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DATA MINING IN OBSERVER'S MATHEMATICS

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Abstract: This work considers Data Mining aspects in a setting of arithmetic provided by Observer's Mathematics (see www.mathrelativity.com). We prove that Data Mining methods based on Observer's Mathematics are more robust than classical methods for certain feature spaces. We further present applications of Observer's Mathematics to data mining problems in Physics and Genetics. Certain theoretical results and communications pertaining to these theorems are also provided.

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1. Introduction

The following discussion is based on the work introduced in [1]. Further information can also be found in [2] and [3]. We consider a finite well-ordered system of observers, where each observer sees the real numbers as the set of all infinite decimal fractions. The observers are ordered by their level of "depth", i.e. each observer has a depth number (hence, we have the regular integer or-

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dering), such that an observer with depth k sees that an observer with depth $n < k$ sees and deals (to be defined below) not with an infinite set of infinite decimal fractions, but, actually, with a finite set of finite decimal fractions. We call this set W_n, i.e. it is the set of all decimal fractions, such that there are at most than n digits in the integer part and n digits in the decimal part of the fraction. Visually, an element in W_n looks like $\underbrace{\_\ \ldots\ \_}_{n}.\underbrace{\_\ \ldots\ \_}_{n}$. Moreover, an observer with a given depth is unaware (or can only assume the existence) of observers with larger depth values and for his purposes, he deals with "infinity". These observers are called *naive*, with the observer with the lowest depth number – the most naive. However, if there is an observer with a higher depth number, he sees that a given observer actually deals with a finite set of finite decimal fractions, and so on. Therefore, if we fix an observer, then this observer sees the sets $W_{n_1}, \ldots, W_{n_k}$ with $n_1 < \ldots < n_k$ indicating the depth level, and realizes that the corresponding observers see and deal with infinity. When we talk about observers, we shall always have some fixed observer (called 'us') who oversees all others and realizes that they are naive. The "W_n-observer" is the abbreviation for somebody who *deals* with W_n while thinking that he deals with infinity.

The following sections describe application of the idea of relativity in mathematics to various mathematical fields.

2. Arithmetic

We begin by defining sets W_n which consist of all finite decimal fractions such that there are at most n digits in the integer part and at most n digits in the decimal part. That is, the set W_n contains all elements of the form $a = a_0.a_1...a_n$ where the integer part can be written as $a_0 = b_{n-1}...b_0$, where $b_{n-1}, ..., b_0, a_1, ..., a_n \in \{0, 1, ..., 9\}$. If $n < m$, then W_n naturally embeds into W_m by placing 0's in the $n + 1$-st through m-th decimal places. We call the embedding $\varphi_{n,m} : W_n \to W_m$. Here are some examples: let $2.34 \in W_2$ and then $\varphi_{2,4}(2.34) = 2.3400 \in W_4$. Similarly, W_m projects onto W_n by cutting off the superfluous digits on the right of the decimal point. Let $\phi_{m,n} : W_m \to W_n$ be the projection, then, for example, if $45.4301 \in W_4$, then $\phi_{4,2}(45.4301) = 45.43 \in W_2$. If the integer part of a fraction contains more than n digits, then $\phi_{m,n}$ is not defined.

Now, given $c = c_0.c_1...c_n$, $d = d_0.d_1...d_n \in W_n$ we endow W_n with the following arithmetic $(+_n, -_n, \times_n, \div_n)$:

Definition 1. Addition and subtraction

$$c \pm_n d = \begin{cases} c \pm d, \text{ if } c \pm d \in W_n \\ \text{not defined, if } c \pm d \notin W_n \end{cases}$$

and we write $((\ldots(c_1 +_n c_2)\ldots) +_n c_N) = \sum_{i=1}^{N} {}^n c_i$ for $c_1, \ldots, c_N$ iff the contents of any parenthesis are in W_n.

Definition 2. Multiplication

$$c \times_n d = \sum_{k=0}^{n} {}^n \sum_{m=0}^{n-k} {}^n 0.\underbrace{0\ldots0}_{k-1} c_k \cdot 0.\underbrace{0\ldots0}_{m-1} d_m\,,$$

where $c, d \geq 0$, $c_0 \cdot d_0 \in W_n$, $0.\underbrace{0\ldots0}_{k-1} c_k \cdot 0.\underbrace{0\ldots0}_{m-1} d_m$ is the standard product, and $k = m = 0$ means that $0.\underbrace{0\ldots0}_{k-1} c_k = c_0$ and $0.\underbrace{0\ldots0}_{m-1} d_m = d_0$. If either $c < 0$ or $d < 0$, then we compute $|c| \times_n |d|$ and define $c \times_n d = \pm |c| \times_n |d|$, where the sign $\pm$ is defined as usual. Note, if the content of at least one parentheses (in previous formula) is not in W_n, then $c \times_n d$ is not defined.

Definition 3. Division

$$c \div_n d = \begin{cases} r, \text{ if } \exists!\ r \in W_n \quad r \times_n d = c \\ \text{not defined, if no such } r \text{ exists or it is not unique.} \end{cases}$$

Let $n = 2$, so we are in W_2. Here are some examples of elements of W_2: $3.14, -99, 0.1 \in W_2$ and $0.115, 123.9, -100000 \notin W_2$. Now, the examples of arithmetic: $2.08 +_2 11.9 = 13.98$; $(-2.08) +_2 11.9 = 9.82$; $80 +_2 24 =$ not defined; $21.36 -_2 0.87 = 20.49$; $1.36 -_2 16.95 = -15.59$; $1.36 -_2 (-99.95) =$ not defined; $11 \times_2 8 = 88$; $(-5) \times_2 19 = -95$; $11 \times_2 12 =$ not defined; $3.41 \times_2 2.64 = 8.98$; $3.41 \times_2 (-2.64) = -8.98$; $3.41 \times_2 42.64 =$ not defined; $99.41 \times_2 1.64 =$ not defined; $0.85 \times_2 0.02 = 0$; $80 \div_2 4 = 20$; $1 \div_n 0.5 =$ not defined (since we get 10 different r's); $1 \div_n 3 =$ not defined (since no r exists).

3. Derivatives

From the point of view of W_n-observer (we will call such observers "naive", since they "think" that they "live" in W and deal with W) a real function y of a real variable x, $y = y(x)$, is called differentiable at $x = x_0$ (see [4]) if there is a derivative

$$y'(x_0) = \lim_{x \to x_0, x \neq x_0} \frac{y(x) - y(x_0)}{x - x_0}.$$

What does the above statement mean from point of view of W_m-observer with $m > n$? It means that

$$|(y(x) -_n y(x_0)) -_n (y'(x_0) \times_n (x -_n x_0))| \leq 0.\underbrace{0\ldots01}_{n},$$

whenever

$$|y(x) -_n y(x_0)| = 0.\underbrace{0\ldots0y_l}_{l} y_{l+1}\ldots y_n \text{ and } |(x -_n x_0)| = 0.\underbrace{0\ldots0x_k}_{k} x_{k+1}\ldots x_n,$$

for $1 \leq k$, $l \leq n$, and x_k – non-zero digit.

We now state the main theorems.

Theorem 1. *From the point of view of a W_m-observer a derivative calculated by a W_n-observer $(m > n)$ is not defined uniquely.*

Proof. Put $y'(x_0) = \pm a_0.a_1\ldots a_p a_{p+1}\ldots a_n$ with $a_0.a_1\ldots a_p a_{p+1}\ldots a_n \geq 0$ and $p \leq n$. Then $0.\underbrace{0\ldots0y_l}_{l} y_{l+1}\ldots y_n = a_0.a_1\ldots a_p a_{p+1}\ldots a_n \times_n 0.\underbrace{0\ldots0x_k}_{k} x_{k+1}\ldots x_n = a_0.a_1\ldots a_p b_{p+1}\ldots b_n \times_n 0.\underbrace{0\ldots0x_k}_{k} x_{k+1}\ldots x_n$ for any digits $b_{p+1}, \ldots, b_n$ and $p = n-k$. Hence $y'(x_0) \in V = \{\pm a_0.a_1\ldots a_p a_{p+1}\ldots a_n | a_{p+1}, \ldots, a_n \in \{0, 1, \ldots, 9\}\}$ and $|V| = 10^k$. □

Theorem 2. *From the point of view of a W_m-observer with $m > n$, $|y'(x_0)| \leq C_n^{l,k}$, where $C_n^{l,k} \in W_n$ is a constant defined only by n, l, k and not dependent on $y(x)$.*

Proof. We have $\pm 0.\underbrace{0\ldots0y_l}_{l} y_{l+1}\ldots y_n = (\pm a_0.a_1\ldots a_n) \times_n (\pm 0.\underbrace{0\ldots0x_k}_{k} x_{k+1}\ldots x_n)$ with x_k - non-zero digit and $a_0.a_1\ldots a_p a_{p+1}\ldots a_n \geq 0$. Now, if $l > k$ then $a_0 = 0$; if $l = k$ then $a_0 \leq 9$ and if $l < k$ then $a_0 < 9 \times 10^{k-1}$. Hence

$$C_n^{l,k} = \begin{cases} 1, \text{ if } l > k, \\ 10, \text{ if } l = k, \\ 9 \times 10^{k-1}, \text{ if } l < k. \end{cases}$$ □

Theorem 3. *From the point of view of a W_m-observer, when a W_n-observer (with $m > n \geq 3$) calculates the second derivative:*

$$y''(x_0) = \lim_{x_1 \to x_0, x_1 \neq x_0, x_2 \to x_0, x_2 \neq x_0, x_3 \to x_1, x_3 \neq x_1} \frac{\frac{y(x_3) - y(x_1)}{(x_3 - x_1)} - \frac{y(x_2) - y(x_0)}{x_2 - x_0}}{x_1 - x_0}$$

we get the following unequality:

$$(|x_2 -_n x_0| \times_n |x_3 -_n x_1|) \times_n |x_1 -_n x_0| \geq 0.\underbrace{0\ldots01}_{n}$$

provided that $y''(x_0) \neq 0$.

Proof. For the W_m-observer existence of $y''(x_0)$ means that $|((y(x_3) -_n y(x_1)) \times_n (x_2 -_n x_0) -_n ((y(x_2) -_n y(x_0)) \times_n (x_2 -_n x_0))) -_n y''(x_0) \times_n ((|x_2 -_n x_0| \times_n |x_3 -_n x_1|) \times_n |x_1 -_n x_0|)| \leq 0.\underbrace{0\ldots01}_{n}$, whenever

$$|(x_2 -_n x_0)| \leq 0.\overbrace{\underbrace{0\ldots0}_{k}p*\ldots*}^{n},$$

and

$$|(x_3 -_n x_1)| \leq 0.\overbrace{\underbrace{0\ldots0}_{l}q*\ldots*}^{n} \quad \text{and} \quad |(x_1 -_n x_0)| \leq 0.\overbrace{\underbrace{0\ldots0}_{s}r*\ldots*}^{n},$$

where p, q, r are non-zero digits, asterisks are any digits and $3 \leq k + l + s \leq n$. Then given $y''(x_0) \neq 0$ we have $(|x_2 -_n x_0| \times_n |x_3 -_n x_1|) \times_n |x_1 -_n x_0| \geq 0.\underbrace{0\ldots01}_{n}$. □

4. Physical Interpretation

The following hypotheses illustrate possible physical interpretation of previous theorems.

Hypotheses 1. Theorem 1 could offer an explanation of why physical speed (or acceleration) is not uniquely defined and, from the point of view of a measurement system (observer), it is possible to consider speed (or acceleration) as a random variable with distribution dependend on the measurement system. Let v be the speed with $v = v_0.v_1 \ldots v_{n-k} + \xi_m^{n,k}$ where $\xi_m^{n,k} \in \{0.\underbrace{0\ldots0}_{n-k}v_{n-k+1}\ldots v_n\}$ – random variable, $m > n$, and the distribution function is $F_m^{n,k}(x) = P(\xi_m^{n,k} < x)$.

Hypotheses 2. Theorem 2 could offer an explanation of why the speed of any physical body cannot exceed some constant (the speed of light, for example). Independence of this constant on explicit expression of space-time function could offer an explanation of why the speed of light does not depend on an inertial coordinate system.

Hypotheses 3. Theorem 3 could offer an explantion of the various uncertainty principles, when a product of a finite number of physical variables has to be not less than a certain constant. This can be seen not just from consideration of second derivatives, but of any derivative.

Hypotheses 4. Theorems 1, 2, and 3 combined may provide an insight into the connection between classical and quantum mechanics.

5. Data Mining

The use of Observer's Mathematics in data mining as it applies to physics, genetics, and control systems allows us to get more accurate and robust results. Below are a couple examples illustrating this fact.

Example 1. Given the following array, fit it with $y = a + bx$:

x	1.23	3.74	-0.65	-5.79	-1.8	3.67	0.81	4.27	-6.38	0.92
y	6.91	16.58	-0.24	-20.09	-4.72	16.29	5.26	18.64	-22.41	5.67

Then we have the following results. Observer's Math Regression: $a = 2.19$ and $b = 3.87$, which produces SAE = SQRT(SSE) = 0, while OLS Regression: $a = 2.181302$ and $b = 3.8489$, which produces SAE = 0.297472 and SQRT(SSE) = 0.117366. Note, SAE is Sum Absolute Error, SSE is Sum Squared Error. Therefore, OLS performs worse then Observer's Mathematics in W_2.

Example 2. Given the following array, fit it with $y = a + bx$:

x	1.23	3.74	-0.65	-5.79	-1.8	3.67	0.81	4.27	-6.38	0.92
y	6.92	16.57	-0.23	-20.1	-4.73	16.28	5.27	18.65	-22.42	5.66

Then we have the following results: Observer's Math Regression: $a = 2.19$ and $b = 3.87$, which produces SAE = 0.1 and SQRT(SSE) = 0.001, while OLS Regression: $a = 2.1793$ and 3.84888, which produces SAE = 0.318841 and SQRT(SSE) = 0.01714056. Note, SAE is Sum Absolute Error, SSE is Sum Squared Error. Therefore, OLS performs worse then Observer's Mathematics in W_2.

References

[1] B. Khots, D. Khots, *Mathematics of Relativity*, www.mathrelativity.com (2004).

[2] B. Khots, D. Khots, An introduction to mathematics of relativity, In: *Lecture Notes in Theoretical and Mathematical Physics* (Ed. A.V. Aminova), **7**, Kazan State University, Russia (2006), 269-306.

[3] B. Khots, D. Khots, Observer's Mathematics – mathematics of relativity, *Applied Mathematics and Computations*, **187**, No. 1 (2007), 228-238.

[4] D. Khots, B. Khots, Quantum theory and observer's mathematics, *American Institute of Physics (AIP)*, QTRF-4 Proceedings, **962** (2007), 261-2007.