International Journal of Pure and Applied Mathematics

Volume 50 No. 2 2009, 269-274

WAVE APPROACH IN NONLINEAR DYNAMIC PROBLEMS OF MULTI-MASS DISCRETE-CONTINUOUS SYSTEMS

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Abstract: The paper deals with the dynamics of nonlinear discrete-continuous systems. These systems consist of elastic elements connected by means of rigid bodies. In the discussion a wave method using the solution of the d'Alembert type is applied, what leads to equations with a retarded argument. Detailed considerations are done for a systems having three rods, two rigid bodies and a local nonlinearity.

AMS Subject Classification: 34C20, 34C25, 93C10
Key Words: dynamics of mechanical systems, waves, nonlinear oscillations, discrete-continuous models

1. Introduction

In the paper vibrations of nonlinear discrete-continuous systems are investigated. They consist of elastic elements connected by means of rigid bodies. In the discussion a wave method using the solution of the d'Alembert type is applied, what leads to solving equations with a retarded argument. The considered systems belong to a certain class of discrete-continuous systems, namely to those where the motion of elastic elements with a constant cross-section is described the classical wave equation. This concerns systems longitudinally and torsionally deformed, strings or systems subject to shear deformations. After a short description of the approach applied, detailed considerations are done for

Received: August 14, 2008

a system having three noncoaxial rods with variable cross-sections, two rigid bodies and a local nonlinearity described by the polynomial of the third degree. The local nonlinearity can have characteristics of a soft as well as of a hard type. The linear rod-rigid element system is discussed in Nadolski et al [2], while a nonlinear system with constant rod cross-sections is studied in Pielorz [3].

2. Wave Approach

Consider multi-mass discrete-continuous systems subject to longitudinal, torsional, shear or certain transversal deformations. These systems consist of elastic elements connected by rigid bodies. They are loaded by an external force $P(t)$. Elastic elements can have variable cross-sections described by the functions

$$A_i(x) = A_{0i}\left(1 - x/b_{0i}\right) , \tag{1}$$

where $A_i(x) = 0$ for $x = b_{0i}$ and $A_i(x) = A_{0i}$ is constant for $b_{0i} \to \pm\infty$. Then the determination of displacements U_i of the i-th elastic element is reduced to solving the following equation of motion

$$\frac{\partial^2 U_i}{\partial t^2} - a^2\left(\frac{\partial^2 U_i}{\partial t^2} - \frac{2}{b_{0i} - x}\frac{\partial U_i}{\partial x}\right) \tag{2}$$

and the solution is sought in the form

$$U_i(x,t) = \frac{1}{x - b_{0i}} f_i(a(t - t_{0i}) - x + x_{0i}) + \frac{1}{x - b_{0i}} g_i(a(t - t_{0i}) + x - x_{0i}, \tag{3}$$

where a is a wave speed, f_i and g_i represent waves propagating in the i-th elastic element in the direction consistent and opposite to x-axis direction. The constants t_{0i} and x_{0i} are the time instant and the end of the i-th rod, respectively, where the first disturbance caused by the external force $P(t)$ occurs. For the constant rod cross-section, equations (2) become classical wave equation and solutions are looking for only by means of the sum of the functions f_i and g_i, i.e., without denominators. To equations (2) one has to add zero initial conditions and appropriate boundary conditions which are conditions for displacements and forces acting in the cross-sections where rigid bodies are located. Upon substituting the assumed solutions into appropriate boundary conditions, ordinary differential equations with a retarded argument are obtained for uknown functions f_i and g_i, see Nadolski et al [2] and Pielorz [3], [4].

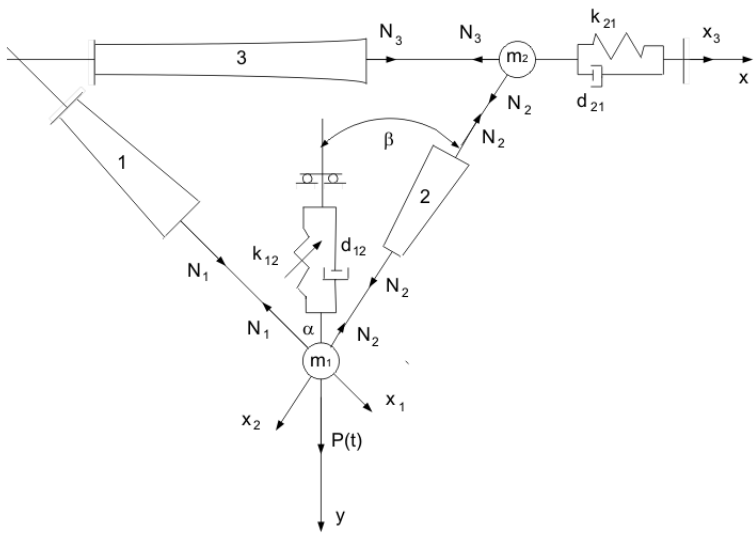

Figure 1: Two-mass rod system

3. Two-Mass Rod-Rigid Element System

Detailed considerations are given for a system shown in Figure 1 consisting of two rigid bodies, three rods and a local nonlinearity. The rods are not coaxial, so in the description a fixed reference system $0xy$ and one-dimensional coordinate systems $0_i x_i$ assigned to an individual i-th rod are used. The rods are characterized by the density ρ, Young's moduli E, variable cross-sections $A_i(x_i)$ and lengths l_i. The displacement for the i-th rod is described by the function $u_i(x_i, t)$ depending on the location of the considered cross-section x_i and on time, whereas the time function V_1 is the displacement of the rigid body having mass m_1 in the y-axis direction, see Nadolski et al [2] and Pielorz [3]. Such systems can represent segments in plane trusses with joints idealized as hinges without friction because truss members are subject then only to longitudinal deformations.

The force acting in a nonlinear spring is assumed to be described by the polynomial of the third degree

$$F_{sp}(V_1) = K_{121} V_1 + K_{123} V_1^3 \,, \tag{4}$$

where K_{121}, K_{123} are constants representing linear and nonlinear terms. This function is widely applied in nonlinear dynamics of discrete systems, Hagedorn

[1], and it includes the soft characteristic case for $K_{123} < 0$, the linear case for $K_{123} = 0$ and the hard characteristic case for $K_{123} > 0$. In the paper the both characteristic cases are taken into account.

In the appropriate nondimensional quantities, given in Nadolski et al [2] and Pielorz [3], the determination of displacements of rod cross-sections is reduced to solving equations (2) for $u_i(x_i, t)$, $i = 1, 2, 3$, with zero initial conditions and with the following boundary conditions

$$\begin{gathered} u_1(x_1,t) = 0 \text{ for } x_1 = 0 \text{ and } u_3(x_3,t) = 0 \text{ for } x_3 = 0, \\ u_1 \cos\beta - u_2 \cos\alpha = 0 \text{ for } x_1 = l_1,\ x_2 = l_2, \qquad (5) \\ -R_1 \frac{\partial^2 V_1}{\partial t^2} - d_{12}\frac{\partial V_1}{\partial t} - K_{121}V_1 - K_{123}V_1^3 \\ - A_1(x_1)K_1\cos\alpha\left(D_1\frac{\partial^2 u_1}{\partial x_1 \partial t} + \frac{\partial u_1}{\partial x_1}\right) \\ - A_2(x_2)K_2\cos\beta\left(D_2\frac{\partial^2 u_2}{\partial x_2 \partial t} + \frac{\partial u_2}{\partial x_2}\right) + P(t) = 0 \text{ for } x_1 = l_1,\ x_2 = l_2, \\ -R_2\frac{\partial^2 u_3}{\partial t^2} - d_{21}\frac{\partial u_3}{\partial t} - k_{21}u_3 - A_3(x_3)K_3\left(D_3\frac{\partial^2 u_3}{\partial x_3 \partial t} + \frac{\partial u_3}{\partial x_3}\right) \\ - A_2(x_2)K_2\sin\beta\left(D_2\frac{\partial^2 u_2}{\partial x_2 \partial t} + \frac{\partial u_2}{\partial x_2}\right) = 0, \text{ for } x_2 = 0,\ x_3 = l_3, \\ u_3\sin\beta + u_2 = 0 \text{ for } x_2 = 0,\ x_3 = l_3, \\ u_3\sin\beta + u_2 = 0 \text{ for } x_2 = 0,\ x_3 = l_3, \end{gathered}$$

where $V_1 = C_2u_1 + C_1u_2$, $C_1 = \sin\alpha/\sin(\alpha+\beta)$, $C_2 = \sin\beta/\sin(\alpha+\beta)$, $R_i = m_i/m_1$ and $K_i = A_{0i}\rho l_i/(m_1a^2)$. Solutions of equations (2) for $u_i(x_i,t)$ are looked for with $t_{01} = t_{02} = 0$, $t_{03} = l_2$, $x_{0i} = l_i$. Upon substituting them into the boundary conditions (5) the following equations for unknown functions f_i and g_i are obtained

$$\begin{gathered} f_1(z) = -g_1(z - 2l_1)\,, \\ f_2(z) = -g_2(z-2l_2) - b_{02}L_3\left[f_3(z-2l_2) + g_3(z-2l_2)\right]\sin\beta, \\ f_3(z) = -g_3(z-2l_3)\,, \qquad (6) \\ r_{11}g_1'' = P(z) + r_{12}g_1'(z) + r_{13}f_1''(z) + r_{14}f_1'(z) + r_{15}f_2''(z) \\ + r_{16}f_2'(z) + r_{17}\left[f_1(z) + g_1(z)\right] + r_{18}\left[f_1(z)+g_1(z)\right]^3\,, \\ g_2(z) = -f_2(z) + L_1L_2^{-1}\left[f_1(z)+g_1(z)\right]\cos\beta/\cos\alpha\,, \end{gathered}$$

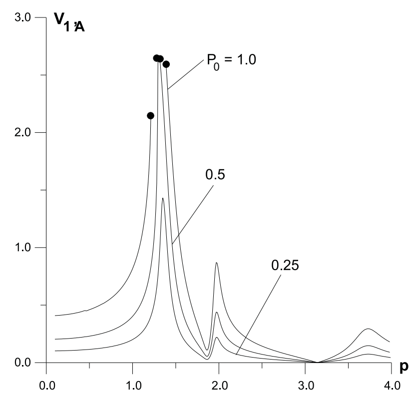

Figure 2: Amplitude-frequency curves for V_1 in a soft characteristic case

$$r_{31}g_3''(z) = r_{32}g_3'(z) + r_{33}f_3''(z) + r_{34}f_3'(z) + r_{35}g_2''(z) + r_{36}g_2'(z)r_{37} + [f_3(z) + g_3(z)] \ ,$$

where $L_i = 1/l_i - b_{0i}$ and coefficients r_{ij} are constant.

Numerical results are exemplary. In Figure 2 amplitude-frequency curves for displacements V_1 of the rigid body m_1 are presented in the soft characteristic case with $K_{123} = -0.05$ and in Figure 3 in the hard characteristic case with $K_{123} = 0.05$ for $P(t) = P_0 \sin pt$, $b_{0i} = -1000$, $R_1 = 0.625$, $R_2 = 0.1$, $K_1 = K_2 = K_3 = 1.0$, $k_{ij} = K_{121} = 1.05$, $D_i = d_{ij} = 0.1$. Diagrams show three resonant regions. Nonlinear effects are observed only in the first resonant region. They are escape phenomenon for a soft characterstic and jump phenomenon for a hard characteristic of the local nonlinearity. Dots in Figure 2 denote intervals of the frequency p of the external force where solutions begin to escape to infinity. These nonlinear effects are known in the dynamics of nonlinear discrete systems, see Hagedorn [1] and Stewart et al [5].

4. Final Remarks

In the paper it is proposed to use the wave approach in the dynamic analysis of certain discrete-continuous systems consisting of rigid bodies and elastic elements the motion of which is described by partial differential equations having solutions of the d'Alembert type. Detailed considerations are presented for the system having three noncoaxial rods with variable cross-sections, two rigid bodies and a local nonlinearity described by the polynomial of the third degree. It is shown that in the system loaded by the external force changing harmonically

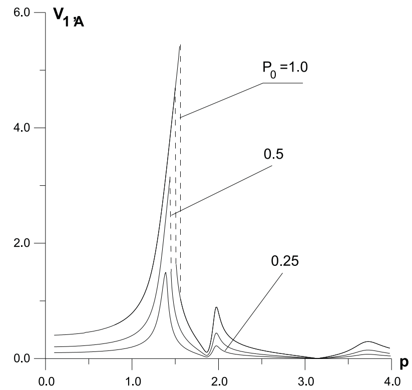

Figure 3: Amplitude-frequency curves for V_1 in a hard characteristic case

in time nonlinear effects occur in the first resonant region: in the form of the escape phenomenon for the local nonlinearity with the characteristic of a soft type and in the form of amplitude jumps for the local nonlinearity having the characteristic of a hard type.

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