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ON FUZZY MAGNIFIED TRANSLATION IN HEMIRINGS

Sanjib Kumar Datta[§]

Department of Mathematics
University of North Bengal
Raja Rammohunpur, District Darjeeling
PIN Code 734013, West Bengal, INDIA
e-mail: sanjib_kr_datta@yahoo.co.in

Abstract: In this paper we introduce the concept of fuzzy magnified translation of a fuzzy bi-ideal in a hemiring and study some properties of it.

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1. Introduction, Definitions and Preliminaries

The concept of a fuzzy subset of a set was first introduced by L.A. Zadeh [8]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belongs to a set. Using this concept Rosenfeld [7] established some results in fuzzy group theory. Later Kuroki [1] initiated the notion of fuzzy ideals in semigroups and Liu [4] studied them in rings. Lajos and Szasz [3] introduced the idea of bi-ideals in a ring. Kandasamy [2] and Majumder and Sarder [5] respectively explored on the idea of fuzzy translation and fuzzy magnified translation in fuzzy group theory.

An algebra $(R;+,.)$ is said to be a semiring if $(R;+)$ and $(R;.)$ are semigroups satisfying $a.(b+c)=a.b+a.c$ and $(b+c).a=b.a+c.a$ for all $a,b,c \in R$. A semiring R is said to be additively commutative if $a+b=b+a$ for all $a,b \in R$. A semiring R may have an identity 1, defined by $1.a=a=a.1$ and a zero 0,

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[§]Correspondence address: 25, School Road, Kalianibash, Barrackpore, P.O.: Nonachandanpukur, Dist.: 24 Parganas (North), P.S.: Titagarh, PIN Code: 743102, West Bengal, INDIA

defined by $0 + a = a = a + 0$ and $a.0 = 0 = 0.a$ for all $a \in R$. A semiring R is said to be a hemiring if it is additively commutative with zero. In this paper we introduce the notion of fuzzy magnified translation of fuzzy bi-ideals in hemirings and develop some results. Henceforth we denote a hemiring by H unless otherwise stated.

We now review some definitions that are used in this paper.

Definition 1. (see [7]) Let X be a non empty set. A fuzzy subset μ of X is a function $\mu : X \to [0,1]$. Let μ_i, $i \in I$ be fuzzy subsets of a ring R. The intersection of the fuzzy sets μ_i is defined as follows:

$$[\cap \mu_i](x) = \inf_{i \in I} [\mu_i(x)], \quad x \in X.$$

Definition 2. (see [6]) Let μ be a fuzzy subset of a hemiring H. Imμ is defined by

$$\text{Im}\mu = \{t \in [0,1] \mid \mu(x) = t \text{ for some } x \in X\}.$$

Let $t \in [0,1]$. The set

$$\mu_t = \{x \in \mid \mu(x) \geq t\}$$

is called a level subset of μ.

Clearly, $\mu_t \subseteq \mu_s$ whenever $t \geq s$.

Definition 3. (see [3]) A sub-ring S of a ring R is called a bi-ideal of R if $SRS \subseteq S$ holds where SRS is the additive subgroup of R generated by the set of all elements of the form srs, $s \in S$ and $r \in R$.

Definition 4. (see [6]) A fuzzy subset μ of a hemiring H is called a fuzzy left(right) ideal of H, if for every $x, y \in H$:

(i) μ is a fuzzy subgroup of $(H, +)$, i.e.,

$$\mu(x+y) \geq \min\{\mu(x), \mu(y)\} \text{ and}$$

(ii) $\mu(xy) \geq \mu(y) \, (\mu(xy) \geq \mu(x))$.

If μ is both a fuzzy left ideal and a fuzzy right ideal of H, then it is called a fuzzy ideal of H.

Definition 5. A non-empty fuzzy subset μ of a hemiring H (i.e. $\mu(x) \neq 0$ for some $x \in H$) is called a fuzzy bi-ideal of H if:

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ and

(iii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in H$.

Definition 6. Let μ be a non-empty fuzzy subset of a hemiring H (i.e.

$\mu(x) \neq 0$ for some $x \in H$) and also let $\alpha \in [0, 1 - \sup\{\mu(x) : x \in H\}]$, $\beta \in [0,1]$. Then the fuzzy magnified translation $\mu^c_{\beta\alpha}$ of μ in H is defined as

$$\mu^c_{\beta\alpha}(x) = \beta.\mu(x) + \alpha \text{ for all } x \in H.$$

It is also a fuzzy subset of H.

In particular if $\beta = 1$ then μ^T_α is called the fuzzy translation of μ, i.e.,

$$\mu^T_\alpha(x) = \mu(x) + \alpha \text{ for all } x \in H.$$

Also when $\alpha = 0$ then μ^M_β is called the fuzzy multiplication of μ, i.e.,

$$\mu^M_\beta(x) = \beta\mu(x) \text{ for all } x \in H.$$

2. Theorems

In this section we present the main results of our paper.

Theorem 1. *Let μ be a non empty fuzzy subset of a hemiring H. Then μ is a fuzzy bi-ideal of H iff the fuzzy magnified translation $\mu^c_{\beta\alpha}$ of μ is a fuzzy bi-ideal of H.*

Proof. Let μ be a fuzzy bi-ideal of a hemiring H.

Now for all $x, y, z \in H$,

$$\begin{aligned} \mu^c_{\beta\alpha}(x+y) &= \beta.\mu(x+y) + \alpha \\ &\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\}, \end{aligned}$$

$$\mu^c_{\beta\alpha}(x+y) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}.$$

Again

$$\begin{aligned} \mu^c_{\beta\alpha}(xy) &= \beta.\mu(xy) + \alpha \\ &\geq \beta \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\}, \end{aligned}$$

$$\mu^c_{\beta\alpha}(xy) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}.$$

Also

$$\begin{aligned} \mu^c_{\beta\alpha}(xyz) &= \beta.\mu(xyz) + \alpha \\ &\geq \beta \min\{\mu(x), \mu(z)\} + \alpha = \min\{\beta.\mu(x) + \alpha, \beta.\mu(z) + \alpha\}, \end{aligned}$$

$$\mu^c_{\beta\alpha}(xyz) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(z)\right\}.$$

Thus $\mu^c_{\beta\alpha}$ is a fuzzy bi-ideal of H.

Conversely let $\mu^c_{\beta\alpha}$ be a fuzzy bi-ideal of H.

Then for all $x, y, z \in H$,

$$\mu(x+y) = \frac{1}{\beta}\left[\mu^c_{\beta\alpha}(x+y) - \alpha\right] \geq \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\} - \alpha\right]$$

$$= \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x) - \alpha, \mu^c_{\beta\alpha}(y) - \alpha\right\}\right] = \min\left\{\frac{\mu^c_{\beta\alpha}(x) - \alpha}{\beta}, \frac{\mu^c_{\beta\alpha}(y) - \alpha}{\beta}\right\},$$

$$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}.$$

Again

$$\mu(xy) = \frac{1}{\beta}\left[\mu^c_{\beta\alpha}(xy) - \alpha\right] \geq \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\} - \alpha\right]$$

$$= \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x) - \alpha, \mu^c_{\beta\alpha}(y) - \alpha\right\}\right] = \min\left\{\frac{\mu^c_{\beta\alpha}(x) - \alpha}{\beta}, \frac{\mu^c_{\beta\alpha}(y) - \alpha}{\beta}\right\},$$

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}.$$

Also

$$\mu(xyz) = \frac{1}{\beta}\left[\mu^c_{\beta\alpha}(xyz) - \alpha\right] \geq \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(z)\right\} - \alpha\right]$$

$$= \frac{1}{\beta}\left[\min\left\{\mu^c_{\beta\alpha}(x) - \alpha, \mu^c_{\beta\alpha}(z) - \alpha\right\}\right] = \min\left\{\frac{\mu^c_{\beta\alpha}(x) - \alpha}{\beta}, \frac{\mu^c_{\beta\alpha}(z) - \alpha}{\beta}\right\},$$

$$\mu(xyz) \geq \min\{\mu(x), \mu(z)\}.$$

Hence μ is a fuzzy bi-ideal of H. □

Corollary 1. *Let μ be a non empty fuzzy subset of a hemiring H. Then μ is a fuzzy bi-ideal of H iff the fuzzy translation μ^T_α of μ is a fuzzy bi-ideal of H.*

Proof. Taking $\beta = 1$ the above corollary follows from Theorem 1. □

Corollary 2. *Let μ be a non empty fuzzy subset of a hemiring H. Then μ is a fuzzy bi-ideal of H iff the fuzzy multiplication μ^M_β of μ is a fuzzy bi-ideal of H.*

Proof. Taking $\alpha = 0$ Corollary 2 follows from Theorem 1. □

Theorem 2. *If μ is a fuzzy left (right, two-sided) ideal of a hemiring H then the fuzzy magnified translation $\mu^c_{\beta\alpha}$ of μ is a fuzzy bi-ideal of H.*

Proof. Let μ be a fuzzy left (right, two-sided) ideal of a hemiring H.

Then for all $x, y \in H$,

$$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$$

and

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}.$$

Also let $x, y, z \in H$. Then

$$\mu(xyz) = \mu((xy)z) \geq \mu(z) \geq \min\{\mu(x), \mu(z)\}.$$

Thus μ will be a fuzzy bi-ideal of H.

Now for all $x, y \in H$,

$$\begin{aligned} \mu^c_{\beta\alpha}(x+y) = \beta.\mu(x+y) + \alpha \geq \beta \min\{\mu(x), \mu(y)\} + \alpha \\ = \min\{\beta\mu(x) + \alpha, \beta\mu(y) + \alpha\} \\ i.e.,\ \mu^c_{\beta\alpha}(xy) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}. \end{aligned}$$

Again

$$\begin{aligned} \mu^c_{\beta\alpha}(xy) = \beta.\mu(xy) + \alpha \geq \beta \min\{\mu(x), \mu(y)\} + \alpha \\ = \min\{\beta.\mu(x) + \alpha, \beta.\mu(y) + \alpha\} \\ i.e.,\ \mu^c_{\beta\alpha}(xy) \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(y)\right\}. \end{aligned}$$

Also let $x, y, z \in H$. Now

$$\begin{aligned} \mu^c_{\beta\alpha}(xyz) = \beta.\mu(xyz) + \alpha \geq \beta.\mu(z) + \alpha = \mu^c_{\beta\alpha}(z) \\ \geq \min\left\{\mu^c_{\beta\alpha}(x), \mu^c_{\beta\alpha}(z)\right\}. \end{aligned}$$

So $\mu^c_{\beta\alpha}$ is a fuzzy bi-ideal of H.

Similarly we can prove the other statements. □

Corollary 3. *Let μ be a fuzzy left (right, two-sided) ideal of a hemiring H. Then the fuzzy translation μ^T_α of μ is a fuzzy bi-ideal of H.*

Proof. Taking $\beta = 1$ Corollary 3 follows from Theorem 2. □

Corollary 4. *If μ be a fuzzy left (right, two-sided) ideal of a hemiring H then the fuzzy multiplication μ^M_β of μ is a fuzzy bi-ideal of H.*

Proof. Taking $\alpha = 0$ the above corollary follows from Theorem 2. □

Theorem 3. *The fuzzy magnified translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a hemiring H is a fuzzy bi-ideal of H if it is not empty.*

Proof. Let $\mu_i\,(i \in I)$ be an arbitrary collection of fuzzy bi-ideals of H and $\mu = \bigcap_{i \in I} \mu_i$ be not empty.

Let $x, y \in H$. Then

$$\begin{aligned} \mu(x+y) &= \left(\underset{i\in I}{\cap}\mu_i\right)(x+y) = \inf_{i\in I}\{\mu_i(x+y)\} \\ &\geq \inf_{i\in I}[\min\{\mu_i(x), \mu_i(y)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i(x)\}, \inf_{i\in I}\{\mu_i(y)\}\right] \\ &= \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x), \left(\underset{i\in I}{\cap}\mu_i\right)(y)\right\} \end{aligned}$$

and

$$\begin{aligned} \left(\underset{i\in I}{\cap}\mu_i\right)(xy) &= \inf_{i\in I}\{\mu_i(xy)\} \\ &\geq \inf_{i\in I}[\min\{\mu_i(x), \mu_i(y)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i(x)\}, \inf_{i\in I}\{\mu_i(y)\}\right] \\ &= \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x), \left(\underset{i\in I}{\cap}\mu_i\right)(y)\right\}. \end{aligned}$$

Again for all $x, y, z \in H$,

$$\begin{aligned} \left(\underset{i\in I}{\cap}\mu_i\right)(xyz) &= \inf_{i\in I}\{\mu_i(xyz)\} \\ &\geq \inf_{i\in I}[\min\{\mu_i(x), \mu_i(z)\}] \\ &= \min\left[\inf_{i\in I}\{\mu_i(x)\}, \inf_{i\in I}\{\mu_i(z)\}\right] \\ &= \min\left\{\left(\underset{i\in I}{\cap}\mu_i\right)(x), \left(\underset{i\in I}{\cap}\mu_i\right)(z)\right\}. \end{aligned}$$

Hence $\mu = \underset{i\in I}{\cap}\mu_i$ is a fuzzy bi-ideal of H. Thus Theorem 3 follows from Theorem 1. □

We may now state the following two corollaries without proof.

Corollary 5. *The fuzzy translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a hemiring H is a fuzzy bi-ideal of H if it is not empty.*

Corollary 6. *The fuzzy multiplication of the intersection of an arbitrary collection of fuzzy bi-ideals of a hemiring H is a fuzzy bi-ideal of H if it is not empty.*

The proofs of the following two theorems are straight forward and therefore are omitted.

Theorem 4. *A subset A of a hemiring H is a bi-ideal of H iff its characteristic function χ_A is a fuzzy bi-ideal of H.*

Theorem 5. *Let μ be a fuzzy subset of a hemiring H. μ is a fuzzy bi-ideal of H iff its level sets μ_t's are bi-ideals of H for all $t \in Im\mu$.*

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