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ON THE LOWER LEVEL SETS OF ANTI FUZZY BI-IDEALS IN RINGS

Sanjib Kumar Datta §

Department of Mathematics
University of North Bengal
Raja Rammohunpur, District Darjeeling
PIN Code 734013, West Bengal, INDIA
e-mail: sanjib_kr_datta@yahoo.co.in

Abstract: In this paper we prove a necessary and sufficient condition for a fuzzy subset μ of a ring R to be anti fuzy bi-ideal of R in terms of its lower level set.

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1. Introduction, Definitions and Preliminaries

The idea of fuzzy subset μ of a set X was primarily introduced by L.A. Zadeh [6] as a function $\mu : X \to [0, 1]$. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belongs to a set. Using this concept Rosenfeld [5] established some results in fuzzy group theory. Later Kuroki [1] introduced the notion of fuzzy ideals in semigroups and Liu [3] studied them in rings. Lajos and Szasz [2] introduced the idea of bi-ideals in a ring. Majumder and Sarder [4] explored on the idea of anti fuzzy bi-ideals in fuzzy group theory. In this paper we introduce the notion of anti fuzzy bi-ideals in rings and characterise them by their lower level subsets.

We now review some definitions that are used in this paper.

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§Correspondence address: 25, School Road, Kalianibash, Barrackpore, P.O.: Nonachandanpukur, Dist.: 24 Parganas (North), P.S.: Titagarh, PIN Code: 743102, West Bengal, INDIA

Definition 1. (see [5]) Let μ be a fuzzy subset of a ring R. $\mathrm{Im}\mu$ is defined as

$$\mathrm{Im}\mu = \{t \in [0,1] \mid \mu(x) = t \text{ for some } x \in X\}.$$

Definition 2. (see [5]) Let μ be a fuzzy subset of a ring R. The set $\mu_t = \{x \in R \mid \mu(x) \leq t\}$ is called a lower level subset of μ.

Definition 3. (see [2]) A subring S of a ring R is called a bi-ideal of R if $SRS \subseteq S$ holds where SRS is the additive subgroup of R generated by the set of all elements of the form srs, $s \in S$ and $r \in R$.

Definition 4. A fuzzy subset μ of a ring R is called an anti fuzzy left (right) ideal of R, if for every $x, y \in R$,

(i) μ is an anti fuzzy subgroup of $(R, +)$, i.e.,

$$\mu(x-y) \leq \max\{\mu(x), \mu(y)\} \text{ and}$$

(ii) $\mu(xy) \leq \mu(y)\,(\mu(xy) \leq \mu(x))$.

If μ is both an anti fuzzy left ideal and an anti fuzzy right ideal of R, then it is called an anti fuzzy ideal of R.

Definition 5. A non-empty fuzzy subset μ of a ring R (i.e. $\mu(x) \neq 0$ for some $x \in R$) is called an anti fuzzy bi-ideal of R if:

(i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$,

(ii) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$, and

(iii) $\mu(xyz) \leq \max\{\mu(x), \mu(z)\}$ for all $x, y, z \in R$.

2. Theorems

In this section we present the main results of our paper.

Theorem 1. *Let μ be a fuzzy subset of a ring R. μ is an anti fuzzy bi-ideal of R iff its lower level sets μ_t's are bi-ideals of R for all $t \in \mathrm{Im}\mu$.*

Proof. Let us suppose that μ is an anti fuzzy bi-ideal of R. Let $t \in \mathrm{Im}\mu$. Then there exists $x \in R$ such that $\mu(x) = t$. So $x \in \mu_t$. Hence μ_t is non-empty. Let $x, y \in \mu_t$. Therefore

$$\mu(x) \leq t\ ,\ \mu(y) \leq t.$$

So

$$\mu(x-y) \leq \max\{\mu(x), \mu(y)\} \leq t$$

and

$$\mu(xy) \leq \max\{\mu(x), \mu(y)\} \leq t.$$

Therefore $x - y \in \mu_t$ and $xy \in \mu_t$ for every $x, y \in \mu_t$. Hence μ_t is a subring of R.

Let $x, x' \in \mu_t$ and $r \in R$. So $\mu(x) \leq t$ and $\mu\left(x'\right) \leq t$.

Now

$$\mu\left(xrx'\right) \leq \max\left\{\mu(x), \mu\left(x'\right)\right\} \leq t.$$

So

$$xrx' \in \mu_t,$$

$$\mu_t R \mu_t \subseteq \mu_t.$$

Hence μ_t is a bi-ideal of R.

Conversely let μ_t's be bi-ideals of R for all $t \in \mathrm{Im}\mu$. We shall show that μ is an anti fuzzy bi-ideal of R.

Let $x, y \in R$. Let $\mu(x) = t_1$ and $\mu(y) = t_2$ Suppose that $t_1 < t_2$. Then $\mu(x) = t_1 < t_2$ and $\mu(y) = t_2$. So $x \in \mu_{t_2}, y \in \mu_{t_2}$. Since μ_{t_2} is a bi-ideal of R, $x - y \in \mu_{t_2}$. Hence

$$\mu(x - y) \leq t_2 = \max\{\mu(x), \mu(y)\}.$$

Similar is the case when $t_2 < t_1$. Again $x \in \mu_{t_2}, y \in \mu_{t_2}$ imply that $xy \in \mu_{t_2}$. Hence

$$\mu(xy) \leq t_2 = \max\{\mu(x), \mu(y)\}.$$

Similar is the case when $t_2 < t_1$. So μ is an anti fuzzy subring of R. Next let $\mu(x) = t_1$ and $\mu(z) = t_2$ and $t_1 < t_2$ and $y \in R$. Then $\mu(x) = t_1 < t_2$, $\mu(z) = t_2$. So $x \in \mu_{t_2}$ and $z \in \mu_{t_2}$. As μ_{t_2} is a bi-ideal of R, therefore $xyz \in \mu_{t_2}$. So

$$\mu(xyz) \leq t_2 \leq \max\{\mu(x), \mu(z)\}.$$

Similar is the case when $t_2 < t_1$. Hence μ is an anti fuzzy bi-ideal of R.

Thus the theorem is established. □

In the line of Theorem 1 we may state the following theorem without proof.

Theorem 2. *Let R be a ring and μ be a fuzzy subset of R. μ is an anti fuzzy left (right) ideal of R iff the lower level sets μ_t's are left (right) ideals of R for all $t \in \mathrm{Im}\mu$.*

As an application of Theorem 1 we may establish the following theorem.

Theorem 3. *If every bi-ideal of a ring R is a left (right) ideal of R then every anti fuzzy bi-ideal of R is an anti fuzzy left (right) ideal of R.*

Proof. Let μ be an anti fuzzy bi-ideal of R. Then by Theorem 1, μ_t's are bi-ideals of R. Therefore μ_t's are left (right) ideals of R. Hence in view of Theorem 2, μ is an anti fuzzy left (right) ideal of R.

This proves the theorem. □

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