluded his famous lecture. He saw the subject called "Mathematics" as a living "Organism" "*whose vitality is conditioned upon the connection of its parts*". He indicated a strong fear that in the near future Mathematics will disperse into separate branches and the connection between these will loosen. He predicted that in the future every Mathematician would work in a specific branch of Mathematics and that separate Mathematical groups will form. He envisaged each group studying a small and specific area of Mathematics. To our great regret, this fear has become reality. Hilbert ended his lecture by sharing his vision of an "Organic unity" between all branches of Mathematics. We felt that in order to turn this vision into reality we have to look at Hilbert's 6-th problem which deals with the relationship between Mathematics and Physics. Physics describes the world of phenomena, for example: Atoms, Galaxies, Clouds, etc. We understand that, in order to fully describe the essence and nature of the relationship between Mathematics and Physics, we should agree that Mathematicians are also part of the world of phenomenon.

This idea is similar to some basic conclusions that derive from Quantum Mechanics; the Scientist (the person who takes the measurements) has the power to influence the results. The person is part of the phenomenon. The Mathematician is part of the world of phenomena, where thoughts are influenced by the thinkers from whom they emanate. This idea might at first appear unusual but we will now explain how we can use this idea to establish certain things mathematically and discover a path that might lead to solving one of the biggest problems of Mathematics.

Using the idea just introduced, the 6-th problem is special and extremely unique since it deals with itself – that is, it deals with the way Mathematicians and Physicists think, and with finding the correlation between them.

This is similar to the invention of Calculus, where not only do the results matter, but also the way they are derived (this is an analog to entries that originate from a function, just like Mathematical theorems that originate from

Mathematicians) – the Mathematicians’ mind (this is similar and analogous to derivatives – which describe the way functions act). This is a hard concept to grasp and might sound rash, but will be described more profoundly further on.

We need to produce and invent an entirely new Mathematical Language that places its effort not only on the Mathematical Results but on the procedure itself, and the way in which Mathematical Knowledge is formulated – *that is, from the Mathematicians themselves!* We will have to investigate the way Mathematicians think. We introduced notions of locality and non-locality in Mathematics. This will be explained later on as well.

Towards the end of the 19-th Century, David Hilbert constructed the foundations of Euclidean Geometry [8] by inventing, stating and organizing the appropriate axioms into 5 Groups. By this method he demonstrated that Euclidean Geometry is a complete Mathematical theory. That is, every statement in Euclidean Geometry is either true or false under the basic axioms. Because of this success Hilbert hoped that one could do a similar thing in Physics. In his discussion of the 6-th problem, he says:

“The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical Sciences in which Mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.”

2. Non-Locality in Nature

Now let us investigate what happened to mechanics after Hilbert’s Lecture. Five years after Hilbert’s Lecture, in 1905, Physics experienced a revolution when Albert Einstein published his Special Relativity Theory [4] and the basis of Quantum Mechanics. In 1915, Einstein published the General Relativity Theory [5]. Einstein, in his work, used Non-Euclidean Geometry (developed by Lobachevsky, Bolyai, Gauss and the more general Riemannian geometry) in order to describe gravitation in terms of curvature of space.

The next step in Modern Physics was the development of Quantum Mechanics. Together with Planck, Bohr and Sommerfeld, Einstein contributed much to the development of Quantum Mechanics. But he disagreed with the Probabilistic Interpretation that became associated with it at a later date. In 1935, Einstein, Podolsky and Rosen jointly published an important scientific article on a Thought Experiment called EPR [6]. The purpose of the article was to disprove the probabilistic interpretation introduced by Bohr to Quan-

tum Mechanics. Einstein said: “*I am convinced that He (God) does not play dice with the Universe*”.

In the Thought experiment electrons are launched in two opposite directions from a common origin and measurements are made of the location and momentum of one of the particles. At this moment, the wave function of the other electron collapses, violating the Locality Principle, according to which information does not move faster than the speed of light. As aforesaid, this article resulted in a serious controversy with Bohr. In 1952, David Bohm published a variation to the EPR Experiment that is based upon spin measurements [3]. Knowing the spin direction on one side determines the spin direction on the other side, and the result is that an interaction faster than the speed of light occurs in two places. Apparently, this fact contradicts the Locality Principle (established by Einstein in the Special Relativity Theory).

In 1964 Bell published an article [2] using probabilistic methods, where he estimates what the correlation needs to be somewhere between the spin measurements in 3 different vector directions, denoted by a, b, c , for two different electrons. He also assumes that all the influences (and interactions) on those electrons are local. If the influences and interactions are local, the correlations should maintain the following inequality:

$$1 + P(b, c) \geq |P(a, b) - P(a, c)|.$$

In 1982 the experiments were actually conducted by Alain Aspect [1] and, somewhat surprisingly, Bell’s Inequality did not hold. This means there was a non-local influence. This also signifies that information passed through those electrons faster than the speed of light.

Scientists and Mathematicians need a new Mathematical language that would fit with the non-local reality in the universe. In order to develop this non-local Mathematical language we suggest trying to continue the procedure that began when Non-Euclidean Geometry was discovered.

3. Organic Thinking

A favorite from amongst the questions asked of pre-school children is: “*Are there more Eyes, or more People?*” The children’s answers and explanations are often surprising. For instance, one child explained that there are more people because “people are bigger than eyes”. Another one explained that “there are more people than eyes because people are more important”. It is

important to really listen to these answers even if some of them appear to be incorrect.

The fact that the children haven't yet been exposed to the formal education systems - hence their thought process is free and unblemished - gave us the feeling that the work with them could be utilized in our research.

We came to the conclusion that kindergarten children have a different way of grasping concepts and a different way of thinking than do adults. While the so-called "adult Mathematical thinking" is based mostly on Logic, children think in a way that is balanced somewhere in-between logic, intuition, emotion and imagination. We called this thought process "Organic Thinking" and tried to characterize it.

After conducting a number of research meetings we were able to understand how it is possible to characterize this thinking mathematically. While working with these children we noticed that they comprehend the line as being something complete, "dis-composite" (that is, not as a sequence of points which is the way most people would grasp the lines). Even if we carefully study Euclid's "Book of Elements", no explicit mention of a line as a sequence of points will be found. (Although, Definition #4 in Book *I* of Elements in some ways alludes to that).

A point for speculation is what directions our thought process can take us in if we assume that Lines and Points are two independent elements that do not derive from each other?

We understand that a new Mathematical framework can be invented if one assumes that Points and Lines are two different independent elements. For instance, we examined the term "belongs to". When we examine the way a point 'belongs to' a line, we can see that the point belongs locally to the line. In this local viewpoint there is an XOR connective between 'belonging' and 'not belonging' that prevents them from being simultaneously truthful. That is: A point can either belong or not belong to a line. Looking at this relationship from the line's viewpoint we see that the line simultaneously belongs AND does NOT belong to the point. This can happen only if we see the line as an indivisible element. This might seemingly appear to be a logical contradiction but, after our investigation during which we demonstrated that a lot of our world is non-local, we understand that the contradiction exists when only the local viewpoint is used.

We will call the point a local element (which can belong XOR not belong), and the line a non-local element (belonging AND not belonging). After examining the relationship between Locality and Non-locality in Mathematics,

we redefined the notion of the Natural Number as being the outcome of the interaction between Locality and Non-locality. Locality on its own is total isolation and Non-locality on its own is total connectivity. Those two extremes are not researchable individually but if they interact with each other they provide a researchable area for discoveries. We describe this interaction by the term ‘bridging’, meaning that Non-locality and Locality do not derive from each other since they are atoms. In order to bridge between the two different worlds – Locality and Non-Locality - one uses the notion of the Natural Number. We can look at the Natural Numbers as Ordinals or Cardinals, but by using a Non-locality/Locality bridge we discover another dimension of the concept of the Natural Number, which is Distinction.

Example: According to the traditional viewpoint of the Natural Number we immediately know its cardinality and the exact identification of each element belonging to the set having this particular cardinal. If Non-locality is represented by a horizontal line, bridging is represented by a vertical line and locality is represented by a point, we find that the traditional viewpoint of the Natural Number is the particular Non-locality/Locality bridging where the identification of each local element is clearly known.

By using Non-locality/Locality bridging, at least two fundamental notions are achieved:

— Non-locality/Locality bridging is actually also used by the traditional ‘Natural Number’ point of view.

— By using Non-locality/Locality bridging as a fundamental property of the concept of the Natural Number we discover two important things:

– No Natural Number > 1 can be found without Non-locality/Locality bridging, because without bridging’s non-local aspect it is impossible that two (or more) local elements are gathered into some cardinality > 1 .

– If some cardinality > 1 is known we cannot immediately conclude what is the exact identification of each gathered local element.

In that case cardinality is known but the identifications of the gathered local elements are in superposition with each other.

For example, let us explore number 4 (see Figure 1):

The lower right bridging (marked by a rectangle) is the traditional Natural Number where Cardinal and Ordinal are clearly known. The upper left bridging is the case where cardinal is clearly known but the identifications of the gathered local elements are in superposition with each other. Between these extreme states we define intermediate states of identification distinction.

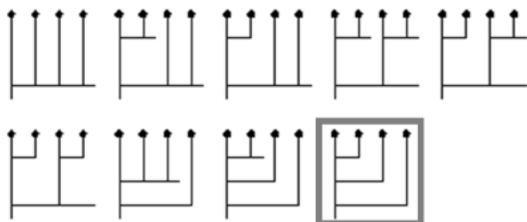


Figure 1:

Some claims are that we actually use distinction as a particular case with some Ordinals, thus Distinction cannot be considered as a fundamental property of the Natural Number.

This conclusion is true only if we continue to use the lower right bridging (marked by a rectangle) as the general viewpoint of the possible result of Non-locality/Locality bridging. However, as we have demonstrated, Non-locality/Locality bridging is not limited to any particular result, and each result can be used both as Non-local AND Local viewpoint of the concept of Natural Number. It is very hard to be understood by any viewpoint that was trained to explore and define things by using serial (step-by-step) thoughts.

Non-locality/Locality bridging can be understood only by using Parallel AND Serial viewpoints of the explored subject simultaneously, and it shows the importance of the Mathematicians' mental training as a significant factor of the Mathematical research. Let us examine bridging cases 1 to 5:

As can be seen in Figure 2, we are using the particular case of clearly distinct identification as a general step-by-step viewpoint of the entire system, but any other case which is not a step-by-step viewpoint, can be used as a general viewpoint of the entire system as well. In order to understand it, the mind has to be trained to think simultaneously in both Parallel AND Serial points of view of the explored subject.

4. Proposing a Solution to Hilbert's 6-th Problem

Now, after briefly describing the key notions and necessary fundamental training (parallel/serial thinking) that will help us establish our new Mathematics, we wish to propose a new way of dealing with the latest discoveries of Quantum Mechanics regarding Locality and Non-locality.

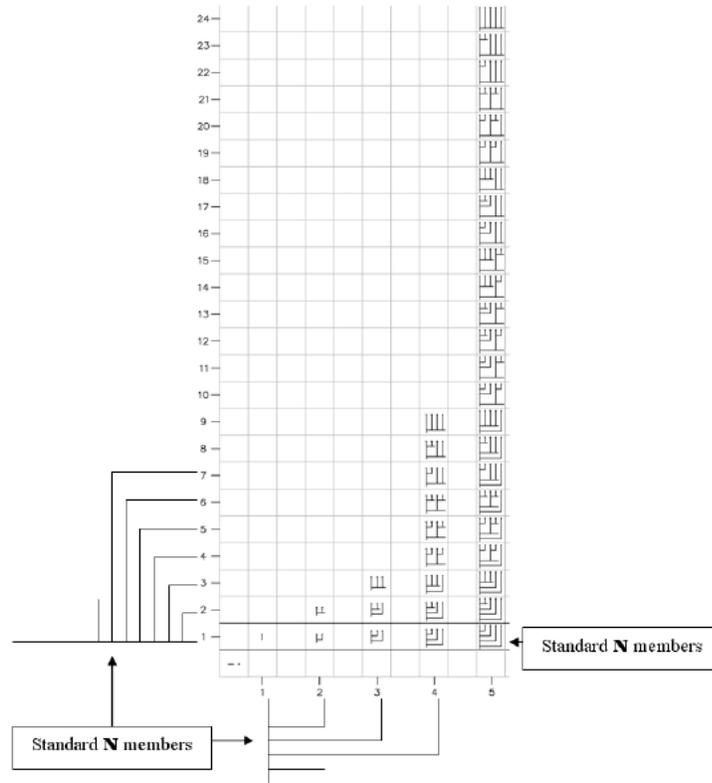


Figure 2:

While explaining the 6-th problem Hilbert, during his lecture, suggested initially examining the task in which the Probability Theory fits into understanding the relationship between Mathematics and Physics. A few years later Einstein succeeded in this and discovered that Probability plays an important role in the way particles act in nature, just as Quantum Mechanics describes it. But he used a language where Distinction is not its first-order property. Organic Mathematics may be the language that Physics uses in order to solve this problematic issue, and it lays the basis for understanding how to deal with Hilbert’s 6-th problem

In our opinion, one of the most important books in the history of Science is Isaac Newton’s “Principia”. This book defines the Mathematical basis needed for the works of Copernicus and Galileo. Newton tried to unite the universe by introducing the Notion of Mass and the Notion of Force as something that interacts with different masses. This theory has governed Science for more than

300 years.

1905 is considered to be one of the most significant years in Scientific History because Einstein published his Special Relativity Theory [6] and laid down the basis of Quantum Mechanics. These two theories were meant to counter Newton's way of thinking. People are not only observing and studying the universe, but they are also an integral part of the world of phenomena. At the end of his life Einstein unsuccessfully tried to unify Gravitation and Electromagnetism. Organic Mathematics, perhaps, might be able to show why Einstein was not able to succeed, and that is because he lacked the Non-local/Local bridging suggested by Organic Mathematics.

5. Discussion Regarding Applications of Organic Mathematics

Following are a number of ideas that arose during work conducted with Children, while dealing with Organic Mathematics. It is our belief that these ideas could be expanded and might result in changes in the way Mathematicians grasp things. This can also explain the way Organic Mathematics works:

Quantum Mechanics: Mathematical Foundations of Non-locality. This has been explained quite thoroughly in the article.

Mathematics: Topology: The Felix Klein Bottle. The Felix Klein Bottle is a 4-th dimensional expansion of the Möbius Strip. The Klein Bottle proves that two languages - which are totally opposite from one another in a local viewpoint but are one in a non-local viewpoint - can be mixed. A line and a point are the geometrical aspects of Locality and Non-locality, but Locality and Non-locality can also be seen in Logic where the logical connective is the non-local aspect of Logic, and a proposition is the local aspect of Logic. Locality and Non-locality are also shown in Arithmetic where the arithmetical operation is its non-local aspect, and a number is its local aspect. In each of these different branches we find Non-locality and Locality to be a common property.

Education: Mathematical Dialogs with pre-school children – Interaction between child and adult. We believe that kindergarten is the natural environment for a growing mind to be trained to think parallel AND serial simultaneously, where Parallel thinking is more intuitive and Serial thinking is more analytical. In order to be developed both ways are needed and Organic Mathematics is focused on the educational methods that have to be developed in order to reinforce the bridging between Intuition (parallel thinking) and Analysis (serial thinking).

Computers: Quantum Computers – that is, computers that are based on Parallel AND Serial bridging.

Philosophy: Organic Thinking: Organic Mathematics bridges, using local and non-local notions, between - for example - 3 Poles using Character Thinking process in general and Mathematical Thought in particular:

Objective and Subjective – This means objective thinking – Looking at and analyzing things from outside of the Object, looking at the whole picture; or subjectively – Looking and analyzing the world from within the object itself.

Logical and Paradoxical – One can see the either the logic in things (as most Mathematicians do) or the paradox - which only some Mathematicians do. This also relates to modes of verification – proving something by finding the paradox (assuming false assumptions) or logically going step-by-step – getting the desired result. In either case Parallel AND Serial points of view are available for systematic use.

Reality and Imagination – These are the two primary worlds dealt with by the mind, both of which involve Mathematical thought.

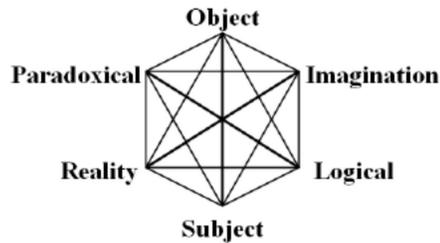


Figure 3:

6. Summary

We started by discussing Hilbert's Unity Problem. He discovered that the solution to this is closely related to understanding the relationship between Mathematics and Physics. Using Locality and Non-Locality we can build a new Mathematical language that fundamentally changes the way that Mathematics deals with Mathematical thinking itself. This will help in Solving Hilbert's 6-th problem and may make Hilbert's Vision come true.

Acknowledgements

Ofir Ben-Tov, thank you for your important suggestion to add *Distinction* as the third property of the *Organic Natural Numbers*, in addition to *Cardinality* and *Ordinality*.

Alon Navot, thank you for your support, and for recognizing the value for education in the application of “Dialogue in Mathematics”; and also for giving us the opportunity to apply it in seven kindergartens in Kiryat Tivon.

Yona Golan, thank you for the opportunity to act as Academic Advisor for your Mathematics experiment in kindergartens, conducted on behalf of the Ministry of Education in Israel.

Yosi Shitrit, thank you for the opportunity to apply “Dialogue in Mathematics” at your Rimolim School.

Shlomo Mendelovitz, thank you for long discussions which brought the concept of Organic thinking to this paper.

Chen Kupperman, thank you for translating this paper from Hebrew.

Susan Gurney, thank you for the English editing of the translated paper.

Lynda Schwartz, thank you for the final and most professional English editing.

Tom Ben-Yehuda, thank you for the Latex editing.

References

- [1] Alain Aspect, Experimental tests of realistic local theories via Bell’s theorem, *Phys. Rev. Lett.*, **47** (1981), 460.
- [2] J.S. Bell, On the Einstein-Podolsky-Rosen paradox, *Physics*, **1**, No. 195 (1964).
- [3] David Bohm, A suggested interpretation of the quantum theory in terms of “hidden variables”, *Physical Review*, **85** (1952), 166-179.
- [4] Albert Einstein, On the electrodynamics of moving bodies, *Annalen der Physik*, **17**, No. 891 (June 30, 1905); English translation by W. Perrett, G.B. Jeffery.
- [5] Albert Einstein, *Die Feldgleichungen der Gravitation (The Field Equations of Gravitation)*, Koniglich Preussische Akademie der Wissenschaften (1915), 844-847.

- [6] Einstein, Podolsky, Rosen, Can quantum-mechanical description of physical reality be considered complete?, *Phys. Rev.*, **47** (1935), 777-780.
- [7] David Hilbert, *The Foundations of Geometry*, Second Edition. Chicago, Open Court (1899).
- [8] David Hilbert, Mathematical problems, *Bulletin of The American Mathematical Society*, **37**, No. 4, 407-436.