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**PARAMETRIC SURFACE CONSTRUCTION FOR
ENGINEERING APPLICATIONS**

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Abstract: This paper presents a parametric surface construction method that is general for numerous engineering applications, including integration of topology and shape optimizations, modeling for bioengineering applications, and reverse engineering for air logistics support. The proposed method employs curve fitting and surface skinning using B-spline curves and surfaces, respectively. This method minimizes the ripple and twist of the surfaces constructed by aligning knot vector across sectional B-spline curves. The proposed method also allows the flexibility of choosing polynomial order and number of control points for support of applications with different requirements. In addition to a very brief presentation on the method itself, examples will be given to illustrate and demonstrate the advantage of the method.

AMS Subject Classification: 70-02

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1. Introduction

Many engineering assignments deal with design and analysis of physical objects. Very often the objects are represented in discrete geometric points that are inadequate to support engineering assignments, such as engineering design using

topology and shape optimizations, modeling and simulation of bioengineering applications, and reverse engineering for air logistics support.

Topology optimization has drawn significant attention in recent development of structural optimization. This method has been proven very effective in determining the initial geometric layout for structural designs. The main drawback of the method, however, is that the topology optimization always leads to a non-smooth structural geometry, while most of the engineering applications require a smooth geometric shape. On the other hand, shape optimization starts with a smooth geometric model. However, a final optimal shape will be confined to the topology of its initial geometry. No holes can be created nor removed during the shape optimization process. The topology and shape optimizations must be combined to support structural design more effectively. The essential step that supports the integration is constructing smooth surfaces that not only approximate the structural layout obtained from topology optimization but also support design parameterization for shape optimization.

In bioengineering modeling for mechanics study, it often involves slicing physical objects, scanning the sliced issues, and tracing the section contours to digitize the geometry of the object, such as bones. This process results in a set of data points that describes the geometric shape of the object in a discrete form, which has to be further processed to support modeling tasks.

Reverse engineering aims at replicating parts to support defense logistics centers to maintain, repair, and overhaul (MRO) aging systems and components. The reverse engineering involves scanning physical objects using advanced scanners, such as laser or optical scanners, to capture the geometry of the object in millions data points. Again, these points must be processed to support engineering tasks, such as manufacturing.

All engineering assignments discussed above require constructing smooth geometric representation for the objects in a manageable form. The quality and efficacy of the surfaces becomes essential to the success of the engineering assignments. In this paper, we propose a curve fitting and surface skinning method to construct smooth parametric surfaces using B-spline curves and surfaces that support general engineering assignments.

2. Curve Fitting Technique

The curve fitting technique employs the least square fitting for discrete points measured on a pre-selected section of an object. The best fitting curve can be

obtained by minimizing the distance sum between the curve and the geometric points. The distance sum f is defined as [4]

$$f = \sum_{j=0}^r \|\mathbf{P}_j - \mathbf{x}(u_j)\|^2 \tag{1}$$

where \mathbf{P}_j is the position vector of the j -th data point, and $r + 1$ is the total number of points in the contour; $\|\bullet\|$ is the norm of the vector \bullet , $\mathbf{x}(u)$ is the fitting B-spline curve, $\mathbf{x}(u_j) = [x_1(u_j), x_2(u_j), x_3(u_j)]^T$ is the position vector of the fitting B-spline curve at u_j , where u is the parametric coordinate of the curve. The u_j in equation 1 is defined by the length ratio of the polygon formed by the geometric points \mathbf{P}_j , as illustrated in Figure 1a. Mathematically, the values of u_j can be calculated by

$$u_0 = 0, u_j = (r + 1) \frac{\sum_{k=0}^{j-1} |\mathbf{P}_{(k+1) \bmod (r+1)} - \mathbf{P}_k|}{\sum_{k=0}^r |\mathbf{P}_{(k+1) \bmod (r+1)} - \mathbf{P}_k|} \tag{2}$$

$(j = 1, r).$

The B-spline curve is defined as,

$$\mathbf{x}(u) = \sum_{i=0}^n \mathbf{B}_i N_{i,k}(u) \tag{3}$$

where \mathbf{B}_i is the i -th control point shown in Figure 1b, $n + 1$ is the number of control points, and $N_{i,k}(u)$ is the basis function of the B-spline curve defined recursively as

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}, \begin{cases} N_{i,1}(u) = 1, & \text{if } t_i \leq u \leq t_{i+1}, \\ N_{i,1}(u) = 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $[t_i, t_{i+1})$ is a knot span formed by the two consecutive knots t_i and t_{i+1} , and $k - 1$ is the polynomial order of the basis functions [2].

In order to minimize f , the derivatives of f with respect to the $n + 1$ control points are set to zero. For simplicity, considering only the j -th-control point, one has

$$\frac{df}{d\mathbf{B}_\ell} = \sum_{j=0}^r \left\| -2\mathbf{P}_j \sum_{i=0}^n N_{i,k}(u_j) + 2 \sum_{i=0}^n N_{i,k}(u_j) \left(\sum_{i=0}^n N_{i,k}(u_j) \mathbf{B}_\ell \right) \right\| = 0. \tag{5}$$

For $l = 0, n$, the above expression can be rewritten in a matrix form as $\mathbf{N}^T \mathbf{N} \mathbf{B} = \mathbf{N}^T \mathbf{P}$, where $\mathbf{N} \in \mathbf{R}^{(r+1) \times (n+1)}$, $\mathbf{B} = \mathbf{R}^{(n+1) \times 3}$, $\mathbf{P} = \mathbf{R}^{(r+1) \times 3}$.

Note that $\mathbf{N}^T \mathbf{N}$ is invertible if $N_{i,k}(u_j) \neq 0$. This is true if and only if $t_{i-k+1} < u_j < t_{i+1}$, for $i = 0, n$, and $j = 0, r$. This implies that there must exist at least one u_j in at least one knot span so that $N_{i,k}(u_j) \neq 0$ for all basis functions. This requirement can be achieved by adjusting the knot values of

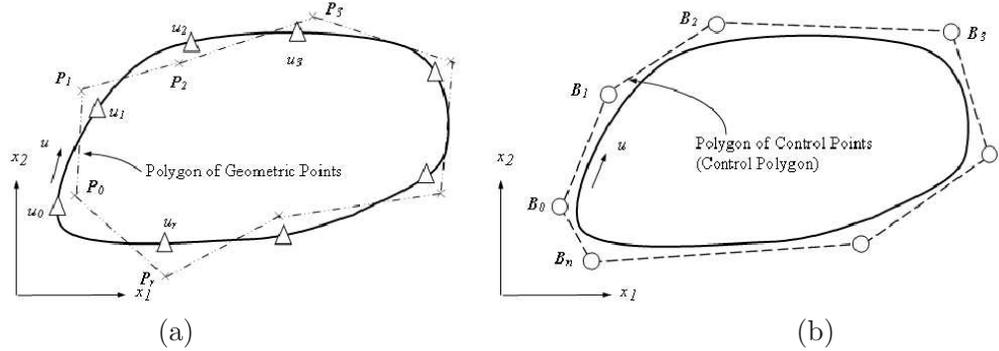


Figure 1: Illustration of B-spline curve fitting: (a) Curve fitting for geometric points P ; (b) B-spline curve with control points B

the basis functions. The curve fitting error can be controlled by adjusting the polynomial order and the number of control points. The output of the curve fitting is a set of control points and basis functions that describe the smoothed section contour.

3. Surface Skinning

The fitting B-spline curves discussed above are then related across sections to form an open B-spline surface, as shown in Figure 2, using the surface skinning technique. Note that in this process, the number of control points of the B-spline curves must be kept identical across sections. In addition, the polynomial order of the basis functions and knot values of the B-spline curves must be identical on all sections. The knot values are adjusted across sectional contour following a detailed computational algorithm [4] in order to minimize the ripples and twists of the B-spline surfaces. This is the major contribution of the proposed method. The control points are connected to their corresponding points across sections in order to create smooth surfaces, as shown in Figure 2a, to form a control polyhedron. The enclosed B-spline surface is then constructed, as shown in Figure 2b, by

$$\mathbf{x}(u, w) = \sum_{i=0}^n \sum_{j=0}^m \mathbf{B}_{ij} N_{i,k}(u) M_{j,\ell}(w), \tag{6}$$

where $n + 1$ and $m + 1$ are the numbers of control points in the u - and w -parametric directions, respectively; and $k - 1$ and $j - 1$ are the polynomial orders

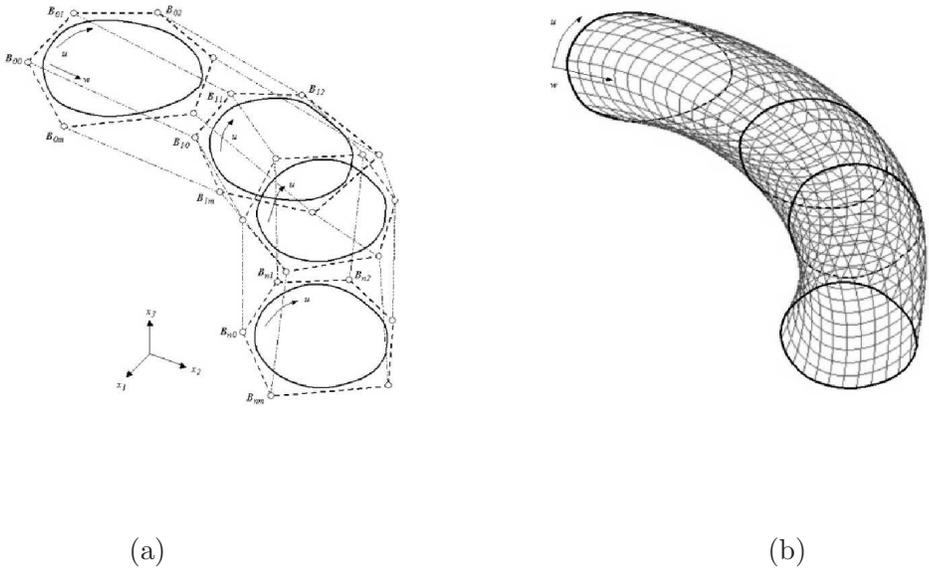


Figure 2: Illustration of B-spline surface skinning: (a) Control polyhedron and section contours (b) B-spline surface enclosed by the control polyhedron

of the basis functions $N_{i,k}(u)$ and $M_{j,\ell}(w)$, respectively. Note that the B-spline surface constructed is C^2 -continuous in both u - and w -parametric directions, if cubic basis functions are assumed. The control points and basis functions of the B-spline surface can be imported into *CAD* tools to support solid modeling.

4. Engineering Applications

The first application is the integration of topology and shape optimization. A tracked vehicle roadarm shown in Figure 3a is optimized using topology optimization from initial shape in Figure 3b to that of Figure 3c [5]. The optimal design is unsmooth and cannot be manufactured. Geometric points of five representative sections of the roadarm are selected and fitted with B-spline curves (Step 2 in Figure 4). Following the surface skinning method, an outer polyhedron formed by the 6×5 control points and the enclosed B-Spline surface are created (Step 4a). Similarly, an inner B-spline surface (4×3 control points) that represents the hole in the roadarm is created (Step 4b). These B-spline surfaces are imported into *SolidWorks* for solid model construction.

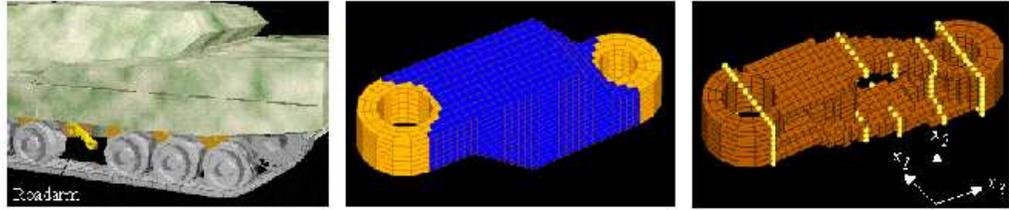


Figure 3: The tracked vehicle roadarm: (a) Physical model; (b) Initial finite element model; (c) Topologically optimized.

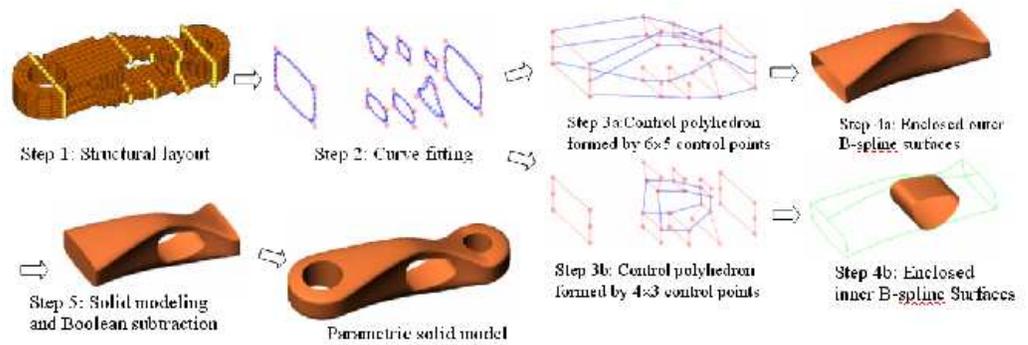


Figure 4: Construction of B-spline surfaces for structural design

In *SolidWorks*, the outer and inner solid models are created by filling up the cavities enclosed by the outer and inner B-spline surfaces, respectively. The final solid model is obtained by subtracting the inner solid from the outer one (Step 5) and uniting the subtracted solid model with two end half cylinders.

The second example is modeling a human middle ear [3]. The proposed method starts with the histological section preparation of human temporal bone. Through tracing outlines of the middle ear components on the sections (Figure 5a), a set of discrete points is obtained and employed to construct B-spline curves that represent the exterior contours of the components using the curve fitting technique (Figure 5b). The surface skinning technique is then employed to quilt the B-spline curves for smooth boundary surfaces of the middle ear components using B-spline surfaces (Figure 5c). The solid models of the middle ear components are constructed using these surfaces and then assembled to create the complete middle ear in *CAD*. The geometric model constructed using the proposed method is smooth and can be used to create finite element models for mechanics study (Figure 5d).

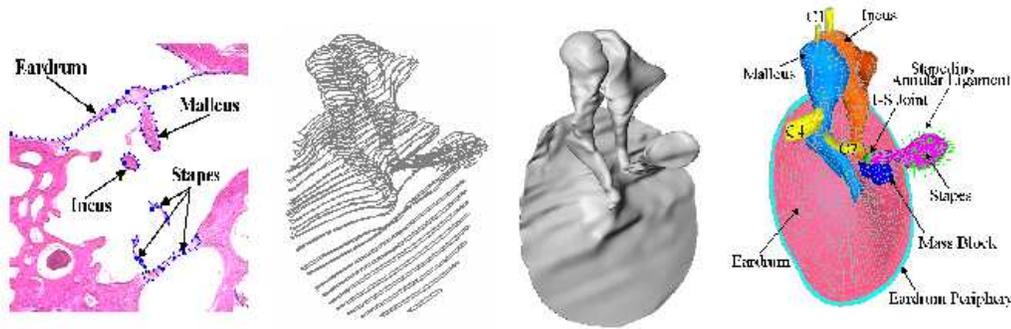


Figure 5: Human middle ear surface and FEA models: (a) Section image; (b) Section contours; (c) Surface model; (d) Finite element model

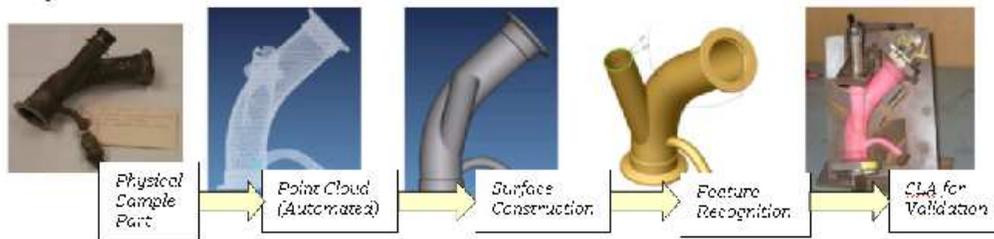


Figure 6: Reverse engineering of an airplane engine tubing

The third example is for reverse engineering. An airplane tubing sample part was first scanned using an industrial CT scanner, capturing both the interior and exterior geometry with 486,107 uniformly spaced data points (Figure 6). B-spline curve fitting and surface skinning approach (and similar capabilities in *Imagware*) was employed to convert the data points into B-spline surfaces [1]. A physical model was produced using StereoLithography Apparatus (SLA) and mounted to the production fixtures to verify the accuracy of the surface model, as shown in Figure 6.

5. Conclusions and Future Research

In this paper a parametric surface construction method that employs curve fitting and surface skinning using B-spline curves and surfaces, has been presented. This method creates smooth surfaces and supports numerous engineer-

ing applications. Even though the method is proven effective, it takes fairly amount of manual labor and certain expertise in geometric modeling to apply the method to engineering applications. More work is needed to automate the surface fitting method for randomly distributed points. In addition, converting B-spline surface models to *CAD* solid features, such as sweep, revolve, etc., for *CAD*-based applications requires further research.

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