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UNIVERSAL CENTRAL EXTENSIONS OF COMPLEX SIMPLE LIE ALGEBRAS EXTENDED OVER $\mathbb{C}[SL_2]$

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Abstract: In this paper, a vector bundle $SL_2(\mathbb{C})\times_{B_2}\mathbb{C}[B_2]$ over $X = SL_2/B_2$ with fiber $\mathbb{C}[B_2] = \mathbb{C}[t_1, t_2^{\pm 1}]$ is constructed, where $\mathbb{C}[SL_2]$ is the algebra of regular functions on the linear algebraic group $SL_2(\mathbb{C})$, and B_2 denotes a Borel subgroup of $SL_2(\mathbb{C})$. As well, an imbedding of $\mathbb{C}[SL_2]$ into $\mathbb{H}^0 = H^0(X, SL_2(\mathbb{C}) \times_{B_2}\mathbb{C}[B_2])$, the algebra of holomorphic sections of the previous bundle, is explicitly given. Moreover, an inclusion of the universal central extension $\widehat{\mathcal{G}}_{\mathbb{C}[SL_2]}$ of the Lie algebra $\mathbb{C}[SL_2]\times_{\mathbb{C}}\mathcal{G}$, with $\mathcal{G}$ a simple complex finite dimensional Lie algebra, into the universal central extension of the Lie algebra $\mathbb{H}^0$ is proved.

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1. Introduction

Let $\mathcal{G}$ be a finite dimensional simple Lie algebra over $\mathbb{C}$ with Killing form $< \cdot, \cdot >$, and let A be a commutative algebra over $\mathbb{C}$. Consider the Lie algebra $\mathcal{G}_A = A \otimes_{\mathbb{C}} \mathcal{G}$ with bracket $[a\otimes x, b\otimes y] = ab\otimes [x, y]$, $a, b \in A$, $x, y \in \mathcal{G}$. Let Ω_A be the A-module of differentials and $d : A \to \Omega_A$ the differential map. Thus d is linear and satisfies $d(ab) = adb + bda$. Let $- : \Omega_A \to \Omega_A/dA$ be the canonical linear map. Then, for $a, b \in A$ we have $\overline{d(ab)} = 0$. The Lie algebra $\widehat{\mathcal{G}}_A = A \otimes_{\mathbb{C}} \mathcal{G} \oplus \Omega_A/dA$ with multiplication defined by $[a\otimes x, b\otimes y] = ab\otimes[x, y] + \overline{(da)b} < x, y >$, and Ω_A/dA central in $\widehat{\mathcal{G}}_A$, is the universal central extension of the Lie algebra $A \otimes_{\mathbb{C}} \mathcal{G}$ (see [4] for details). We would like to in-

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vestigate the universal central extensions of these complex simple Lie algebras extended over the commutative algebra A, when A is the algebra of regular functions $\mathbb{C}[G]$ of a linear algebraic group G. If G is the additive group G_a, i.e. the subgroup of $SL_2(\mathbb{C})$ consisting of the matrices $\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$, $x \in \mathbb{C}$, then $A = \mathbb{C}[t]$, and Ω_A is a free A-module with basis dt (see [5]). Consequently, the description of Ω_A/dA is immediate, and so the universal central extensions of $\mathbb{C}[t] \otimes_{\mathbb{C}} \mathcal{G}$ is completely determined. If G is the multiplicative group $G_m = GL_1(\mathbb{C})$, then $A = \mathbb{C}[t, t^{-1}]$, and the untwisted affine Lie algebras can be realized as the universal central extension of the loop algebra $\mathbb{C}[t, t^{-1}] \otimes_{\mathbb{C}} \mathcal{G}$, the center being one dimensional (see [3]). Since a one dimensional connected linear algebraic group G is isomorphic to either G_a or G_m (see [1]), we get that the universal central extensions of Lie algebras of type $\mathbb{C}[G] \otimes_{\mathbb{C}} \mathcal{G}$ are completely described if G is a one dimensional connected linear algebraic group. From [1] we also know that any connected semisimple linear algebraic group G of rank one is isomorphic to either $SL_2(\mathbb{C})$ or $PGL_2(\mathbb{C})$, the quotient of $GL_2(\mathbb{C})$ by the one dimensional torus of scalar multiplications. The aim of this paper is to investigate the universal central extensions of Lie algebras of type $\mathbb{C}[SL_2] \otimes_{\mathbb{C}} \mathcal{G}$. Let B_2 be the Borel subgroup of $SL_2(\mathbb{C})$ defined by $\begin{bmatrix} t_2 & t_1 \\ 0 & t_2^{-1} \end{bmatrix}$, $t_1, t_2 \in \mathbb{C}$, $t_2 \neq 0$. Then $\mathbb{C}[B_2] = \mathbb{C}[t_1, t_2^{\pm 1}]$ is a B_2-module, the action given by conjugation. Thus, we can construct a vector bundle $SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]$ over $SL_2(\mathbb{C})/B_2 \simeq \mathbb{P}^1$ with fiber $\mathbb{C}[B_2]$, and prove the existence of an embedding of $\mathbb{C}[SL_2]$ into $\mathbb{H}^0 = H^0\left(SL_2/B_2, SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]\right)$, the algebra of holomorphic sections of this vector bundle. As a consequence, there is a description of the elements of $\mathbb{C}[SL_2]$ in term of those of $\mathbb{C}[t_1, t_2^{\pm 1}]$. Moreover, an imbedding of the universal central extension of the Lie algebra $\mathbb{C}[SL_2] \otimes_{\mathbb{C}} \mathcal{G}$ into the universal central extension of the Lie algebra $\mathbb{H}^0 \otimes_{\mathbb{C}} \mathcal{G}$ is proved.

2. The Imbedding of $\mathbb{C}[SL_2]$ into $H^0\left(SL_2/B_2, SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]\right)$

A group G is said to be a linear algebraic group if there is a structure of an affine variety on G and the maps $mult : G \times G \to G$ and $inv : G \to G$, corresponding to multiplication and taking inverses, are morphisms of varieties. This definition is intrinsic. A non-intrinsic one is as follows. A linear algebraic group G is a subgroup of some GL_n which is a closed subset of GL_n. Among the subgroups of a linear algebraic group G there are the so-called Borel subgroups B. They are defined as the closed, connected, solvable subgroups of G which are maximal for these properties. It is known [1] that if B is a Borel subgroup of G, then G/B is projective. Since all Borel subgroups of G are conjugate, G/B is independent of the choice of B (up to isomorphism). Now, let G be the linear algebraic group $SL_2(\mathbb{C}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc = 1,\ a, b, c, d \in \mathbb{C} \right\}$ and let B be the Borel subgroup $B_2 = \left\{ \begin{bmatrix} t_2 & t_1 \\ 0 & t_2^{-1} \end{bmatrix} : t_1, t_2 \in \mathbb{C},\ t_2 \neq 0 \right\}$.

Lemma 1. *Let* $\mathbb{C}[B_2] = \left\{ f : B_2 \to \mathbb{C},\ \begin{bmatrix} t_2 & t_1 \\ 0 & t_2^{-1} \end{bmatrix} \to f \begin{bmatrix} t_2 & t_1 \\ 0 & t_2^{-1} \end{bmatrix} \right\}$. *Then* $\mathbb{C}[B_2]$ *is a* B_2*-module, the action given by conjugation.*

Proof.

$$\begin{bmatrix} a & b \\ 0 & a^{-1} \end{bmatrix} \cdot f \begin{bmatrix} t_2 & t_1 \\ 0 & t_2^{-1} \end{bmatrix} = f \begin{bmatrix} t_2 & a^{-1} b\, t_2 + a^{-2} t_1 - b a^{-1} t_2^{-1} \\ 0 & t_2^{-1} \end{bmatrix}. \qquad \square$$

Since $\mathbb{C}[B_2] = \mathbb{C}[t_1, t_2^{\pm 1}]$, the action can be described as

$$\begin{bmatrix} a & b \\ 0 & a^{-1} \end{bmatrix} \cdot \begin{cases} t_1 \to a^{-1} b t_2 + a^{-2} t_1 - b a^{-1} t_2^{-1}, \\ \\ t_2 \to t_2. \end{cases}$$

Let $E = SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]$ be the vector bundle over $X = SL_2(\mathbb{C})/B_2$ with fiber $V = \mathbb{C}[B_2]$, and let U denote a neighborhood of X (see [6] for the notion of vector bundle). We call $s : U \to E$, a local section of E over U, if for each $x \in U$, $\pi \circ s =$identity on U. If $U = X$, we say that s is a (global) section of E. Let $\{U_i\}$ be a covering of X. A section s of E can be identified with a family $\{\rho_i\}$ of maps $\rho_i : U_i \to V$ such that $\rho_i = b_{ij}\rho_j$ on $U_i \cap U_j$. In fact, the maps $s_i = \varphi_i \circ s : U_i \to U_i \times V$ define maps $\rho_i : U_i \to V$. Since $\varphi_i^{-1} \circ s_i = s$, we have $s_i = \varphi_i \circ \varphi_j^{-1} \circ s_j$ which gives $\rho_i(x) = b_{ij}(x)\rho_j(x)$, for $x \in U_i \cap U_j$, the $\{b_{ij}(x)\}$ being the defining cocycle of the bundle E. Conversely, maps $\rho_i : U_i \to V$ with $\rho_i(x) = b_{ij}(x)\rho_j(x)$ for $x \in U_i \cap U_j$, define a section s of E by $s = \varphi_i^{-1} \circ s_i$ on U_i, $s_i(x) = (x, \rho_i(x))$. Identifying $X = SL_2/B_2$ with the complex projective space $\mathbb{P}^1$, and using the standard covering of $\mathbb{P}^1$, $\mathbb{P}^1 = U^+ \cup U^-$, where $U^+ = \mathbb{P}^1 \backslash \{\infty\} \simeq \mathbb{C}$ (with the coordinate z) and $U^- = \mathbb{P}^1 \backslash \{0\} \simeq \mathbb{C}$ (with the coordinate $\zeta = 1/z$), we have the following description by gluing of the vector bundle E: $SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2] = (U^+ \times \mathbb{C}[B_2]) \cup (U^- \times \mathbb{C}[B_2])$, $X = SL_2(\mathbb{C})/B_2 = U^+ \cup U^-$, with

$$s^+ : \mathbb{C} \simeq U^+,\ s^+(z) = \begin{bmatrix} 1 & 0 \\ z & 1 \end{bmatrix} B_2, \quad s^- : \mathbb{C} \simeq U^-,\ s^-(\zeta) = \begin{bmatrix} \zeta & 1 \\ -1 & 0 \end{bmatrix} B_2.$$

The gluing conditions, i.e. how elements of $U^+ \times \mathbb{C}[B_2]$ and $U^- \times \mathbb{C}[B_2]$ have to be identified over $U^+ \cap U^-$, are given by

$$(z,v) \sim (\zeta,w) \Leftrightarrow [(s^+(z),v)] = [(s^-(\zeta),w)] \Leftrightarrow z = -1/\zeta,\ v = \begin{bmatrix} \zeta & 1 \\ 0 & \zeta^{-1} \end{bmatrix} \cdot w,$$

i.e. $v(-1/\zeta) = \begin{bmatrix} \zeta & 1 \\ 0 & \zeta^{-1} \end{bmatrix} \cdot w(\zeta),\quad v,w \in \mathbb{C}[t_1,t_2^{\pm 1}]$. Since v and w are holomorphic functions, and the action of B_2 on $\mathbb{C}[B_2]$ is by conjugation, this means

$$\sum_{M,N} a_{N,M}(-1/\zeta)\, t_1^N\, t_2^M = \sum_{m,n} b_{n,m}(\zeta) \left(\zeta^{-1}(t_2 - t_2^{-1}) + \zeta^{-2} t_1\right)^n t_2^m, \qquad (1)$$

where $N,n \in \mathbb{Z}^{\geq 0}$, $M,m \in \mathbb{Z}$.

Proposition 1. *Let N_0 be the highest power of t_1 in the expression $\sum_{M,N} a_{N,M}(z) t_1^N t_2^M$ of (1). For all $M \in \mathbb{Z}$, we have $deg\, a_{K,M} \leq N_0 + K,\ 0 \leq K \leq N_0$.*

Proof. Let $N_0 = 0$. Then (1) gives

$$\begin{aligned} a_{0,M}(-1/\zeta) &= b_{0,M}(\zeta), \text{ for all } M \in \mathbb{Z}, \qquad (2) \\ b_{n,m}(\zeta) &= 0, \text{ for all } n \geq 1 \text{ and } m \in \mathbb{Z}. \end{aligned}$$

The equality in (2) implies that $deg\, a_{0,M} = 0$ for all $M \in \mathbb{Z}$. Moreover, since $z = -1/\zeta$, it follows from $a_{0,M}(z) = b_{0,M}(-1/z)$ that $deg\, b_{0,M} = 0$ for all $M \in \mathbb{Z}$. Let $N_0 = 1$. In this case (1) gives

$$\begin{aligned} a_{1,M}(-1/\zeta) &= \zeta^{-2} b_{1,M}(\zeta), \qquad (3) \\ a_{0,M}(-1/\zeta) &= b_{0,M}(\zeta) + (b_{1,M-1}(\zeta) - b_{1,M+1}(\zeta))\, \zeta^{-1}, \qquad (4) \\ b_{n,m}(\zeta) &= 0, \end{aligned}$$

for all $M,m \in \mathbb{Z}$ and $n \geq 2$. Writing (3) as $\zeta^2 a_{1,M}(-1/\zeta) = b_{1,M}(\zeta)$, we see that $deg\, a_{1,M} \leq 2$ for all $M \in \mathbb{Z}$. This fact and $a_{1,M}(z) = z^2 b_{1,M}(-1/z)$ imply $deg\, b_{1,M} \leq 2$ for all $M \in \mathbb{Z}$. Multiplying (4) by ζ we have $\zeta a_{0,M}(-1/\zeta) = P(\zeta)$. So $deg\, a_{0,M} \leq 1$ for all $M \in \mathbb{Z}$. Finally, working with z, and using (3) in (4), we get $Q(z) = z b_{0,M}(-1/z)$. Consequently, $deg\, b_{0,M} \leq 1$ for all $M \in \mathbb{Z}$. More generally, working with an arbitrary N_0, it follows from (1) that

$$\begin{aligned} a_{N_0,M}(-1/\zeta) &= \zeta^{-2N_0} b_{N_0,M}(\zeta), \qquad (5) \\ a_{N_0-j,M}(-1/\zeta) &= P_j(\zeta) + \zeta^{-2N_0+j} Q_j(\zeta),\ j = 1,2,\ldots,N_0, \qquad (6) \\ b_{n,m}(\zeta) &= 0, \text{ for all } n \geq N_0 + 1, \end{aligned}$$

for all $M,m \in \mathbb{Z}$, the $P_j(\zeta)$ and $Q_j(\zeta)$ being combinations of $b_{i,M}(\zeta)$. From (5), we obtain $deg\, a_{N_0,M} \leq 2N_0$ for all $M \in \mathbb{Z}$. Therefore, using $z = -1/\zeta$, we have $deg\, b_{N_0,M} \leq 2N_0$. Using these informations in (6), with $j = 1$, we derive $deg\, a_{N_0-1,M} \leq 2N_0 - 1$, and $deg\, b_{N_0-1,M} \leq 2N_0 - 1$. Proceeding in this way, we will have $deg\, a_{N_0-j,M} \leq 2N_0 - j$, with $j = 1, 2, \ldots, N_0$, and also $deg\, b_{N_0-j,M} \leq 2N_0 - j$. $\square$

Since a given section s of the vector bundle E becomes s^+ on U^+ and s^- on U^-, we get that s is described by expressions of the form $\sum\limits_{M,N} a_{N,M}(z)t_1^N t_2^M$ $(N \in \mathbb{Z}^{\geq 0},\ M \in \mathbb{Z})$, with the conditions of Proposition 1 on the $a_{N,M}$ if we are on the intersection $U^+ \cap U^-$.

Theorem 1. *Let* $\mathbb{H}^0 = H^0\left(X, SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]\right)$ *denote the space of holomorphic sections of the bundle* $SL_2(\mathbb{C}) \times_{B_2} \mathbb{C}[B_2]$. *Then there exists an imbedding* $\varphi : \mathbb{C}[SL_2] \hookrightarrow \mathbb{H}^0$.

Proof. Recall that $\mathbb{C}[SL_2] = \mathbb{C}[a,b,c,d]/(ad-bc-1)$. The map φ is defined by $\mathbb{C}[SL_2] \to \mathbb{C}[n(z)^{-1} \cdot B_2] \to \pi^{-1}(p),\ f \to f^{n(z)^{-1}}|_{B_2} \to f\left(n(z)\, C\, n(z)^{-1}\right)$, where $p \in X$, $C \in B_2$ and $n(z) = \begin{bmatrix} 1 & 0 \\ z & 1 \end{bmatrix}$. This yields $a_{11} \to \{z \to t_2 - zt_1\}$, $a_{12} \to \{z \to t_1\}$, $a_{21} \to \left\{z \to -z^2 t_1 + \left(t_2 - t_2^{-1}\right) z\right\}$, $a_{22} \to \left\{z \to zt_1 + t_2^{-1}\right\}$, i.e. $Im\, \varphi = \mathbb{C}\left[t_2 - zt_1, t_1, -z^2t_1 + (t_2 - t_2^{-1})z, zt_1 + t_2^{-1}\right]$. $\square$

3. The Imbedding of $\widehat{\mathcal{G}}_{\mathbb{C}[SL_2]}$ into $\widehat{\mathcal{G}}_{\mathbb{H}^0}$

Given a Lie algebra $\mathcal{G}$, a central extension of $\mathcal{G}$ consists of a pair $(\widehat{\mathcal{G}}, \pi)$, with $\widehat{\mathcal{G}}$ a Lie algebra and $\pi : \widehat{\mathcal{G}} \to \mathcal{G}$ a surjective homomorphism whose kernel lies in the center of $\widehat{\mathcal{G}}$. If it happens that for every central extension $(\tilde{\mathcal{G}}, \varphi)$ of $\mathcal{G}$ there exists a unique homomorphism $\psi : \widehat{\mathcal{G}} \to \tilde{\mathcal{G}}$ for which $\varphi \circ \psi = \pi$, then $\widehat{\mathcal{G}}$ is called a universal central extension of $\mathcal{G}$. In general, it is not true that a given Lie algebra admits a universal central extension. However, in the case the Lie algebra is perfect, i.e. equals to its derived algebra, this is always true (see [2]). Let $\mathcal{G}$ be a finite dimensional simple Lie algebra over $\mathbb{C}$ with Killing form $< \cdot, \cdot >$ and let A be a commutative algebra over $\mathbb{C}$. Let $\mathcal{G}_A = A \otimes_{\mathbb{C}} \mathcal{G}$ be the Lie algebra with bracket $[a \otimes x, b \otimes y] = ab \otimes [x,y], a, b \in A, x, y \in \mathcal{G}$. Since $\mathcal{G}$ is a finite dimensional simple Lie algebra, $\mathcal{G}_A$ is perfect and so it has a universal central extension $\widehat{\mathcal{G}}_A$. By Kassel's Theorem [3] this is given by the pair $(\widehat{\mathcal{G}}_A, \pi)$, $\widehat{\mathcal{G}}_A = A \otimes_{\mathbb{C}} \mathcal{G} \oplus \Omega_A/dA, \pi : \widehat{\mathcal{G}}_A \to A \otimes_{\mathbb{C}} \mathcal{G}$ (the canonical projection) with Lie bracket $[a \otimes x, b \otimes y]_{\widehat{\mathcal{G}}_A} = ab \otimes [x,y]_{\mathcal{G}} + \overline{(da)b} < x, y >$, $[\mathcal{G}, \Omega_A/dA]_{\widehat{\mathcal{G}}_A} = [\Omega_A/dA, \mathcal{G}]_{\widehat{\mathcal{G}}_A} = [\Omega_A/dA, \Omega_A/dA]_{\widehat{\mathcal{G}}_A} = 0$, where Ω_A/dA is the space of Kähler differentials of A modulo exact differentials. The module of differentials (Ω_A, d) of A is defined in the following way (see [5]). Let $\varphi : A \otimes_{\mathbb{C}} A \to A$ be the homomorphism $\varphi(a \otimes b) = ab$, and $I = Ker \varphi$. This is an ideal of $A \otimes A$ generated by the elements $a\otimes 1-1\otimes a$ ($a\in A$). The quotient algebra $A\otimes A/I$ is isomorphic to A. We define the module of differentials Ω_A by $\Omega_A = I/I^2$. This is an $A\otimes_{\mathbb{C}} A$-module, but since $I\Omega_A = 0$, we may view it as an A-module. Denote by da the image of $a\otimes 1 - 1\otimes a$ in Ω_A. This is the differential of a. It is immediate to check that d is a derivation of A in Ω_A, i.e. $d : A \to \Omega_A$ is a $\mathbb{C}$-linear map such that $d(ab) = adb + bda$, and that the da generate the A-module Ω_A. Moreover, $d(1) = 0$, whence $d(\lambda\cdot 1) = 0$ for all $\lambda \in \mathbb{C}$. Up to isomorphism (Ω_A, d) is characterized by the property that for any A-module M and every derivation $D : A \to M$ there is a unique A-module map $\varphi : \Omega_A \to M$ such that the diagram $\begin{array}{ccc} A & \xrightarrow{d} & \Omega_A \\ {}_D\searrow & & \swarrow_\varphi \\ & M & \end{array}$ commutes. In this way $Der_{\mathbb{C}}(A, M) \simeq Hom_A(\Omega_A, M)$.

Let $- : \Omega_A \to \Omega_A/dA$ be the canonical linear map. Note that $\overline{d(ab)} = 0$ implies $\overline{adb} = -\overline{(da)b} = -\overline{bda}$, for all $a, b \in A$.

Proposition 2. *Let $\mathcal{A}$ be the category of commutative algebras A over $\mathbb{C}$, and let $\mathcal{B}$ be the category of Lie algebras over $\mathbb{C}$ of the form $\widehat{\mathcal{G}}_A = A\otimes_{\mathbb{C}} \mathcal{G} \oplus \Omega_A/dA$. The map $F : \mathcal{A} \to \mathcal{B}$ defined by*

i) $A \rightsquigarrow \widehat{\mathcal{G}}_A$,

ii) $\{f : A \to B \text{ morphism}\} \rightsquigarrow \{F(f) : \widehat{\mathcal{G}}_A \to \widehat{\mathcal{G}}_B \text{ Lie morphism}\}$, *where* $F(f)\left[a\otimes x \oplus \overline{udv}\,\right] = f(a)\otimes x \oplus \overline{f(u)df(v)}$,

is a covariant functor.

Proof. Recall that an arbitrary element $a\otimes x \oplus \overline{udv}$ of $\widehat{\mathcal{G}}_A$ is of the form $\sum\limits_i \alpha_i a_i \otimes \sum\limits_j \beta_j x_j \oplus \sum\limits_k \overline{u_k dv_k}$, with $\{a_i\}$ a basis of A, $\{x_j\}$ a basis of $\mathcal{G}$ and $u_k, v_k \in A$ such that $\overline{d(u_k v_k)} = 0$. By the linearity of f, and the definition of $F(f)$, we obtain

$$F(f)\left[a\otimes x \oplus \overline{udv}, c\otimes y \oplus \overline{u'dv'}\,\right] = \left[F(f)\left(a\otimes x\oplus \overline{udv}\right), F(f)\left(c\otimes y \oplus \overline{u'dv'}\right)\right],$$

i.e. $F(f)$ is a Lie morphism. Similarly, we get $F(id_A) = id_{\widehat{\mathcal{G}}_A}$. So F is a functor. Moreover, it is covariant since $F(g\circ f) = F(g)\circ F(f)$. □

Proposition 3. *Let $f : A \to B$ be a surjective (resp. injective) morphism. Then $F(f) : \widehat{\mathcal{G}}_A \to \widehat{\mathcal{G}}_B$ is a surjective (resp. injective) Lie morphism.*

Proof. The statement follows from definitions and Proposition 2. □

Corollary 2. $\widehat{\mathcal{G}}_{\mathbb{C}[SL_2]} \hookrightarrow \widehat{\mathcal{G}}_{\mathbb{H}^0}$.

Proof. Immediate from Theorem 1 and Proposition 3. □

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