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CONJUGATE DUALITY FOR SET-VALUED VECTOR OPTIMIZATION IN FINITE DIMENSIONAL SPACES

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Abstract: In this paper, we consider conjugate duality theorem in set-valued vector optimization. New perturbation function is presented for a class of set-valued vector optimization, in order to obtain corresponding conjugate duality optimization and duality theorems. We show under a stability criteria that the form of weak and strong duality becomes simple, general and convenient in applications.

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1. Introduction

Under the idea of using perturbations to conjugate duality, our paper considers the prime problem by presenting a class of new perturbation function. This gives raise to a new conjugate dual and allows the formulation of weak and strong duality theorems in finite dimensional spaces. After some transformation

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the form of conjugate duality is obtained. We isolate the variables and disrupt variables of perturbation function by considering constrained set. Finally, with the stability assumptions, we obtain well results for set-valued vector optimization problem. It should be mentioned that in the general conjugate duality theory the weak duality assertion is fulfilled without any assumptions. But in our weak duality theorem it requires the externally stability in order to simplify the result. Corresponding the results in [1], our results become simple, general and convenient in application. Meanwhile, the conditions (externally stability) used in this paper are not stronger than in [1].

2. Preliminaries

Let C be a pointed closed and convex cone in R^n, for any $\xi, \mu \in R^n$, we have the following ordering relations

$$\xi \leq_C \mu \Leftrightarrow \mu - \xi \in C, \quad \xi \leq_{C\setminus\{0\}} \mu \Leftrightarrow \mu - \xi \in C \setminus \{0\},$$

$$\xi \nleq_{C\setminus\{0\}} \mu \Leftrightarrow \mu - \xi \notin C \setminus \{0\}.$$

The notions $\geq_{C\setminus\{0\}}$, $\geq_C$ and $\ngeq_{C\setminus\{0\}}$ are used in an alternative way.

Definition 2.1. (see [5]) Let $Y \subseteq R^n$, $y \in R^n$. If $y \in Y$, and there is no $y' \in Y$, such as $y \leq_{C\setminus\{0\}} y'$, then y is said to be a maximal point of a set Y. The set of all maximal points of Y is called the maximum of Y and is denoted by $\max\limits_{C\setminus\{0\}} Y$. The minimum of Y is defined analogously. If $y \in Y$, and there is no $y' \in Y$, such as $y \geq_{C\setminus\{0\}} y'$, then y is said to be a minimum point of a set Y, the set of all minimum points of Y is called the minimum of Y and is denoted by $\min\limits_{C\setminus\{0\}} Y$. Further we take the cone C being the nonnegative quadrant $R^n_+ = \{x = (x_1, \cdots, x_n)^\top \in R^n \mid x_i \geq 0, i = \overline{1,n}\}$.

Lemma 2.1. (see [2]) *Let $Y_1, Y_2 \subset R^n$, then*

$$(i) \qquad \max_{R^n_+\setminus\{0\}} (Y_1 + Y_2) \subseteq \max_{R^n_+\setminus\{0\}} Y_1 + \max_{R^n_+\setminus\{0\}} Y_2,$$

$$(ii) \qquad \min_{R^n_+\setminus\{0\}} (Y_1 + Y_2) \subseteq \min_{R^n_+\setminus\{0\}} Y_1 + \min_{R^n_+\setminus\{0\}} Y_2.$$

Definition 2.2. (see [3]) Let $h : R^n \rightrightarrows R^p$ be a set-valued map.

(i) The minimum set of set-valued map h, $\min\limits_{R^n_+\setminus\{0\}} h(x)$ is said to be externally stable if $h(x) \subseteq \min\limits_{R^n_+\setminus\{0\}} h(x) + R^p_+, \quad \forall x \in R^n$.

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(ii) The maximum set of set-valued map h, $\max_{R_+^n\backslash\{0\}} h(x)$ is said to be externally stable if $h(x) \subseteq \max_{R_+^n\backslash\{0\}} h(x) - R_+^p, \quad \forall x \in R^n$.

Lemma 2.2. (see [2]) *Let $F_1 : R^n \rightrightarrows R^p, F_2 : R^n \rightrightarrows R^p$ be set-valued maps and $X \subseteq R^n$, then $\max_{R_+^p\backslash\{0\}} \bigcup_{x\in X}[F_1(x)+F_2(x)] \subseteq \max_{R_+^p\backslash\{0\}} \bigcup_{x\in X}[F_1(x)+ \max_{R_+^p\backslash\{0\}} F_2(x)]$. If $\max_{R_+^p\backslash\{0\}} F_2(x)$ is externally stable, then the converse inclusion also holds*

$$\max_{R_+^p\backslash\{0\}} \bigcup_{x\in X}[F_1(x) + F_2(x)] = \max_{R_+^p\backslash\{0\}} \bigcup_{x\in X}[F_1(x) + \max_{R_+^p\backslash\{0\}} F_2(x)].$$

Corollary 2.1. (see [2]) *Let $F_1 : R^n \rightrightarrows R^p$ be a set-valued map and $X \subseteq R^n$. If $\max_{R_+^p\backslash\{0\}} F(x)$ is externally stable, then*

$$\max_{R_+^p\backslash\{0\}} \bigcup_{x\in X} F(x) = \max_{R_+^p\backslash\{0\}} \bigcup_{x\in X} \max_{R_+^p\backslash\{0\}} F(x).$$

Before describing the conjugate duality for vector optimization, let us recall the concepts of conjugate maps and set-valued subgradient.

Definition 2.3. (see [4]) Let $h : R^n \rightrightarrows R^p$ be a set-valued map.

(i) The set-valued map $h^*(U) = \max_{R_+^p\backslash\{0\}} \bigcup_{x\in R^n}[Ux - h(x)]$, $U \in R^{p\times n}$ is called the conjugate map of h.

(ii) U is said to be a subgradient of the set-valued map h at $(\bar{x}, \bar{y})$, if $\bar{y} \in h(\bar{x})$ and $\bar{y} - U\bar{x} \in \min_{R_+^p\backslash\{0\}} \bigcup_{x\in R^n}[h(x) - Ux]$ holds. The set of all subgradients of h at (x, y) is denoted by $\partial h(x, y)$, and is called the subdifferential of for all h at (x, y). If $\forall y \in h(x)$ there is $\partial h(x, y) \neq \emptyset$, then h is said to be subdifferentiable at x.

Now we consider this problem

$$\min_{R_+^p\backslash\{0\}} \{f(x) \mid x \in G\}, \qquad (VO)$$

where $G = \{x \in X \mid g(x) \leq_{R_+^n} 0\}$, $x \in R^n$, $f(x) \in R^p$, $u \in R^n$, $g(x) \in R^n$.

For discussing the conjugate duality of (VO), we introduce the concept of perturbation function. $\phi : R^n \times R^m \to R^p \cup \{\infty\}$ is vector-valued function, such that $\phi(x, 0) = f(x)$, for all $x \in R^n$. That is the so-called perturbation function.

Definition 2.4. (see [4]) If $w(0)$ is subdifferentiable at 0, then (VO) is said to be stable, where $w(0) = \min\{f(x)\}$.

Theorem 2.1. (see [4]) *The vector optimization problem* (VO) *is said to be stable if and only if:*

(i) $\min(VO) = w(0) \subset w^{**}(0) = \max(D)$, *where* $w^{**}(0) = \max\{\bigcup_T [-w^*(T)]\}$ $= \max(D)$, $\max(D)$ *is dual problem of primal problem,* $w^*(T)$ *is conjugate mapping to perturbation function of* $f(x)$.

(ii) In other words for each solution $\bar{x}$ *to the primal problem* (VO), *there exists a solution* $\bar{V}$ *to the dual problem, such that* $\phi(\bar{x}, 0) \in -\phi^*(0, \bar{V})$, *and* $f(\bar{x}) \in -w^*(\bar{V})$ *or* $\bar{x} \in \partial w(0, f(\bar{x}))$, *where* $f(\bar{x}) = \phi(\bar{x}, 0), -\phi^*(0, \bar{V}) = -w^*(\bar{V})$.

Theorem 2.2. (see [4]) *For any* $x \in R^n$ *and* $V \in R^{n\times n}$, *there exists* $\phi(x, 0)\bar{\in} - \phi^*(0, V) - R^p_+ \setminus \{0\}$

3. Main Results

Consider the following set-valued vector optimization problem

$$\min_{R^p_+\setminus\{0\}} \{\phi(x, u) \mid x \in R^n\}. \qquad (P_u)$$

Setting $u = 0$, (P_u) becomes

$$\min_{R^p_+\setminus\{0\}} \{\phi(x, 0) \mid x \in R^n\}. \qquad (P_0)$$

That implies $(P_0) = (VO)$. Then problem (VO) can be translated into (P_u).

In order to get better results of duality, we propose a new class of perturbation function $\phi : R^n \times R^n \longrightarrow R^p \cup \{\infty\}$,

$$\phi(x, u) = \begin{cases} f(x) + [< u, x >]_p, & x \in X, \quad g(x) \leq_{R^n_+} u \\ \infty, & \text{otherwise}, \end{cases}$$
$$\phi(x, 0) = f(x)\,,$$

where $[< u, x >]_p$ denotes p-dimensional vector, and every component is $< u, x >$.

Let set-valued map ϕ^* be the conjugate map ϕ in the usual form, where $\phi^* : R^{p\times n} \times R^{p\times m} \rightrightarrows R^p \cup \{\infty\}$. In order to facilitate the separation u and x, we take $m = n$. That means, we only consider the following conjugate mapping $\phi^* : R^{p\times n} \times R^{p\times n} \rightrightarrows R^p \cup \{\infty\}, \phi^*(U, V) = \max_{R^p_+\setminus\{0\}} \{Ux - Vu - \phi(x, u)|x \in R^n, u \in R^n\}$.

We know that the conjugate duality problem of (P_0) is

$$\max_{R^p_+\backslash\{0\}} \bigcup_{V\in R^{p\times n}} [-\phi^*(0,V)], \tag{D_0}$$

where

$$\phi^*(0,V) = \max_{R^p_+\backslash\{0\}} \{Vu - f(x) - [< u,x >]_p \mid x \in X, g(x) \leq_{R^n_+} u, u \in R^n_+\}$$
$$= \max_{R^p_+\backslash\{0\}} \bigcup_{x\in X} \{Vu - f(x) - [< u,x >]_p \mid g(x) \leq_{R^n_+} u, u \in R^n_+\}. \tag{3.1}$$

Since $g(x) \leq_{R^n_+} u$ holds if and only if $u - g(x) \in R^n_+$, letting $\bar{u} = u - g(x)$, we have that

$$\phi^*(0,V) = \max_{R^p_+\backslash\{0\}} \bigcup_{x\in X} \{V\bar{u} + Vg(x) - f(x) - [< \bar{u} + g(x), x >]_p \mid \bar{u} \in R^n_+\}. \tag{3.2}$$

Now, we consider the optimization problem (3.2) under the following constraint set,

$$L = \{x \in X| < \bar{u},\ x >= 0, \bar{u} \in R^n_+\} = \{x| < u, x >=< g(x),\ x >\}$$

By (3.1), we have

$$\phi^*(0,V)_L = \max_{R^n_+\backslash\{0\}} \{\{V\bar{u} \mid \bar{u} \in R^n_+\} + \{Vg(x) - f(x) - [< g(x), x >]_p \mid x \in L\}\}. \tag{3.3}$$

From Lemmas 2.1 and 3.3, we get

$$\phi^*(0,V)_L \subseteq \max_{R^p_+\backslash\{0\}} \{V\bar{u} \mid \bar{u} \in R^n_+\} + \max_{R^p_+\backslash\{0\}} \{Vg(x) - f(x) - [< g(x), x >]_p\}. \tag{3.4}$$

Let $-V \in R^{p\times n}$, replace $V \in R^{p\times n}$ in (3.4), we can deduce from Lemma 2.2

$$\phi^*(0,-V)_L \subseteq -\min_{R^p_+\backslash\{0\}} \{V\bar{u}|\bar{u} \in R^n_+\} + \max_{R^p_+\backslash\{0\}} \{-Vg(x) - f(x) - [< g(x), x >]_p\}. \tag{3.5}$$

To obtain our results, we attach a feasible restriction set L' to the optimization problem (3.3), where

$$L' = \{V \in R^{p\times n} \mid V\bar{u} \geqslant_{R^p_+} 0,\ \forall \bar{u} \in R^n_+\} = \{V \in R^{p\times n}_+ \mid VR^n_+ \subseteq R^p_+\}. \tag{3.6}$$

From the equation above, we have

$$\min_{R^p_+\backslash\{0\}} \{V\bar{u} \mid \bar{u} \in R^n_+\} = \{0\}, \quad \forall V \in L'. \tag{3.7}$$

By (3.7), we get

$$\max_{R^p_+\backslash\{0\}}\{-V\bar{u} \mid \bar{u}\in R^n_+\} = \{0\}, \quad \forall V\in L'.$$

By Definition 2.2, we can prove that $\max_{R^p_+\backslash\{0\}}\{-V\bar{u} \mid \bar{u}\in R^n_+\}$ is externally stable. Following Lemma 2.2,

$$\phi^*(0,-V)_L = \max_{R^p_+\backslash\{0\}}\{-V\bar{u} \mid \bar{u}\in R^n_+\} + \max_{R^p_+\backslash\{0\}}\{-Vg(x)-f(x)-[< g(x),x >]_p\}$$

$$= \max_{R^p_+\backslash\{0\}}\{-Vg(x)-f(x)-[< g(x),x >]_p\}. = \widetilde{\phi}(V). \quad (3.8)$$

Then

$$\max_{R^p_+\backslash\{0\}} \bigcup_{V\in R^{p\times n}} [-\phi^*(0,V)]$$

$$= \max_{R^p_+\backslash\{0\}} \bigcup_{-V\in R^{p\times n}} [-\phi^*(0,-V)] = \max_{R^p_+\backslash\{0\}} \bigcup_{V\in R^{p\times n}} [-\phi^*(0,-V)]$$

$$= \max_{R^p_+\backslash\{0\}} \bigcup_{V\in L'} [-\widetilde{\phi}(V)]$$

$$= \max_{R^p_+\backslash\{0\}} \bigcup_{V\in L'} [\min_{R^p_+\backslash\{0\}}\{Vg(x)+f(x)+[< g(x),x >]_p \mid x\in L\}].$$

Therefore, the dual problem can be formulated as follows:

$$(D^{VO}_L)\ \max_{R^p_+\backslash\{0\}} \bigcup_{V\in L'} [\min_{R^p_+\backslash\{0\}}\{Vg(x)+f(x)+[< g(x),x >]_p \mid x\in L\}].$$

In the general conjugate duality theory the weak duality assertion is fulfilled without any assumptions. But in our weak duality theorem it requires the externally stability in order to simplify the result.

Theorem 3.1. (Weak Duality Theorem) *If* $\min_{R^p_+\backslash\{0\}}\{-Vg(x)+f(x)+[< g(x),x >]_p \mid x\in L\}$ *is externally stable, then*

$$0\bar{\in} - Vg(x)+[< g(x),x >]_p - R^p_+\setminus\{0\}, \forall \quad x\in L.$$

Proof. By Theorem 2.2, we have $\phi(x,0)\bar{\in} - \phi^*(0,V)_L - R^p_+\setminus\{0\}$. By (3.8) $\phi^*(0,-V)_L = \widetilde{\phi}(V)$, we obtain $-\phi^*(0,V)_L = -\widetilde{\phi}(-V)$, that implies

$$f(x)\bar{\in} \min_{R^p_+\backslash\{0\}}\{-Vg(x)+f(x)+[< g(x),x >]_p\} - R^p_+\setminus\{0\}. \quad (3.9)$$

That is $f(x) \not\leq_{R^p_+\backslash\{0\}} \min_{R^p_+\backslash\{0\}}\{-Vg(x)+f(x)+[< g(x),x >]_p\}$. Since

$$\min_{R^p_+\backslash\{0\}}\{-Vg(x)+f(x)+[< g(x),x >]_p \mid x\in L\}$$

is externally stable, thus

$$\{-Vg(x)+f(x)+[<g(x),x>]_p \mid x\in L\} \subseteq \min_{R^p_+\setminus\{0\}}\{-Vg(x)+f(x)+[<g(x),x>]_p \mid x\in L\}+R^p_+$$

holds. By (3.9), we have that $f(x)\bar{\in}\{-Vg(x)+f(x)+[<g(x),x>]_p \mid x\in L\}-R^p_+\setminus\{0\}$ and furthermore

$$0\bar{\in}-Vg(x)+[<g(x),x>]_p-R^p_+\setminus\{0\}, \quad \forall\, x\in L. \tag{3.10}$$

Since $x\in L$ and $<g(x),x>=<u,x>$, (3.10) is equivalent to $0\bar{\in}-Vg(x)+[<u,x>]_p-R^p_+\setminus\{0\},\ x\in L$. The proof is finished. □

In order to obtain strong duality theorem, we assume that $\max\{Vg(x)-[<g(x),x>]_p-f(x)-R^p_+\}$ is externally stable.

Theorem 3.2. (Strong Duality Theorem) *(i) (VO) is said to be stable with respect to the perturbation function ϕ if and only if for each solution $\bar{x}$ of the primal problem (VO), there exists a solution $\bar{V}$ of the dual problem (D^{VO}_L), such that*

$$0\in \bar{V}g(\bar{x})-[<g(\bar{x}),\bar{x}>]_p-R^p_+=\bar{V}g(\bar{x})-[<u,\bar{x}>]_p-R^p_+.$$

If $\max\{Vg(x)-[<g(x),x>]_p\}$ *is externally stable, then*

$$0\in\max\{\bar{V}g(\bar{x})-[<g(\bar{x}),\bar{x}>]_p\}=\max\{\bar{V}g(\bar{x})-[<u,\bar{x}>]_p\}.$$

(ii) If $x\in L$, $V\in R^{p\times n}$, satisfy the condition (i), then x is a solution of (VO) and V is a solution of (D^{VO}_L).

Proof. (i) Firstly, we prove the "only if". By the necessity condition of Theorem 2.1, we have $f(x)\in-\phi^*(0,V)_L$, where $V\in R^{p\times n}_+$. From (3.8) $\phi^*(0,-V)_L=\widetilde{\phi}(V)$, we get that there exists $f(x)\in-\widetilde{\phi}(-V)$, where $V\in R^{p\times n}_+$, such that $-f(x)\in\widetilde{\phi}(-V)$. Also, we can get easily that

$$\begin{aligned}\widetilde{\phi}(-V) &= \max_{R^p_+\setminus\{0\}}\{Vg(x)-f(x)-[<g(x),x>]_p \mid x\in L\}\\ &= \max_{R^p_+\setminus\{0\}}\bigcup_{x\in L}\{Vg(x)-[<g(x),x>]_p-f(x)\} \qquad (3.11)\\ &\subseteq Vg(x)-[<g(x),x>]_p-f(x)-R^p_+, \quad x\in L.\end{aligned}$$

The equation above can be obtained from $\max\{Vg(x)-[<g(x),x>]_p-f(x)\}$ closedness. It follows that $-f(x)\in Vg(x)-[<g(x),x>]_p-f(x)-R^p_+$, $x\in L$. Since if $x\in L$, we have $<g(x),x>=<u,x>$. Therefore $0\in Vg(x)-[<u,x>]_p-R^p_+$.

Secondly, to see the "if" part. Since $0\in Vg(x)-[<g(x),x>]_p-R^p_+$, we have

$$\begin{array}{rcl} -f(x) & \subseteq & Vg(x) - [< g(x), x >]_p - f(x) - R^p_+ \\ & \subseteq & Vg(x) - [< g(x), x >]_p - f(x) - R^p_+ + R^p_+. \end{array} \tag{3.12}$$

We let $G = -Vg(x) - [< g(x), x >]_p - f(x) - R^p_+$, then (3.12) becomes $-f(x) \in G + R^p_+$. By Lemma 2.1, we obtain

$$\begin{array}{rcl} \max G & \subseteq & \max \{Vg(x) - [< g(x), x >]_p - f(x)\} + \max \{-R^p_+\} \\ & \subseteq & \max \{Vg(x) - [< g(x), x >]_p - f(x)\} = \widetilde{\phi}(-V). \end{array}$$

Since $\max G$ is externally stable, by Definition 2.2, we have the following inequality $G \subseteq \max G - R^p_+$, and $G + R^p_+ \subseteq \max G$. Then $-f(x) \in \max G \subseteq \widetilde{\phi}(-V)$. Furthermore, $f(x) \in -\widetilde{\phi}(-V) = -\phi^*(0, V)_L$. By sufficient conditions of Theorem 2.1, (VO) is said to be stable with respect to the perturbation function ϕ.

If $\max \{Vg(x) - [< g(x), x >]_p\}$ is externally stable, by (3.11) and Lemma 2.2, we get

$$\begin{array}{rcl} \widetilde{\phi}(-V) & = & \max\limits_{R^p_+ \setminus \{0\}} \bigcup\limits_{x \in L} \{\max \{Vg(x) - [< g(x), x >]_p\} - f(x)\} \\ & \subseteq & \max \{Vg(x) - [< g(x), x >]_p\} - f(x), \forall x \in L. \end{array}$$

From $-f(x) \in \widetilde{\phi}(-V)$, one has

$$-f(x) \in \max \{Vg(x) - [< g(x), x >]_p\} - f(x), \quad \forall\, x \in L.$$

That means

$$0 \in \max \{Vg(x) - [< g(x), x >]_p\}.$$

Since $x \in L$ and $< g(x), x >=< u, x >$, therefore $0 \in \max\{Vg(x) - [< u, x >]_p\}$.

(ii) By Theorem 2.2, we have $f(x) = \phi(x, 0) \bar{\in} - \phi^*(0, V)_L - R^p_+ \setminus \{0\}$. By (3.8) $\phi^*(0, -V)_L = \widetilde{\phi}(V)$, we obtain $-\phi^*(0, V)_L = -\widetilde{\phi}(-V)$. That implies

$$f(x) \bar{\in} - \widetilde{\phi}(-V) - R^p_+ \setminus \{0\},$$

$$f(x) \nleqslant_{R^p_+ \setminus \{0\}} -\widetilde{\phi}(-V), \quad \forall x \in L. \tag{3.13}$$

By the condition of (i), one obtains $0 \in -Vg(x) - [< g(x), x >]_p - R^p_+$. It follows that there exists $\bar{x}$ and $\bar{V}$, such that $0 = -\bar{V}g(\bar{x}) + [< g(\bar{x}), \bar{x} >]_p$.

If $\bar{x}$ is not the solution of optimization problem (VO), i.e., $f(\bar{x}) \nleqslant \min f(x)$, then there exists $x' \in L$, such that $f(\bar{x}) \nleqslant f(x')$. One knows that

$$\begin{array}{rcl} f(\bar{x}) & = & \min f(\bar{x}) = \min[f(\bar{x} + 0)] = \min\{f(\bar{x}) - \bar{V}g(\bar{x}) + [< g(\bar{x}), \bar{x} >]_p\} \\ & \in & -\widetilde{\phi}(-\bar{V}), \end{array}$$

That means if $f(\bar{x}) \in -\widetilde{\phi}(-\bar{V})$, then $f(\bar{x}) \nleqslant f(x')$. This contradicts to (3.13), hence $\bar{x}$ is the solution of optimization problem (VO).

If $\bar{V}$ is not the solution of duality problem (D_L^{VO}), then $\max\{-\widetilde{\phi}(-V)\} \nleq -\widetilde{\phi}(-\bar{V})$. That implies there exists $V' \in L'$, such that $-\widetilde{\phi}(-V') \nleq -\widetilde{\phi}(-\bar{V})$. From the proof above, if $f(\bar{x}) \in -\widetilde{\phi}(-\bar{V})$, we have $-\widetilde{\phi}(-V') \nleq f(\bar{x})$. This contradicts to (3.13), therefore, $\bar{V}$ is the solution of optimization problem (D_L^{VO}).

The proof is completed. □

The novelty of the paper is to be found in the way the perturbation function ϕ is defined. This gives raise to a new conjugate dual and allows the formulation of weak and strong duality theorems in finite dimensional spaces. For infinite dimensional situation, we need to structure a class of perturbation functions to get the different dual problems.

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