

HEAT TRANSFER IN THREE DIMENSIONAL
HYDROMAGNETIC FLOW ALONG A POROUS INFINITE
PLATE IN THE PRESENCE OF VISCOUS DISSIPATIVE HEAT

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Abstract: Gebhast has observed that in natural convection flow fields of extreme size or extremely low temperatures or in high gravity field, the heat due to viscous dissipation plays a dominant role. Hence taking the dissipative heat into account, the heat transfer in the hydromagnetic boundary layer flow of viscous incompressible and finitely conducting fluid along an infinite porous plate with slightly transverse sinusoidal suction has been analyzed. Expressions for velocity profile and temperature distribution are obtained. The rate of heat transfer coefficient on the surface of the plate is obtained and numerically computed to get physical insight of the problem.

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Key Words: heat transfer, dissipative heat, sinusoidal suction, Nusselt number

1. Introduction

Study of heat transfer in porous medium has paramount importance because of its potential applications in soil physics, geohydrology, filtration of solids from liquids, chemical engineering and biological systems. The heat transfer in saturated porous medium in presence of transverse magnetic field was studied by Rao et al [15] and brought out the effects of porous parameter on temperature

and Nusselt number. They have shown that the effects of increasing porous parameter is to increase the temperature and Nusselt number. But the effect of magnetic field on temperature is not discussed. Rao et al [14] made a similar analysis in a rotating straight pipe and derived the effects of porous parameter on the Nusselt number. It is shown that Nusselt number increases with increase in porosity. Gulab et al [8] investigated the unsteady flow in MHD porous media and concluded that the increase in porosity accelerates the flow. According to them unsteady motion converts into steady flow after considerable lapse of time and the flow becomes more stable near the wall. Gupta et al [9] studied three dimensional flow past a porous plate and established the effects of Hartmann number and suction parameter on velocity and skin friction.

Sikiadis [17] has shown that the flow past a continuously moving plate is different from the flow past a stationary plate studied by Blasius [2]. Goldstein et al [7] studied the heat transfer aspect of continuously moving plate without considering viscous dissipative heat. But when the velocity of the plate is rather higher in an incompressible fluid or when the Prandtl number of an incompressible fluid is high, the viscous dissipative heat does play an important role in heat transfer problem.

Gebhast [4] has observed that in natural convection flow fields of extreme size or extremely low temperature or in high gravity field, the heat due to viscous dissipation plays a dominant role. Soundalgekar [19] studied the effects of transverse magnetic field in the viscous dissipative effects on unsteady free convection in the literature of heat transfer. Gersten et al [6] studied the influence of slightly sinusoidal transverse transpiration (suction) velocity distribution on the flow and heat transfer over a plane wall in absence of magnetic field and dissipative heat. Haldavanekar et al [10] made an analysis of viscous dissipative heat on the two dimensional unsteady free convection on magnetohydrodynamic flow past an infinite vertical porous plate moving in its own plane. Taking the viscous dissipative heat into consideration Singh et al [18], investigated flow under magnetic field along an infinite porous plate with transverse sinusoidal suction and discussed the effects of heat transfer. Soundalgekar et al [20] investigated free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux. Geindreau et al [5] studied magnetohydrodynamic flows in porous media. Sahoo et al [16] studied magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Patangi et al [13] made heat transfer analysis of free convection of a non-Newtonian power law fluid over a vertical plate with constant heat flux.

Muthucumaraswamy et al [11] studied MHD flow past an impulsively started vertical plate with variable temperature and uniform mass diffusion. Amakiri et al [1] studied the effect of viscous dissipative heat and uniform magnetic field on the free convective flow through a porous medium with heat generation/Absorption.

Chaudhari et al [3] investigated combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium.

Muthucumaraswamy et al [12] studied effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature.

The present paper deals with heat transfer aspect in three dimensional hydromagnetic flow along a porous infinite plate in the presence of viscous dissipative heat. Expression for velocity profile and temperature distribution are obtained. With the help of these the Nusselt number is obtained and numerically computed for various values of suction parameter S , Hartmann number M and Reynolds number Re .

Results of numerical calculations are analysed to observe the influence of these parameters on the flow.

2. Mathematical Model Equations

We consider the steady free convection laminar flow of an electrically conducting viscous, incompressible fluid past an infinite porous flat plate. The plate is subjected to a slightly sinusoidal transverse suction velocity distribution. The suction velocity profile leads to a cross-flow and consequently to a three dimensional flow over the surface. Let L be the wavelength of the weak superimposed suction velocity distribution. The plate is taken horizontally as x - z plane. The x -axis is assumed to be along the horizontal porous infinite plate in the horizontal direction and z axis is taken perpendicular to it. The origin of coordinate system is taken to be at a point on the plate. Since the length of the plate is very large, all the physical variables are independent of x . The uniform magnetic field is applied perpendicularly to the plane x - z of the plate. The magnetic Reynolds number is taken very small. So the induced magnetic field is neglected. Hall effects, electrical and polarizations effect, Joule heating in the energy equation are also neglected. With these assumptions, the governing equations for the problem under consideration after boundary layer

simplifications in present notations in dimensionless form are:

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{\omega}}{\partial \bar{z}} = 0, \quad (1)$$

$$\bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{\omega} \frac{\partial \bar{u}}{\partial \bar{z}} = \frac{1}{Re} \left[\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{u}}{Re}, \quad (2)$$

$$\bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{\omega} \frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{Re} \left[\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right], \quad (3)$$

$$\therefore \bar{v} \frac{\partial \bar{\omega}}{\partial \bar{y}} + \bar{\omega} \frac{\partial \bar{\omega}}{\partial \bar{z}} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{Re} \left(\frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{\omega}}{\partial \bar{z}^2} \right) - \frac{M^2 \bar{\omega}}{Re}, \quad (4)$$

$$\bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} + \bar{\omega} \frac{\partial \bar{T}}{\partial \bar{z}} = \frac{1}{Pr Re} \left[\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right] + \frac{Ec \phi}{Re}. \quad (5)$$

Here

$$\phi = 2 \left[\left(\frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 + \left(\frac{\partial \bar{\omega}}{\partial \bar{z}} \right)^2 \right] + \left[\left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \left(\frac{\partial \bar{\omega}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{z}} \right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 \right]. \quad (6)$$

The boundary conditions are:

$$\begin{aligned} \bar{y} = 0 : \bar{u} = 0, \quad \bar{v} = -S(1 + \cos \pi \bar{z}), \quad \bar{\omega} = 0, \quad \bar{T} = 1, \\ \bar{y} \rightarrow \infty : \bar{u} = 1, \quad \bar{v} = -S, \quad \bar{\omega} = 0, \quad \bar{p} = \bar{p}_\infty, \quad \bar{T} = 0, \end{aligned} \quad (7)$$

where

$$\bar{y} = \frac{y}{L}; \quad \bar{z} = \frac{z}{L}; \quad \bar{u} = \frac{u}{U}; \quad \bar{v} = \frac{v}{U}; \quad \bar{\omega} = \frac{\omega}{U}; \quad \bar{p} = \frac{p}{\rho U^2}; \quad \bar{T} = \frac{T - T_\infty}{T_\omega - T_\infty}. \quad (8)$$

$\epsilon (\ll 1)$ = Amplitude of the suction velocity distribution; $\bar{u}, \bar{v}, \bar{\omega}$ are the dimensionless velocity components in x, y, z directions respectively, \bar{T} the non-dimensional temperature, ρ the constant density, ν ($= \frac{\mu}{\rho}$) Kinematic viscosity, μ the coefficient of viscosity, Re ($= \frac{UL}{\nu}$) the Reynolds number, Pr ($= \frac{\mu c_p}{k}$) the Prandtl number, c_p specific heat at constant pressure, assumed constant, Ec ($= \frac{U^2}{c_p(T_\omega - T_\infty)}$) the Eckert number, M [$= -B_0 L \left(\frac{\sigma}{\mu} \right)^{-\frac{1}{2}}$] the Hartmann number, σ the electrical conductivity, $\bar{v} > 0$ the basic steady suction velocity, s ($= \frac{\bar{v}}{U}$) suction parameter, U the free streamline velocity, L characteristic length (wave length).

3. Method of Solution

For the solution of equations (1) to (5) we assume the following expressions for \bar{u} , \bar{v} , \bar{w} , \bar{p} and \bar{T} near the plate:

$$\begin{aligned}\bar{u}(\bar{y}, \bar{z}) &= \bar{u}_0(\bar{y}) + \epsilon \bar{u}_1(\bar{y}, \bar{z}) + \epsilon^2 \bar{u}_2(\bar{y}, \bar{z}) + \dots, \\ \bar{v}(\bar{y}, \bar{z}) &= \bar{v}_0(\bar{y}) + \epsilon \bar{v}_1(\bar{y}, \bar{z}) + \epsilon^2 \bar{v}_2(\bar{y}, \bar{z}) + \dots, \\ \bar{w}(\bar{y}, \bar{z}) &= \bar{w}_0(\bar{y}) + \epsilon \bar{w}_1(\bar{y}, \bar{z}) + \epsilon^2 \bar{w}_2(\bar{y}, \bar{z}) + \dots, \\ \bar{p}(\bar{y}, \bar{z}) &= \bar{p}_0(\bar{y}) + \epsilon \bar{p}_1(\bar{y}, \bar{z}) + \epsilon^2 \bar{p}_2(\bar{y}, \bar{z}) + \dots, \\ \bar{T}(\bar{y}, \bar{z}) &= \bar{T}_0(\bar{y}) + \epsilon \bar{T}_1(\bar{y}, \bar{z}) + \epsilon^2 \bar{T}_2(\bar{y}, \bar{z}) + \dots.\end{aligned}\quad (9)$$

Substitute (9) in equation (1) to (5) and compare the coefficients of identical power of ϵ . The zeroth order equations give following ordinary differential equations:

$$\frac{d\bar{v}_0}{d\bar{y}} = 0, \quad (10)$$

$$\bar{v}_0 \frac{d\bar{u}_0}{d\bar{y}} = \frac{1}{Re} \frac{d^2\bar{u}_0}{d\bar{y}^2} - \frac{M^2\bar{u}_0}{Re}, \quad (11)$$

$$\bar{v}_0 \frac{d\bar{v}_0}{d\bar{y}} = \frac{d\bar{P}_0}{d\bar{y}} + \frac{1}{Re} \frac{d^2\bar{v}_0}{d\bar{y}^2}, \quad (12)$$

$$\bar{v}_0 \frac{d\bar{w}_0}{d\bar{y}} = \frac{1}{Re} \frac{d^2\bar{w}_0}{d\bar{y}^2} - \frac{M^2\bar{w}_0}{Re}, \quad (13)$$

$$\bar{v}_0 \frac{d\bar{T}_0}{d\bar{y}} = \frac{1}{ReP_r} \frac{d^2\bar{T}_0}{d\bar{y}^2} + \frac{Ec}{Re} \left[\frac{d\bar{u}_0}{d\bar{y}} \right]^2 \quad (14)$$

whose solutions give $\bar{u}_0(\bar{y})$ and $\bar{T}_0(\bar{y})$.

The first order equations are partial differential equations which describe three dimensional MHD flow. These equations are as follows:

$$\frac{\partial \bar{v}_1}{\partial \bar{y}} + \frac{\partial \bar{w}_1}{\partial \bar{z}} = 0, \quad (15)$$

$$\bar{v}_1 \frac{\partial \bar{u}_0}{\partial \bar{y}} + \bar{v}_0 \frac{\partial \bar{u}_1}{\partial \bar{y}} = \frac{1}{Re} \left[\frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{u}_1}{Re}, \quad (16)$$

$$\bar{v}_1 \frac{\partial \bar{w}_0}{\partial \bar{y}} - S \frac{\partial \bar{u}_1}{\partial \bar{y}} = \frac{1}{Re} \left[\frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{u}_1}{Re}, \quad (17)$$

$$\bar{v}_0 \frac{\partial \bar{v}_1}{\partial \bar{y}} = -\frac{\partial \bar{p}_1}{\partial \bar{y}} + \frac{1}{Re} \left[\frac{\partial^2 \bar{v}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}_1}{\partial \bar{z}^2} \right], \quad (18)$$

$$-S \frac{\partial \bar{w}_1}{\partial \bar{y}} = -\frac{\partial \bar{p}_1}{\partial \bar{z}} + \frac{1}{Re} \left[\frac{\partial^2 \bar{w}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{w}_1}{Re}, \quad (19)$$

$$\bar{v}_1 \frac{\partial \bar{T}_0}{\partial \bar{y}} - S \frac{\partial \bar{T}_1}{\partial \bar{y}} = \frac{1}{P_r Re} \left[\frac{\partial^2 \bar{T}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}_1}{\partial \bar{z}^2} \right] + \frac{2Ec}{Re} \frac{\partial \bar{u}_0}{\partial \bar{y}} \cdot \frac{\partial \bar{u}_1}{\partial \bar{y}}. \quad (20)$$

For the solution of above partial differential equations we assume:

$$\begin{aligned} \bar{u}_1(\bar{y}, \bar{z}) &= \bar{u}_{11}(\bar{y}) \cos \pi \bar{z}, & \bar{v}_1(\bar{y}, \bar{z}) &= \bar{v}_{11}(\bar{y}) \cos \pi \bar{z}, \\ \bar{\omega}_1(\bar{y}, \bar{z}) &= -\frac{1}{\pi} \bar{v}_{11}'(\bar{y}) \sin \pi \bar{z}, \\ \bar{P}_1(\bar{y}, \bar{z}) &= \bar{P}_{11}(\bar{y}) \cos \pi \bar{z}, & \bar{T}_1(\bar{y}, \bar{z}) &= \bar{T}_{11}(\bar{y}) \cos \pi \bar{z}, \end{aligned} \quad (21)$$

where the prime in v_{11} denotes the differentiation with respect to \bar{y} .

4. Solution

Solving zeroth and first order equations, we get

$$\bar{u}_0(\bar{y}) = 1 - e^{-A_1 \bar{y}}, \quad (22)$$

$$\bar{T}_0(\bar{y}) = (1 + h_1) e^{-sP_r Re \bar{y}} - h_1 e^{-2A_1 \bar{y}}, \quad (23)$$

$$\begin{aligned} \bar{T}_1(\bar{y}, \bar{z}) &= \frac{sP_r Re \cos \pi \bar{z}}{A_3 - A_4} \left[\frac{2A_1}{A_7} \{A_3 h_1 + E_c A_{12} (A_1 + A_4)\} \left(e^{-A_6 \bar{y}} - e^{-(2A_1 + A_4) \bar{y}} \right) \right. \\ &\quad - \frac{2A_1}{A_8} \{A_4 h_1 + E_c A_3 (A_1 + A_3)\} \left\{ e^{-A_6 \bar{y}} - e^{-(2A_1 + A_3) \bar{y}} \right\} \\ &\quad + \frac{sP_r Re A_4 (1 + h_1)}{A_9} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + sP_r Re) \bar{y}} \right\} \\ &\quad - \frac{sP_r Re A_3 (1 + h_1)}{A_{10}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + sP_r Re) \bar{y}} \right\} \\ &\quad \left. + \frac{2A_1 A_5 E_c (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + A_5) \bar{y}} \right\} \right], \quad (24) \end{aligned}$$

where

$$h_1 = \frac{P_r E_c A_1}{2(2A_1 - SP_r Re)}. \quad (25)$$

From equation (9) taking $\bar{T}(\bar{y}, \bar{z}) = \bar{T}_0(\bar{y}) + \in \bar{T}_1(\bar{y}, \bar{z})$ and substituting (23) and (24) in it, we get

$$\begin{aligned} \bar{T}(\bar{y}, \bar{z}) &= (1 + h_1) e^{-SP_r Re \bar{y}} - h_1 e^{-2A_1 \bar{y}} \\ &\quad + \frac{\epsilon SP_r Re \cos \pi \bar{z}}{A_3 - A_4} \left[\frac{2A_1}{A_7} \{A_3 h_1 + E_c A_{12} (A_1 + A_4)\} \left\{ e^{-A_6 \bar{y}} - e^{-(2A_1 + A_4) \bar{y}} \right\} \right. \end{aligned}$$

$$\begin{aligned}
 & -\frac{2A_1}{A_8} \{A_4 h_1 + E_c A_3 (A_1 + A_3)\} \left\{ e^{-A_6 \bar{y}} - e^{-(2A_1 + A_3) \bar{y}} \right\} \\
 & + \frac{SP_r Re A_4 (1 + h_1)}{A_9} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + SP_r Re) \bar{y}} \right\} \\
 & - \frac{SP_r Re A_3 (1 + h_1)}{A_{10}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + SP_r Re) \bar{y}} \right\} \\
 & + \left. \frac{2A_1 A_5 E_c (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + A_5) \bar{y}} \right\} \right]. \tag{26}
 \end{aligned}$$

This gives the temperature field. Knowing the temperature profile, the rate of heat transfer coefficient at the surface of the plate in terms of the Nusselt number $N\bar{u}$ is obtained as:

$$N\bar{u} = SP_r Re \left[1 + h_1 - \frac{2A_1 h_1}{SP_r Re} + \epsilon (1 - H) \cos \pi \bar{z} \right], \tag{27}$$

where

$$\begin{aligned}
 H & = 1 - \frac{1}{A_3 - A_4} \left[\frac{2A_1}{A_7} \{A_3 h_1 + E_c A_{12} (A_1 + A_4)\} (A_6 - 2A_1 - A_4) \right. \\
 & - \frac{2A_1}{A_8} \{A_4 h_1 + E_c A_{13} (A_1 + A_4)\} (A_6 - 2A_5 - A_3) \\
 & + \frac{SP_r Re A_4}{A_9} (1 + h_1) (A_6 - SP_r Re - A_3) \\
 & + \frac{SP_r Re A_3}{A_{10}} (A_6 - SP_r Re - A_4) \\
 & \left. + \frac{2A_1 A_5 E_c (A_{13} - A_{12})}{A_{11}} (A_6 - A_1 - A_5) \right], \tag{28}
 \end{aligned}$$

and

$$\begin{aligned}
 A_1 & = \frac{1}{2} \left[SRe + (S^2 Re^2 + 4M^2)^{\frac{1}{2}} \right], \\
 A_2 & = \frac{1}{2} \left[SRe - (S^2 Re^2 + 4M^2)^{\frac{1}{2}} \right], \\
 A_3 & = \frac{1}{2} \left[A_1 + (A_1^2 + 4\pi^2)^{\frac{1}{2}} \right], \\
 A_4 & = \frac{1}{2} \left[A_2 + (A_2^2 + 4\pi^2)^{\frac{1}{2}} \right], \\
 A_5 & = \frac{1}{2} \left[SRe + \{S^2 Re^2 + 4(\pi^2 + M^2)\}^{\frac{1}{2}} \right], \\
 A_6 & = \frac{1}{2} \left[SP_r Re + \{(SP_r Re)^2 + 4\pi^2\}^{\frac{1}{2}} \right], \\
 A_7 & = (2A_1 + A_4)^2 - SP_r Re (2A_1 + A_4) - \pi^2,
 \end{aligned}$$

	$M \rightarrow$		$N\bar{u}$			
	S	Re	0	3	5	10
0.71	0.5	0.1	0.03531895	0.0247577	0.0176577	0.00008875
0.71	0.5	1	0.3531895	0.3433915	0.336327	0.318577
0.71	0.5	10	3.531895	3.526925	3.521245	3.504205
0.71	1	0.1	0.0706379	0.0601654	0.0530654	0.0353154
0.71	1	1	0.706379	0.697362	0.690333	0.672654
0.71	1	10	7.06379	7.06095	7.05669	7.04249
0.71	1.5	0.1	0.10595685	0.0955731	0.0884802	0.07072665
0.71	1.5	1	1.0595685	1.0512615	1.044339	1.02666
0.71	1.5	10	10.59462	10.59462	10.595685	10.578645

Table 1: Values of Nu for different values of Pr, S, Re when $Ec = 0.01, \epsilon = 0$

$$\begin{aligned}
 A_8 &= (2A_1 + A_3)^2 - SP_r Re(2A_1 + A_3) - \pi^2, \\
 A_9 &= A_3(A_1 + SP_r Re), \\
 A_{10} &= A_4(A_2 + SP_r Re), \\
 A_{11} &= (A_1 + A_5)^2 - SP_r Re(A_1 + A_5) - \pi^2, \\
 A_{12} &= \frac{A_3}{A_4}, \\
 A_{13} &= \frac{A_1 A_4}{A_3(3A_1 - SRe)}.
 \end{aligned}$$

5. Results

For $\epsilon = 0$, equation (27) reduces to

$$N\bar{u} = SP_r Re \left[1 + h_1 - \frac{2A_1 h_1}{SP_r Re} \right], \tag{29}$$

and it is numerically computed for different values of magnetic parameter M , Reynolds number Re , suction parameter S , prandtl number Pr and Eckert number Ec .

The calculated values are entered in Tables 1, 2 and 3.

Pr	$M \rightarrow$	Re	Nu			
	S		0	3	5	10
0.0328	0.5	0.1	1.631636×10^{-3}	1.143736×10^{-3}	8.15736×10^{-4}	4.92×10^{-6}
0.0328	0.5	1	0.01631636	0.01586372	0.01553736	0.01471736
0.0328	0.5	10	0.1631636	0.162934	0.1626716	0.1618844
0.0328	1	0.1	0.003263272	0.002779472	0.002451472	0.001631472
0.0328	1	1	0.03263272	0.03221616	0.03189144	0.03107472
0.0328	1	10	0.3263272	0.326196	0.3259992	0.3253432
0.0328	1.5	0.1	0.004894908	0.004415208	0.004087536	0.003267372
0.0328	1.5	1	0.04894908	0.04856532	0.04824552	0.0474288
0.0328	1.5	10	0.4894908	0.4894416	0.4894908	0.4894908

Table 2: Values of Nu for different values of Pr, S, Re when $Ec = 0.01$ and $\epsilon = 0$

$Re \rightarrow$	$Ec \downarrow$	$N\bar{u}$		
		10	20	50
0		7.1	14.2	35.5
0.02		6.985119585	14.02818	35.13132668
0.04		6.870240251	13.8571817	34.7626551
0.06		6.755358772	13.68577255	34.39398
0.08		6.640478358	13.51436337	34.02530664
0.10		6.525597958	13.34295421	33.65663332

Table 3: Variation of $N\bar{u}$ against Ec when $S = 1, M = 10, Pr = 0.71, \epsilon = 0$

6. Conclusions

A steady free convection laminar flow of an electrically conducting viscous incompressible fluid past an infinite porous plate in presence of a transverse magnetic field is analyzed numerically and the following conclusions are obtained:

1. The rate of heat transfer decreases with increase in the strength of magneticity.
2. Nusselt number decreases with increase in Eckert number. That is rate of heat transfer decreases with increase in dissipative heat.
3. For fixed value of Hartmann number, Nusselt number increases with increase in Reynolds number.
4. For fixed value of Hartmann number, Nusselt number increases with increase in suction parameter.

5. For fixed value of Eckert number, Nusselt number increases with increase in Reynolds number.

6. For fixed value of Prandtl number, Reynolds number and suction parameter, Nusselt number decreases with increase in Hartmann number.

7. Dissipation heat plays a dominant role in the hydromagnetic flow.

8. Effects of dissipation heat as rate of heat transfer is quite important.

9. For correct prediction of temperature field the influence of dissipation heat should also be taken into account.

10. In absence of magnetic field (Hartmann number zero) and dissipative heat (Eckert number zero), the expression for suction parameter $S = 1$ reduces to

$$H = \frac{1}{A_{14} - \pi} \left[\left\{ \frac{A_{14}}{\pi} - \frac{\pi P_r}{A_{14}(1 + P_r)} \right\} A_{15} + \frac{\pi P_r^2 Re}{A_{14}(1 + P_r)} + \frac{\pi P_r}{1 + P_r} - \frac{A_{14} P_r Re}{\pi} - \pi \right], \quad (30)$$

where

$$A_{14} = \frac{1}{2} \left[Re + (Re^2 + 4\pi^2)^{\frac{1}{2}} \right],$$

$$A_{15} = \frac{1}{2} \left[P_r Re + (P_r^2 Re^2 + 4\pi^2)^{\frac{1}{2}} \right].$$

In the notation of this paper the expression (30) is same as obtained by Gersten and Gross (1974). This confirms the correctness of the present method in which they did not consider magnetic field and dissipative heat.

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