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QUANTUM FIELDS IN A COMBINED SPACE-TIME 4-MANIFOLD – A PRELIMINARY HEURISTIC EXPOSITION

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Abstract: This present paper connects to quantum mechanics our previous geometric construct of a combined space-time 4-manifold "M3" published earlier in this journal. Here we resolve the (particle, wave) duality by establishing the proposition that 0.75 of a known particle is manifested as a point particle in the recognized space-time 4-manifold "M1" of matter and light, and the remaining 0.25 is manifested as an electromagnetic wave of energy in a complex space-time (sub) 4-manifold "M2" that is contained in a larger space-time 4-manifold "M2+" comprising of electromagnetic waves only; i.e., we have identified the quantum wave function of a particle in "M1" with the complex norm of the electric field in "M2" as associated with the particle. In addition to settling the particle/wave energy ratio, we have also derived the other previously undetermined parameter, i.e., the gravitational constant of "M2+," which turns out to be so astronomically large as to cause the Schwarzschild singularity "M2," accounting for the quantum mechanics therein.

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1. Introduction

In a previous paper [9] (also cf. [10]) we proposed the idea of a combined space-time 4-manifold

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$$\mathcal{M}^{[3]} : \left\{ \left(p^{[1]}, p^{[2]}\right) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]} \mid h\left(p^{[1]}\right) = p^{[2]}, \right. \\ \left. h = \text{any diffeomorphism} \right\}, \quad (1.1)$$

where $\mathcal{M}^{[i]}$, $i = 1, 2$, is determined by the metric $g^{[i]}$ in Einstein field equations,

$$R^{[i]}_{\mu\nu} - \frac{1}{2} R^{[i]} g^{[i]}_{\mu\nu} = -\frac{8\pi G^{[i]}}{c^2} T^{[i]}_{\mu\nu}; \tag{1.2}$$

$\mathcal{M}^{[1]} \equiv$ the space-time of matter and light, and $\mathcal{M}^{[2]} \equiv$ the space-time of "invisible energies" (energies that only exert gravitational influences upon themselves. We proved (by a consideration of the form invariance of $g^{[i]}_{11} = 1 - \frac{2G^{[i]}M^{[i]}}{rc^2}$ $\forall i \in \{1, 2, 3\}$) that the gravitational motions in $\mathcal{M}^{[3]}$ are determined by the geodesics as measured by the weighted-average metric

$$g^{[3]} = \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \cdot g^{[1]} + \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \cdot g^{[2]}, \tag{1.3}$$

which has the Newtonian approximation

$$\left(m^{[1]} + m^{[2]}\right) \mathbf{a}^{[3]} \\ = -\frac{G^{[2]}}{G^{[1]} + G^{[2]}} \cdot \left(\frac{G^{[1]} M^{[1]} m^{[1]}}{r^2} \right) - \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \cdot \left(\frac{G^{[2]} M^{[2]} m^{[2]}}{r^2} \right); \quad (1.4)$$

here, $m^{[1]} + m^{[2]} \equiv m^{[3]}$ is the total mass of a "combined particle" $\Omega \equiv \left(m^{[1]}, m^{[2]}\right)$ at $r > 0$, which is attracted to another combined particle $\Lambda \equiv \left(M^{[1]}, M^{[2]}\right)$ at $r = 0$.

Rewriting the above equation, we have

$$\mathbf{a}^{[3]} = -\frac{G^{[3]} M^{[3]}}{r^2} \left(\frac{M^{[1]}}{M^{[3]}} \cdot \frac{m^{[1]}}{m^{[3]}} + \frac{M^{[2]}}{M^{[3]}} \cdot \frac{m^{[2]}}{m^{[3]}} \right), \tag{1.5}$$

where

$$\begin{aligned} G^{[3]} &\equiv \frac{G^{[1]} G^{[2]}}{G^{[1]} + G^{[2]}} && (1.6) \\ &\equiv \text{the recognized universal gravitational constant} && (1.7) \\ &\approx 6.7 \times 10^{-11} \times \frac{\text{meter}^3}{\text{kilogram} \times \text{second}^2}. && (1.8) \end{aligned}$$

By assuming $\frac{M^{[1]}}{M^{[3]}} = \frac{m^{[1]}}{m^{[3]}} = 0.95$ in the previous paper, we gave an example of the gravity of Earth at $r \approx 6.37 \times 10^6$ meter, i.e., on the surface of the planet:

$$\left| \mathbf{a}^{[3]} \right| \approx 9.8 \times \frac{\text{meter}}{\text{second}^2} \tag{1.9}$$

$$= \frac{6.7 \times 10^{-11} \times \frac{\text{meter}^3}{\text{kilogram}\times\text{second}^2} \times M^{[3]}}{\left(6.37 \times 10^6 meter\right)^2} \times \left(0.95^2 + 0.05^2\right), \quad (1.10)$$

where $M^{[3]} \times 0.905 \equiv$ the recognized Earth mass $\approx 5.98 \times 10^{24}$ kilograms, implying that

$$\begin{aligned} M^{[3]} &= 6.61 \times 10^{24} \text{ kilograms} && (1.11)\\ &= M^{[1]} + M^{[2]} && (1.12)\\ &= 6.28 \times 10^{24} \text{ kilograms } + 0.33 \times 10^{24} \text{ kilograms.} && (1.13) \end{aligned}$$

From this example, we see that the ratio $\frac{M^{[1]}}{M^{[3]}}$ is yet to be determined; also, the weight

$$w^{[1]} \equiv \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \equiv 1 - w^{[2]} \quad (1.14)$$

in the determination of $g^{[3]}$, or equivalently the values of $G^{[1]}$ and $G^{[2]}$, remain to be fixed. In the following Section 2, we will settle these undetermined quantities. As a logical consequence, we will then show in Section 3 that the combined particle $\Omega \equiv \left(m^{[1]}, m^{[2]}\right) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]}$, where $m^{[1]}$ manifests itself as a particle $p \in \mathcal{M}^{[1]}$ and $m^{[2]}$ manifests itself as an energy wave $\varpi \in \mathcal{M}^{[2]}$, with $\mathcal{M}^{[2]} = \mathbf{B}$ (a black hole contained in a larger space-time $\tilde{\mathcal{M}}^{[2]}$ of invisible energies, cf. e.g., [5], for how a black hole may give rise to a macroscopic universe, i.e., our $\mathcal{M}^{[1]}$) = a complex (sub) 4-manifold comprising of quantum fields. Lastly in Section 4 we will make some summary remarks. In line with the trend (cf. e.g., [15, 17]) of a return to 4-dimensional analyses from the previous elaborate programs such as supersymmetry, this paper only seeks to introduce our combined 4-manifold as a frame of quantum mechanics, deferring technical details until a sequel.

Notation 1. Throughout this paper: (1) The letter "M" denotes mass by appearing as M, and manifold by appearing as $\mathcal{M}$. (2) The letter "E" denotes energy by appearing as E, and electric field by appearing as $\mathbb{E}$. (3) Although a particle p, a wave ϖ, a mass m, or energy E, is not a geometric object, for expository simplicity we will nevertheless use set symbols as in $\Omega \equiv \left(m^{[1]}, m^{[2]}\right) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]}$, $p \in \mathcal{M}^{[1]}$, $\varpi \in \mathcal{M}^{[2]}$, or $E^{[2]} \in \mathcal{M}^{[2]}$.

2. The Particle/Wave Energy Ratio and the Three Gravitational Constants

Postulate. $\mathcal{M}^{[1]}$ contains point particles, which engage in gravitational forces and may engage in electromagnetic, strong or weak nuclear forces; $\mathcal{M}^{[2]}$

contains electromagnetic waves, which only engage in gravitational forces; any particle $p \in \mathcal{M}^{[1]}$ carries a distinct electromagnetic energy wave $\varpi \in \mathcal{M}^{[2]}$ (as a historical note: Einstein actually envisaged the idea of a particle carrying a "ghost wave – Gespensterfelder;" see, e.g., [14], 287-288) and exists as a combined particle $\Omega \equiv (p, \varpi) \in \mathcal{M}^{[3]}$; Ω carries energy $E^{[3]} = E^{[1]} + E^{[2]}$, with $E^{[1]}$ from p and $E^{[2]}$ from ϖ.

Remark 1. By Maxwell equations for spaces free from electric charges, electromagnetic fields, which carry energy, can still exist (cf. e.g., [3], II Chapter 20). Also, the particles $\{p_i\} \subset \mathcal{M}^{[1]}$ engage in non-gravitational forces via sending and receiving virtual mediating particles.

Proposition 1. (The Particle/Wave Energy Ratio) *For any electric charge of mass $m^{[3]}$ $\left(\equiv m^{[1]} + m^{[2]}\right)$, we have*

$$m^{[1]} = \frac{3}{4} m^{[3]} \in \mathcal{M}^{[1]}, \text{ and} \tag{2.1}$$

$$m^{[2]} = \frac{1}{4} m^{[3]} \in \mathcal{M}^{[2]}. \tag{2.2}$$

Proof. Refer to [3], II-28-2 to 4, for the detailed calculation; otherwise, see the following explanatory remark. □

Remark 2. In the above referred [3] Feynman calculated the mass of a stationary charge by a consideration of the electrostatic force exerted by a single electron q,

$$m_{stationary} = \frac{1}{2} \left(\frac{q^2}{4\pi\varepsilon_o r_e c^2} \right), \tag{2.3}$$

where $\varepsilon_o \equiv$ the permittivity constant $\lesssim 10^{-11} \cdot \frac{\text{coulomb}^2 \cdot \text{second}^2}{\text{kilogram} \cdot \text{meter}^3}$, $r_e \equiv 2.82 \times 10^{-15} \cdot$meter $\equiv$ the classical electron radius, and $c \equiv 3 \times 10^8 \cdot \frac{\text{meter}}{\text{second}} \equiv$ the speed of light in empty spaces. He then calculated the mass of a nonrelativistically moving electron to be

$$m_{moving} = \frac{2}{3} \left(\frac{q^2}{4\pi\varepsilon_o r_e c^2} \right) = \frac{4}{3} m_{stationary}, \tag{2.4}$$

and he pointed out that the above discrepancy between $m_{stationary}$ and m_{moving} could not be resolved by quantum mechanics ([3] , II-28-4 to 12). We now resolve this discrepancy (see, e.g., [13], for this celebrated discrepancy) by our setup of the combined particle. From the postulate above, electric forces take place only in $\mathcal{M}^{[1]}$; as such, we attribute $m_{stationary}$ to $m^{[1]}$ of q. However, the motions of q necessarily take place in $\mathcal{M}^{[3]}$; thus, we set $m_{moving} = m^{[3]} \equiv m^{[1]} + m^{[2]}$, with $m^{[1]} = \frac{3}{4} m^{[3]} =$ the mass of the electron existing as a point particle p in $\mathcal{M}^{[1]}$, and $m^{[2]} = \frac{1}{4} m^{[3]} = E^{[2]}/c^2 =$ the mass (energy) of the electron existing as an electromagnetic wave of energy ϖ in $\mathcal{M}^{[2]}$, where ϖ is concentrated within a sphere of radius of r_e. Of course, there are particles such as photons and neutrons that do not possess electric charge, but for our present paper we will apply the same ratio $\frac{3}{4} : \frac{1}{4}$ to all particles about their energy distribution over $\mathcal{M}^{[1]}$ and $\mathcal{M}^{[2]}$.

Corollary 1. $\forall \Omega \in \mathcal{M}^{[3]}$ *of total mass (energy)* $M^{[3]} \equiv M^{[1]} + M^{[2]}$,

$$\begin{aligned} M^{[3]} &= 1.6 \times \text{ the measured (or known) mass,} && (2.5)\\ M^{[1]} &= 1.2 \times \text{ the measured (or known) mass, and} && (2.6)\\ M^{[2]} &= 0.4 \times \text{ the measured (or known) mass.} && (2.7) \end{aligned}$$

Proof. Any measured mass (energy) $M_{measured}$ necessarily observes

$$\begin{aligned} \mathbf{a}^{[3]} &= -\frac{G^{[3]}M^{[3]}}{r^2}\left(\frac{M^{[1]}}{M^{[3]}}\cdot\frac{m^{[1]}}{m^{[3]}} + \frac{M^{[2]}}{M^{[3]}}\cdot\frac{m^{[2]}}{m^{[3]}}\right) \quad \text{(equation (1.5))} && (2.8)\\ &= -\frac{G^{[3]}M^{[3]}}{r^2}\left(\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right) && (2.9)\\ &= -\frac{G^{[3]}M^{[3]}\cdot\left(\frac{10}{16}\right)}{r^2} && (2.10)\\ &\equiv -\frac{G^{[3]}M_{measured}}{r^2}; && (2.11) \end{aligned}$$

thus, we have the conclusion. □

Proposition 2. (The Gravitational Constant of the Invisible Space-Time $\tilde{\mathcal{M}}^{[2]}$)

$$G^{[2]} \approx \frac{c^5 \times \text{ second}^2}{1.6h} \approx 2.3 \times 10^{75} \times \frac{\text{meter}^3}{\text{kilogram} \times \text{ second}^2}, \qquad (2.12)$$

where $h \equiv$ *Planck constant* $\approx 6.6 \times 10^{-34} \times$ *joule* $\times$ *second.*

Proof. $\tilde{\mathcal{M}}^{[2]}$ is determined by Einstein field equations

$$R^{[2]}_{\mu\nu} - \frac{1}{2}R^{[2]}g^{[2]}_{\mu\nu} = -\frac{8\pi G^{[2]}}{c^2}T^{[2]}_{\mu\nu}, \qquad (2.13)$$

which has the Schwarzschild solution for the $(\text{time}, \text{time})$ component of $g^{[2]}$

$$\begin{aligned} g^{[2]}_{11} &= \left(\frac{t^{[2]}_o \equiv \text{ a proper time in } \mathcal{M}^{[2]}}{t^{[1]}_o \equiv \text{ a proper time in } \mathcal{M}^{[1]}}\right)^2 && (2.14)\\ &= 1 - \frac{2G^{[2]}M^{[2]}}{rc^2}, && (2.15) \end{aligned}$$

where $M^{[2]} = E^{[2]}/c^2$ has the center of gravity at $r = 0$. By Planck's formula the measured energy of a photon satisfies

$$E_{measured} = h\nu \tag{2.16}$$

$$\forall \nu \equiv \frac{\text{the (pure) number of the traveled electromagnetic wave cycle}}{1 \text{ second}} \gg \frac{1}{\text{second}} \tag{2.17}$$

(cf. [11], 206, 219, for how Planck derived his formula). Thus, by Corollary 1 we have

$$E^{[3]} = 1.6\, h\nu, \text{ and } E^{[2]} = 0.4\ h\nu. \tag{2.18}$$

Let $\lambda \equiv$ the wave length $= d$ meter, $d > 0$ being a pure number; then

$$\nu = \frac{c}{\lambda}, \tag{2.19}$$

so that

$$\begin{aligned}
\sqrt{-1} \cdot \text{second} \cdot \nu &= \sqrt{-1} \cdot \text{second} \cdot \frac{3 \times 10^8 \times \frac{\text{meter}}{\text{second}}}{d \times \ \text{meter}} && (2.20)\\
&= \frac{\left(\frac{3 \times 10^8}{d}\right) \times \sqrt{-1} \text{ second}}{1 \text{ second}} && (2.21)\\
&= \frac{t_o^{[2]}}{t_o^{[1]}}, && (2.22)
\end{aligned}$$

where $t_o^{[2]} = \left(\frac{3\times 10^8}{d}\right) \times \sqrt{-1}$ second $=$ the proper time as registered by a clock in $\mathcal{M}^{[2]}$ at a distance of radius $r = \frac{\lambda}{2}$ to the center of the ball that contains one cycle of the electromagnetic wave energy (cf. [4], 34, and [8] for the analytic continuation of time), and $t_o^{[1]} = 1$ second $=$ the proper time as registered by a clock at a point in a parameter domain in $\mathcal{M}^{[1]}$. Thus,

$$\begin{aligned}
-\nu^2 \times \text{second}^2 &= \left(\frac{t_o^{[2]}}{t_o^{[1]}}\right)^2 = g_{11}^{[2]} && (2.23)\\
&= 1 - \frac{2G^{[2]}E^{[2]}}{\frac{\lambda}{2} \cdot c^4} \ \text{(by equation (2.15))} && (2.24)\\
&= 1 - \frac{4G^{[2]} \times 0.4 h\nu^2}{c^5} && (2.25)\\
&\quad \text{(by equations (2.18), (2.19))} &&\\
&\approx -\frac{1.6 G^{[2]}\, h\nu^2}{c^5} \equiv -b && (2.26)\\
\forall b &> > 1; && (2.27)
\end{aligned}$$

then we have

$$1 \text{ second}^2 \approx \frac{1.6G^{[2]}\, h}{c^5}, \tag{2.28}$$

i.e.,

$$\begin{aligned} G^{[2]} &\approx \frac{c^5 \times \text{second}^2}{1.6h} && (2.29)\\ &= \frac{\left(3\times 10^8\right)^5 \times \left(\frac{\text{meter}}{\text{second}}\right)^5 \times \text{second}^2}{1.6\times 6.6\times 10^{-34}\times \frac{\text{kilogram}\times\text{meter}^2}{\text{second}^2}\times\ \text{second}} && (2.30)\\ &\approx 2.3\times 10^{75}\times \frac{\text{meter}^3}{\text{kilogram}\times\ \text{second}^2}. \qquad \square && (2.31) \end{aligned}$$

Remark 3. For the above proof to be valid, we must check back if indeed the assumption (inequality (2.27)) of $b \equiv \frac{1.6G^{[2]}\, h\nu^2}{c^5} \gg 1$ is observed: By equation (2.28), we have $b = (\nu\cdot\text{second})^2$, but from inequality (2.17) we have $(\nu\cdot\text{second}) \gg 1$; thus, $b \gg 1$.

Corollary 2. $G^{[1]} \approx G^{[3]}$; $w^{[1]} \lessapprox 1$ $\left(\textit{or } w^{[2]} \gtrapprox 0\right)$.

Proof.

$$\begin{aligned} G^{[3]} &\equiv 6.7\times 10^{-11}\times \frac{\text{meter}^3}{\text{kilogram}\times\text{second}^2} && (2.32)\\ &= \frac{G^{[1]}G^{[2]}}{G^{[1]}+G^{[2]}} \quad \text{(from equation (1.6))} && (2.33)\\ &\approx \frac{6.7\times 10^{-11}\times 2.3\times 10^{75}}{6.7\times 10^{-11}+2.3\times 10^{75}}\times \frac{\text{meter}^3}{\text{kilogram}\times\text{second}^2}, && (2.34) \end{aligned}$$

i.e., $G^{[1]} \approx G^{[3]}$.

$$\begin{aligned} w^{[1]} &= \frac{G^{[2]}}{G^{[1]}+G^{[2]}} \quad \text{(from equation (1.14))} && (2.35)\\ &\lessapprox 1, \text{ or} && (2.36)\\ w^{[2]} &\equiv 1 - w^{[1]} \gtrapprox 0. \qquad \square && (2.37) \end{aligned}$$

3. The Quantum Fields in $\mathcal{M}^{[2]}$

As is well known, the singularity of $g_{11} = 0 = 1 - \frac{2GM}{rc^2}$ in the Schwarzschild metric can be removed by a change of the coordinate system, but in view of the fact that the physical existence of black holes in space has been generally accepted, we will illustrate the formation of the black hole $\mathbf{B} = \mathcal{M}^{[2]} \subset \tilde{\mathcal{M}}^{[2]}$

with the Schwarzschild radius

$$r_{\mathbf{B}} = \frac{2G^{[2]}\mathcal{E}^{[2]}}{c^4}, \tag{3.1}$$

where $\mathcal{E}^{[2]}$ is the entire energy contained in $\mathbf{B}$. We claim that the Big Bang creating $\mathcal{M}^{[1]}$ was the result of $g_{11}^{[2]} = 0$ in $\tilde{\mathcal{M}}^{[2]}$ (again, cf. [5]); then any combined particle $\Omega \equiv \left(E^{[1]}, E^{[2]}\right) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]}$ is such that the electromagnetic wave energy $E^{[2]}$ is contained in the complex space-time submanifold $\mathbf{B} = \mathcal{M}^{[2]}$ in $\tilde{\mathcal{M}}^{[2]}$ with $r < r_{\mathbf{B}}$ and consequently $g_{11}^{[2]} < 0$, consistent with the derivation of $G^{[2]}$ in Section 2 (cf. equation (2.23); for general singularity formations in Einstein manifolds, see, e.g., [2, 16]).

Remark 4. In our previous paper [9], we identified the "dark matter" with $\left(0, M^{[2]}\right)$ and defined

$$G^{[3]} = G^{[2]} \text{ if } M^{[1]}m^{[1]} = 0 \quad ([9]\,, 311)\,; \tag{3.2}$$

as such, a combined particle $\left(m^{[1]}, m^{[2]}\right)$ would be subject to a much stronger gravity toward $\left(0, M^{[2]}\right)$ than toward $\left(\frac{3}{4}M^{[2]}, \frac{1}{4}M^{[2]}\right)$, of the same total mass, as shown below:

$$\begin{aligned} \left(m^{[1]} + m^{[2]}\right)\left|\mathbf{a}^{[3]}\right| &= \frac{G^{[2]}M^{[2]}m^{[2]}}{r^2} && (3.3)\\ \gg\ & \frac{G^{[3]}\left(\frac{3}{4}M^{[2]}\cdot m^{[1]} + \frac{1}{4}M^{[2]}\cdot m^{[2]}\right)}{r^2} && (3.4)\\ =\ & \frac{G^{[3]}\left(\frac{3}{4}M^{[2]}\cdot 3m^{[2]} + \frac{1}{4}M^{[2]}\cdot m^{[2]}\right)}{r^2} && (3.5)\\ =\ & \frac{2.5G^{[3]}M^{[2]}m^{[2]}}{r^2}. && (3.6) \end{aligned}$$

Proposition 3. (The Complex Electric Field) *Let $f^{[1]} : U \longrightarrow \mathcal{M}^{[1]}$ be a local parametrization of $\mathcal{M}^{[1]}$, where $U \subset \mathbb{R}^{1+3}$ ($\equiv$ the Minkowski space) and $f^{[1]}(U) \approx$ a free space (from forces). Let photon γ_j of angular frequency $-\omega_j$ ($\equiv -2\pi\nu_j$, where "frequency" is defined clockwise) be propagated along any arbitrary direction through $(0, \mathbf{0})$ to $(t, \mathbf{x_j} \equiv (\hat{x}_j, \hat{y}_j, \hat{z}_j))$ as recorded in U. Then the associated electric field is*

$$\mathbb{E}_j\left(t, \mathbf{x_j}\right) = \mathbb{E}_{oj} \cdot e^{-i\left(\omega_j t - \mathbf{k}_j\cdot\mathbf{x_j} + \theta_j\right)}, \tag{3.7}$$

where $\theta_j \in [0, 2\pi)$, $\mathbb{E}_{oj} \in \mathbb{R}^3$ is such that

$$\begin{aligned} \mathbb{E}_{oj}\cdot\mathbf{x_j} &= 0, \text{ and} && (3.8)\\ \left\|\mathbb{E}_{oj}\right\| &\approx \sqrt{\frac{3h\nu_j}{4\pi\epsilon_o r_{\mathbf{B}}^3}}\ \ \forall \nu_j \gg \frac{1}{second}, && (3.9) \end{aligned}$$

and $\mathbf{k}_j \cdot \mathbf{x_j} = \left(\frac{2\pi}{\lambda_j}\right) \cdot \|\mathbf{x_j}\| =$ *the number of radians for* γ_j *to travel from* $(0, \mathbf{0})$ *to* $(t, \mathbf{x_j} \equiv (\hat{x}_j, \hat{y}_j, \hat{z}_j))$.

Proof. By choosing the Lorentz gauge, the solutions of Maxwell equations in free space reduce to solving the scalar wave equation (cf. e.g., [3], II-18-11)

$$u_{tt} = c^2 u_{xx}, \tag{3.10}$$

which has solutions of the general form

$$u(x,t) = f(x-ct) + g(x+ct) \quad \forall f, g \in C^2(\mathbb{R}, \mathbb{R}). \tag{3.11}$$

Thus, we can consider the solution

$$\frac{2\pi}{\lambda_j}(x-ct) \equiv \left(\frac{2\pi}{\lambda_j} \cdot x - \omega_j t\right) \equiv (k_j x - \omega_j t). \tag{3.12}$$

By identifying with $x > 0$ any arbitrary direction of propagation $\mathbf{x}_j \equiv (\hat{x}_j, \hat{y}_j, \hat{z}_j) \in \mathbb{R}^3$, we have

$$\begin{aligned} k_j x &\equiv k_j \left(\frac{\hat{x}_j}{\|\mathbf{x_j}\|}, \frac{\hat{y}_j}{\|\mathbf{x_j}\|}, \frac{\hat{z}_j}{\|\mathbf{x_j}\|}\right) \cdot (\hat{x}_j, \hat{y}_j, \hat{z}_j) && (3.13)\\ &\equiv \mathbf{k}_j \cdot \mathbf{x_j} && (3.14)\\ &\equiv k_j \|\mathbf{x_j}\| && (3.15)\\ &\equiv \frac{2\pi}{\lambda_j} \|\mathbf{x_j}\|, && (3.16) \end{aligned}$$

i.e., $(\mathbf{k}_j \cdot \mathbf{x_j} - \omega_j t)$ is a solution of Maxwell equations, but by equation (2.22)

$$\begin{aligned} t_o^{[2]} &= i\ \nu_j\ t_o^{[1]} \text{ second} && (3.17)\\ &\equiv i\left(\frac{\omega_j}{2\pi}\right) t \text{ second}; && (3.18) \end{aligned}$$

thus, the physical wave of γ_j in $\mathcal{M}^{[2]}$ is necessarily a function of $i(\mathbf{k}_j \cdot \mathbf{x_j} - \omega_j t)$ (cf. e.g., [1], for the inherent necessity of the imaginary time in quantum mechanics). Since the function is periodic, we arrive at $\mathbb{E}_j(t, \mathbf{x_j}) = \mathbb{E}_{oj} \cdot e^{-i(\omega_j t - \mathbf{k}_j \cdot \mathbf{x_j} + \theta_j)}$, for some $\mathbb{E}_{oj} \in \mathbb{R}^3$ and some $\theta_j \in [0, 2\pi)$.

Since the known energy density over all the space of the electromagnetic field associated with γ_j is

$$\epsilon_o \|\mathbb{E}_j(t, \mathbf{x_j})\|^2 \quad \text{(cf. e.g., [3], II-27-7)}, \tag{3.19}$$

we have the total energy of γ_j in $\mathcal{M}^{[3]}$ equal to

$$\begin{aligned} E_j^{[3]} &= 1.6 h\nu_j \left(\forall \nu_j \gg \frac{1}{\text{second}}, \text{ recall equations (2.17), (2.18)}\right) && (3.20)\\ &= \int_{\mathbf{B}=\mathcal{M}^{[2]}} 1.6\ \epsilon_o \|\mathbb{E}_j(t, \mathbf{x_j})\|^2\, dxdydz && (3.21)\\ &\equiv 1.6\ \epsilon_o \|\mathbb{E}_{oj}\|^2 \cdot \frac{4\pi r_{\mathbf{B}}^3}{3}, && (3.22) \end{aligned}$$

implying that (by equation (2.18)) $E_j^{[2]} = 0.4\ h\nu_j = 0.4\ \epsilon_o \left\|\mathbb{E}_{oj}\right\|^2 \cdot \frac{4\pi r_{\mathbf{B}}^3}{3}$, i.e.,

$$\left\|\mathbb{E}_{oj}\right\| \approx \sqrt{\frac{3h\nu_j}{4\pi\epsilon_o r_{\mathbf{B}}^3}}\ \ \forall \nu_j \gg \frac{1}{\text{second}}. \tag{3.23}$$

Finally we have $\mathbb{E}_{oj} \cdot \mathbf{x_j} = 0$ by the standard property that $\mathbb{E}\ \times$ the magnetic field $\mathbb{B}$ gives the direction of propagation of the electromagnetic wave. □

Remark 5. Since $\mathbb{R}/\left\langle 2\pi \right\rangle \approx$ the group of rotations, we have the corresponding isomorphism

$$\begin{aligned} & \mathbb{E}_{oj} \cdot \cos\left(\omega_j t - \mathbf{k}_j \cdot \mathbf{x_j} + \theta_j\right) && (3.24) \\ \approx\ & \mathbb{E}_{oj} \cdot e^{-i\left(\omega_j t - \mathbf{k}_j \cdot \mathbf{x_j} + \theta_j\right)} = \mathbb{E}_j\left(t, \mathbf{x_j}\right). && (3.25) \end{aligned}$$

In this way, the complex quantum electrodynamics can assume a real classical form.

Claim 1. Any particle $p \in \mathcal{M}^{[1]}$ is formed by a superposition of electromagnetic fields in $\mathcal{M}^{[2]}$; i.e., p has its distinct identity $\mathbb{E}_p\left(t, \mathbf{x}\right)$, with

$$\mathbb{E}_p\left(t, \mathbf{x}\right) = \sum_j \mathbb{E}_j\left(t, \mathbf{x_j}\right) = \begin{pmatrix} z_1\left(t, \mathbf{x}\right) \\ z_2\left(t, \mathbf{x}\right) \\ z_3\left(t, \mathbf{x}\right) \end{pmatrix} \in \mathbb{C}^3\left(t, \mathbf{x}\right), \tag{3.26}$$

i.e., composed of electromagnetic propagations through $\left(t, \mathbf{x}\right)$ of multiple directions $\{\mathbf{x_j}\} \subset \mathbb{R}^3$, multiple frequencies $\{\omega_j\}$, and multiple phases $\{\theta_j\}$ as introduced in Proposition 3.

Remark 6. We back up the above claim by the following considerations: (1) As is well known, traveling waves can sum to standing waves, and the sum of standing waves can approximate arbitrary functions by Fourier series. (2) Physically, the pair creation process of anti particles by photons such as

$$\gamma + \gamma \longrightarrow \textit{electron } e^- + \textit{positron } e^+, \tag{3.27}$$

has been well established cf. [7], pp. 164. In passing, we also note the possibility of engendering a new particle $\tilde{p}$ from an existing particle p via a field transformation

$$\Phi : \mathbb{E}_p\left(t, \mathbf{x}\right) \in \mathbb{C}^3\left(t, \mathbf{x}\right) \longmapsto \mathbb{E}_{\tilde{p}}\left(t, \mathbf{x}\right) \in \mathbb{C}^3\left(t, \mathbf{x}\right), \tag{3.28}$$

especially by the general principle of symmetry as associated with electric charge, spatial parity, and time direction.

Remark 7. As a reminder, the wave function Ψ of a particle p in quantum mechanics has the property

$$\begin{aligned} \left|\Psi\left(t, \mathbf{x}\right)\right|^2 &\equiv \left|e^{-i\theta} \cdot \Psi\left(t, \mathbf{x}\right)\right|^2 \quad \forall \theta \in [0, 2\pi) && (3.29) \\ &= \text{the probability density } \rho \text{ of } p \text{ at } \left(t, \mathbf{x}\right). && (3.30) \end{aligned}$$

Proposition 4. (Meaning of the Wave Function) Ψ *Invoking the above Claim 1, let a particle p of identity $\mathbb{E}_p(t,\mathbf{x}) \in \mathcal{M}^{[3]}$. Then the wave function Ψ_p of p is*

$$\begin{aligned} \Psi_p(t,\mathbf{x}) &= \alpha \cdot \|\mathbb{E}_p(t,\mathbf{x})\| \in \mathbb{C}, \\ \textit{for some constant } \alpha &\in \mathbb{C}. \end{aligned} \tag{3.31}$$

Proof. By Claim 1 and equation (3.7), we have

$$\mathbb{E}_p(t,\mathbf{x}) = \sum_j \mathbb{E}_j(t,\mathbf{x_j}) \tag{3.32}$$

$$= \sum_j \mathbb{E}_{oj}\ e^{-i\left(\omega_j t - \mathbf{k}_j \cdot \mathbf{x_j} + \theta_j\right)} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}_{(t,\mathbf{x})} \in \mathbb{C}^3; \tag{3.33}$$

thus,

$$\|\mathbb{E}_p(t,\mathbf{x})\| = \left(z_1^2 + z_2^2 + z_3^2\right)^{\frac{1}{2}}_{(t,\mathbf{x})} \in \mathbb{C} \tag{3.34}$$

(cf. e.g., [6], 221, and [12] for metrics on complex manifolds), but as indicated earlier in equation (3.19),

$$\epsilon_o \left|\|\mathbb{E}_p(t,\mathbf{x})\|\right|^2 \in \mathbb{R}$$

is known as the electromagnetic field energy density. Now considering it as an axiom that the probability density ρ of p at $(t,\mathbf{x})$ is proportional to the electromagnetic field energy density, we have by the above Remark 7

$$\rho = \left|e^{-i\theta} \cdot \Psi_p(t,\mathbf{x})\right|^2 = \beta \cdot \epsilon_o \left|\|\mathbb{E}_p(t,\mathbf{x})\|\right|^2, \tag{3.35}$$

for some $\theta \in [0, 2\pi)$ and some $\beta > 0$; i.e.,

$$\Psi_p(t,\mathbf{x}) = \alpha \cdot \|\mathbb{E}_p(t,\mathbf{x})\|, \tag{3.36}$$

$$\text{with } \alpha \equiv \sqrt{\beta \cdot \epsilon_o} \cdot e^{i\theta} \in \mathbb{C}. \tag{3.37}$$

□

Remark 8. As a well-known historical fact, Schrödinger had initially interpreted his $|\Psi_p(t,\mathbf{x})|^2$ as the electric charge density (cf. e.g., [3], III-21-6). Now the above Proposition 4 shows that his interpretation was not too different from ours. In fact, the vector potential $\mathbb{A}$ in classical electrodynamics is the same as the wave function Ψ in quantum mechanics, so that the solutions of Maxwell equations are identical to those of Schrödinger's equation (cf. [3], II-15-8 and 20-3, also III-21-6). Here we contend that with his equations as applied to free space, Maxwell in fact already gave a description of quantum fields $\subset \mathcal{M}^{[2]}$, even though the way by which he derived his equations in 1861 was based on the electrodynamics of charges in $\mathcal{M}^{[1]}$ (see, e.g., [11], 40-47); in short, his electromagnetic fields $(\mathbb{E}, \mathbb{B})$ have always been in the complex $\mathcal{M}^{[2]}$.

Remark 9. Denote by $\mathbb{F}(t,\mathbf{x}) \equiv \sum_i \mathbb{E}_{p_i}(t,\mathbf{x})$ = the aggregate quantum field in $\mathcal{M}^{[2]}$. Then as $\{p_i\} \subset \mathcal{M}^{[1]}$ engage in all the fundamental four forces, $\mathbb{F}(t,\mathbf{x})$ undergoes evolution, for which Schrödinger wrote down his equation in 1926, Dirac gave a (special) relativistic wave equation for the electron in 1928, introducing spin and positron, and Feynman formulated his path integral as based on his principle of least action in 1942. As is well known, the wave function $\Psi_{p_i}(t,\mathbf{x})$ of a single particle p_i is inadequate to account for the complexities of $\mathbb{F}(t,\mathbf{x})$; a more logical approach is to solve for $\mathbb{F}(t,\mathbf{x})$ first and then separate $\mathbb{E}_{p_i}(t,\mathbf{x})$ from $\mathbb{F}(t,\mathbf{x})$ (analogous to the idea of diffraction grating) to arrive at $\Psi_{p_i}(t,\mathbf{x}) = \alpha \cdot \|\mathbb{E}_{p_i}(t,\mathbf{x})\|$.

Remark 10. As a reminder, we note that the probability or uncertainty aspect of quantum mechanics refers to observations in $\mathcal{M}^{[1]}$; the wave dynamics in $\mathcal{M}^{[2]}$ is exact and deterministic.

4. Summary Remarks

In this paper we have settled the (particle, wave) duality as attributes of the combined particle, of energy

$$E^{[3]} = \frac{3}{4}E^{[3]} + \frac{1}{4}E^{[3]} = E^{[1]} + E^{[2]}. \tag{4.1}$$

$\mathcal{M}^{[2]}$, being a black hole in $\tilde{\mathcal{M}}^{[2]}$, is a complex 4-manifold. Moreover, the complex electric field as proved in Proposition 3, $\mathbb{E}_j(t,\mathbf{x_j}) = \mathbb{E}_{oj} \cdot e^{-i\left(\omega_j t - \mathbf{k}_j \cdot \mathbf{x_j} + \theta_j\right)}$, results in a quotient space, i.e., $\forall (t,\mathbf{x}) \in U - \{0,\mathbf{0}\}$ we have

$$t \equiv t_0 \left(\bmod \frac{2\pi}{\omega_j} \equiv \frac{1}{\nu_j} \right) \tag{4.2}$$

for some $t_0 \in \left[0, \frac{1}{\nu_j}\right]$, and

$$\mathbf{x} \equiv \mathbf{x_0} \left(\bmod \left(\frac{2\pi}{k_j}\right) \left(\frac{\mathbf{x}}{\|\mathbf{x}\|}\right) \equiv \lambda_j \left(\frac{\mathbf{x}}{\|\mathbf{x}\|}\right) \right) \tag{4.3}$$

for some $\mathbf{x}_0$ with $\|\mathbf{x}_0\| \in [0, \lambda_j]$; as such, $\forall \lambda_j \gtrapprox 0$ we have

$$t \tilde{\equiv} 0 \text{ and } \mathbf{x} \tilde{\equiv} \mathbf{0}, \tag{4.4}$$

resulting in "instantaneous communication" across U, which, among other things, accounts for the double-slit phenomenon: That is, to propagate γ_j along the direction of $(0,\mathbf{0})$ to, say,

$$\left(\frac{\sqrt{1+d^2}}{c}, 1 \text{ meter}, |d| \text{ meter (the "upper slit")}, 0 \right) \tag{4.5}$$

is nearly the same as via $(0, \mathbf{0})$ to $(0, 0, y > 0, 0)$, with

$$\left|\left|\left|\mathbb{E}_{oj} \cdot e^{-i(\omega_j t - k_j y)}\right|\right|\right|^2 = \|\mathbb{E}_{oj}\|^2 > 0, \tag{4.6}$$

i.e., a nonzero (constant) probability density ρ along the y-axis for γ_j to be observed; similarly, a switch to the lower slit

$$\left(\frac{\sqrt{1+d^2}}{c}, 1 \text{ meter}, -|d| \text{ meter}, 0\right) \tag{4.7}$$

is to result in the same conclusion. However, if both slits are open, then there exists a superposition of fields

$$\cos(\omega_j t - k_j y) + \cos(\omega_j t + k_j y) = 2\cos\omega_j t \ \cos k_j y \tag{4.8}$$

and the probability density of γ_j equals zero $\forall y$ such that $\cos k_j y = 0$.

Also, $\mathbb{F}(t, \mathbf{x}) \equiv \sum_i \mathbb{E}_{p_i}(t, \mathbf{x})$ exists in $\mathcal{M}^{[2]}$ as *one* quantum field; as such, $\{\mathbb{E}_{p_i}(t, \mathbf{x})\}$ are correlated or "entangled," displaying global behavior such as the celebrated Einstein-Podolski-Rosen ("EPR") phenomenon.

In closing, we contend that our construct of the combined space-time 4-manifold $\mathcal{M}^{[3]}$ provides quantum mechanics with a more complete geometric framework, which can resolve many outstanding conceptual and analytical problems. At the same time, we do envision a further development of our theory, to furnish more detailed analyses such as when a dark matter or energy $\left(0, E^{[2]}\right)$ becomes a combined particle of $\left(\frac{3}{4}E^{[2]}, \frac{1}{4}E^{[2]}\right) \in \mathcal{M}^{[3]}$.

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