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FUZZY MAGNIFIED TRANSLATION IN RINGS

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Abstract: The aim of this paper is to introduce the notion of fuzzy magnified translation of a fuzzy bi-ideal in a ring and to characterize it.

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1. Introduction, Definitions and Preliminaries

Zadeh [8] introduced the notion of a fuzzy subset A of a set X as a function from X into $[0,1]$. Rosenfeld [7] used this concept and developed some results in fuzzy group theory. Later Kuroki [2] introduced the notion of fuzzy ideals in semi-groups and Liu [5] studied them in rings. Lajos and Szasz [4] initiated the idea of bi-ideals in a ring. Kandasamy [3] and Majumder and Sardar [6] respectively used the concept of fuzzy translation and fuzzy magnified translation in fuzzy group theory. In this paper we introduce the notion of fuzzy magnified translation of fuzzy bi-ideals in rings and improve some earlier results.

We now review some definitions that are used in this paper.

Definition 1. (see [7]) Let μ_i, $i \in I$ be fuzzy subsets of a ring R. The intersection of the fuzzy sets μ_i is defined as follows:

$$[\cap \mu_i](x) = \inf_{i \in I} [\mu_i(x)], \quad x \in X.$$

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Definition 2. (see [7]) Let μ be a fuzzy subset of a ring R. Imμ is defined as

$$\mathrm{Im}\mu = \{t \in [0,1] \mid \mu(x) = t \text{ for some } x \in X\}.$$

Definition 3. (see [4]) A sub-ring B of a ring A is called a bi-ideal of A if $BAB \subseteq B$ holds where BAB is the additive subgroup of A generated by the set of all elements of the form bab, $b \in B$ and $a \in A$.

Definition 4. (see [7]) A fuzzy subset μ of a ring R is called a fuzzy left(right) ideal of R, if for every $x, y \in R$,:

(i) μ is a fuzzy subgroup of $(R, +)$, i.e.,

$$\mu(x-y) \geq \min\{\mu(x), \mu(y)\} \text{ and}$$

(ii) $\mu(xy) \geq \mu(y)\,(\mu(xy) \geq \mu(x))$.

If μ is both a fuzzy left ideal and a fuzzy right ideal of R, then it is called a fuzzy ideal of R.

Definition 5. (see [1]) A non-empty fuzzy subset μ of a ring R (i.e. $\mu(x) \neq 0$ for some $x \in R$) is called a fuzzy bi-ideal of R if:

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ and

(iii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in R$.

Example 1. Let R be the ring of all 2×2 matrices over the ring of integers with respect to the matrix addition and multiplication. Let μ be a fuzzy subset of R defined as follows:

$$\mu\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = 1 \text{ if } a = b = c = d = 0$$

$$= \frac{1}{2} \text{ if } a \text{ is a non-zero even integer and } b = c = d = 0$$

$$= \frac{1}{3} \text{ if } a \text{ is a non-zero odd integer and } b = c = d = 0$$

$$= 0 \text{ in all other cases.}$$

Then μ is a fuzzy bi-ideal of R.

Definition 6. Let μ be a non-empty fuzzy subset of a ring R (i.e. $\mu(x) \neq 0$ for some $x \in R$) and also let $\alpha \in [0, 1 - \sup\{\mu(x) : x \in R\}]$, $\beta \in [0,1]$. Then the fuzzy magnified translation $\mu^c_{\beta\alpha}$ of μ in R is defined as

$$\mu^c_{\beta\alpha}(x) = \beta.\mu(x) + \alpha \text{ for all } x \in R.$$

It is also a fuzzy subset of R.

In particular if $\beta = 1$ then μ_{α}^{T} is called the fuzzy translation of μ, i.e.,

$$\mu_{\alpha}^{T}(x) = \mu(x) + \alpha \text{ for all } x \in R.$$

Also when $\alpha = 0$ then μ_{β}^{M} is called the fuzzy multiplication of μ, i.e.,

$$\mu_{\beta}^{M}(x) = \beta\mu(x) \text{ for all } x \in R.$$

2. Theorems

In this section we present the main results of our paper.

Theorem 1. *Let μ be a fuzzy bi-ideal of a ring R. Then the fuzzy magnified translation $\mu_{\beta\alpha}^{c}$ is also a fuzzy bi-ideal of R.*

Proof. Let μ be a fuzzy bi-ideal of a ring R. Now for all $x, y, z \in R$,

$$\mu_{\beta\alpha}^{c}(x-y) = \beta.\mu(x-y) + \alpha$$
$$\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\},$$
$$\text{i.e., } \mu_{\beta\alpha}^{c}(x-y) \geq \min\left\{\mu_{\beta\alpha}^{c}(x), \mu_{\beta\alpha}^{c}(y)\right\}.$$

Again

$$\mu_{\beta\alpha}^{c}(xy) = \beta.\mu(xy) + \alpha$$
$$\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\},$$
$$\text{i.e., } \mu_{\beta\alpha}^{c}(xy) \geq \min\left\{\mu_{\beta\alpha}^{c}(x), \mu_{\beta\alpha}^{c}(y)\right\}.$$

Also

$$\mu_{\beta\alpha}^{c}(xyz) = \beta.\mu(xyz) + \alpha$$
$$\geq \beta \min\{\mu(x), \mu(z)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(z) + \alpha\},$$
$$\text{i.e., } \mu_{\beta\alpha}^{c}(xyz) \geq \min\left\{\mu_{\beta\alpha}^{c}(x), \mu_{\beta\alpha}^{c}(z)\right\}.$$

Thus $\mu_{\beta\alpha}^{c}$ is a fuzzy bi-ideal of R. □

Corollary 1. *If μ be a fuzzy bi-ideal of a ring R then the fuzzy translation μ_{α}^{T} of μ is also a fuzzy bi-ideal of R.*

Proof. Taking $\beta = 1$ the above corollary follows from Theorem 1. □

Corollary 2. *Let μ be a fuzzy bi-ideal of a ring R. Then the fuzzy multiplication μ_{β}^{M} of μ is also a fuzzy bi-ideal of R.*

Proof. Taking $\alpha = 0$ Corollary 2 follows from Theorem 1. □

Theorem 2. *If μ be a fuzzy left (right, two-sided) ideal of a ring R then* *the fuzzy magnified translation* $\mu^c_{\beta\alpha}$ *of* μ *is a fuzzy bi-ideal of* R.

Proof. Let μ be a fuzzy left (right, two-sided) ideal of a ring R. Then for all $x, y \in R$,

$$\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$$

and

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}.$$

Also let $x, y, z \in R$. Then

$$\mu(xyz) = \mu((xy)z) \geq \mu(z) \geq \min\{\mu(x), \mu(z)\}.$$

Thus μ will be a fuzzy bi-ideal of R (cf. [1], Proposition 2.3.4).

Now for all $x, y \in R$,

$$\begin{aligned}\mu^c_{\beta\alpha}(x-y) &= \beta.\mu(x-y) + \alpha \\ &\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta\mu(x) + \alpha, \beta\mu(y) + \alpha\}, \\ &\text{i.e., } \mu^c_{\beta\alpha}(xy) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}.\end{aligned}$$

Again

$$\begin{aligned}\mu^c_{\beta\alpha}(xy) &= \beta.\mu(xy) + \alpha \\ &\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\}, \\ &\text{i.e., } \mu^c_{\beta\alpha}(xy) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}.\end{aligned}$$

Also let $x, y, z \in R$. Now

$$\begin{aligned}\mu^c_{\beta\alpha}(xyz) &= \beta.\mu(xyz) + \alpha \\ &\geq \beta.\mu(z) + \alpha = \mu^c_{\beta\alpha}(z) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(z)\right\}.\end{aligned}$$

So $\mu^c_{\beta\alpha}$ is a fuzzy bi-ideal of R.

Similarly we can prove the other statements. □

Corollary 3. *Let* μ *be a fuzzy left (right, two-sided) ideal of a ring* R. *Then the fuzzy translation* μ^T_α *of* μ *is a fuzzy bi-ideal of* R.

Proof. Taking $\beta = 1$ Corollary 3 follows from Theorem 2. □

Corollary 4. *If* μ *be a fuzzy left (right, two-sided) ideal of a ring* R *then the fuzzy multiplication* μ^M_β *of* μ *is a fuzzy bi-ideal of* R.

Proof. Taking $\alpha = 0$ the above corollary follows from Theorem 2. □

Remark 1. The converses of Theorem 2, Corollary 3 and Corollary 4 are not true. Taking $\beta = 1$ and $\alpha = 0$ the fuzzy subset μ given in Example 1 is a fuzzy bi-ideal of R but it is not a fuzzy left ideal of R.

Theorem 3. *The fuzzy magnified translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a ring R is a fuzzy bi-ideal of R if it is not empty.*

Proof. Let $\mu_i\,(i \in I)$ be an arbitrary collection of fuzzy bi-ideals of R and $\mu = \underset{i\in I}{\cap}\mu_i$ be not empty. Let $x, y \in R$. Then

$$\begin{aligned}\mu\,(x-y) &= \left(\underset{i\in I}{\cap}\mu_i\right)(x-y) = \inf_{i\in I}\{\mu_i\,(x-y)\} \geq \inf_{i\in I}[\min\{\mu_i\,(x)\,,\mu_i\,(y)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i\,(x)\}\,,\inf_{i\in I}\{\mu_i\,(y)\}\right] = \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x)\,,\left(\underset{i\in I}{\cap}\mu_i\right)(y)\right\},\end{aligned}$$

and

$$\begin{aligned}\left(\underset{i\in I}{\cap}\mu_i\right)(xy) &= \inf_{i\in I}\{\mu_i\,(xy)\} \geq \inf_{i\in I}[\min\{\mu_i\,(x)\,,\mu_i\,(y)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i\,(x)\}\,,\inf_{i\in I}\{\mu_i\,(y)\}\right] = \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x)\,,\left(\underset{i\in I}{\cap}\mu_i\right)(y)\right\}.\end{aligned}$$

Again for all $x, y, z \in R$,

$$\begin{aligned}\left(\underset{i\in I}{\cap}\mu_i\right)(xyz) &= \inf_{i\in I}\{\mu_i\,(xyz)\} \geq \inf_{i\in I}[\min\{\mu_i\,(x)\,,\mu_i\,(z)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i\,(x)\}\,,\inf_{i\in I}\{\mu_i\,(z)\}\right] = \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x)\,,\left(\underset{i\in I}{\cap}\mu_i\right)(z)\right\}.\end{aligned}$$

Hence $\mu = \underset{i\in I}{\cap}\mu_i$ is a fuzzy bi-ideal of R. (cf. [1], Proposition 2.3.6). Thus Theorem 3 follows from Theorem 1. □

We may now state the following two corollaries without proof.

Corollary 5. *The fuzzy translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a ring R is a fuzzy bi-ideal of R if it is not empty.*

Corollary 6. *The fuzzy multiplication of the intersection of an arbitrary collection of fuzzy bi-ideals of a ring R is a fuzzy bi-ideal of R if it is not empty.*

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