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NON-ISOTHERMAL EFFECT ON THE MOTION OF A PARTICULATE FLOW BETWEEN TWO PARALLEL PLATES

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Abstract: The non-isothermal flow of dusty, viscous, incompressible conducting fluid between two oscillatory parallel plates is studied. This flow is described by the continuity, momentum and the energy equation, which accounts for the heat and mass transfer. Closed-form solutions method initiated by the boundary conditions were obtained for zero and non-zero pressure gradient. These solutions show that the non-isothermal nature of the flow has led to a dramatic departure from the isothermal case (see Ajadi [1], Ganguly and Lahiri [6]). The solutions were further demonstrated graphically to elucidate the significance of parameters such as Prandtl number (P_r), the magnetic parameter (B_0) and the Grashoff number (G_r) on the velocity profiles.

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1. Introduction

The study of non-isothermal convective flow of a fluid-particles system between

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two parallel plates is still receiving attentions. This may be due to its wide range of application in diverse area of science and technology, such as aerospace technology, geothermal energy and area which involves heat and mass transfer. However, little is known about the effect of temperature on the flow behaviour of the particulate flow. In the absence of a radiative heat source (isothermal case), Saffman [8] examined the stability of laminar flow of a dusty gas, in order to see how dust may affect the critical Reynold's number for transition from laminar to turbulent flow. They derived the equation describing the motion of a gas carrying small dust particles and the equation satisfied by small disturbance of a steady laminar flow.

Furthermore, the motion of an isothemal dusty viscous incompressible fluid between two infinite parallel plates, where both plates are assumed to be oscillating harmonically with different amplitude and frequency in their own planes were considered by Ganguly and Lahiri [6]. They obtained a closed-form solutions for the velocities. Recently, Ajadi [1] examined the isothermal flow of dusty viscous incompressible conducting fluid under the influence of gravitational force. By using realistic approximation, closed-form solutions for the velocity of the fluid and particles were obtained. It was observed that only the velocity of the fluid is affected by gravity.

The influence of temperature resulting from heat transfer on a fluid particle system embedded between parallel plates is well documented. For instance, Uwanta [12] considered the oscillatory free convection flow of incompressible rigid conducting fluid, which contains suspended inert rigid spherical particles between two infinite plates. Using the Laplace transform, they showed that temperature has significant effect on the fluid velocity but has no effect on the particle velocity. Furthermore, Fareo [5] obtained closed-form solutions for isothermal and non-isothermal motion of dusty viscous incompressible fluid between two infinite oscillating plates. The solution for the non-isothermal case is a departure from the isothermal case. Soundalgekar and Bhat [9] also obtained an exact solution of fully developed flow of a viscous incompressible fluid in a porous medium between two vertical parallel plates. They showed the temperature and velocity profiles graphically. Lyubimov et al [7] considered the non-isothermal two phase flow in a closed cavity, where one of the phase is gas (or liquid) and another phase consist of solid particles. They examined the linear stability of the plane parallel flow between differently heated vertical plates for the constant gravity field. Singh et al [10] also examined the laminar convective flow of an incompressible, conducting, viscous fluid embedded with non-conducting dust particles through a vertical parallel plate channel in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of dust particles into account, when one plate of the channel is fixed and the other is oscillating in time. They examined the effect of various parameters on the velocities, skin friction and heat transfer.

The main objective of this paper is to examine the effect of a radiative heat source on a system of laminar, convective particulate flow through a parallel plate channel of non-conducting and oscillatory walls. By using an analytical framework, the effect of temperature on the fluid and particle velocity have been investigated for different parameter regimes and also demonstrated graphically.

2. Governing Equations

In cartesian coordinate system, we consider a two dimensional unsteady, incompressible plane viscous fluid between two parallel plates distance d apart. Let x-axis be along the flow of liquid at the fixed wall and y-axis perpendicular to it. A uniform magnetic field of strength $B_0(=\mu_e H_0)$ is applied perpendicular to the flow region.

In addition, the following assumptions are essential:

(i) the dust particles are spherical solid, non-conducting, equal size and uniformly distributed in the flow,

(ii) the interactions between the particles, chemical reactions between the solid and fluid are neglected,

(iii) insignificant particle concentration with constant density.

Thus, the conservation laws for the mass, momentum and energy can be written as

$$\frac{\partial U}{\partial t}+(U.\nabla)U=-\frac{1}{\rho}\nabla p-\frac{\sigma B_0^2 U}{\rho}+\nu\nabla^2 U+\frac{KN}{\rho}(V-U)+g\beta(T-T_0), \quad (2.1)$$

$$\nabla.U=0\,, \quad (2.2)$$

$$\frac{\partial V}{\partial t}+(V.\nabla)V=g\beta(T-T_0)+\frac{K}{m}(U-V), \quad (2.3)$$

$$\frac{\partial N}{\partial t}+\nabla.(NV)=0\,, \quad (2.4)$$

$$\frac{\partial T}{\partial t}+(U.\nabla)T=\lambda\nabla^2 T-\gamma(T-T_0), \quad (2.5)$$

where U and V denote the local velocity vectors of fluid and dust particles respectively, ρ is the density, p is the static fluid pressure, ν is the kinetic viscosity, N is the number of dust particles per unit volume and K is a resistance coefficient, B_0 is a constant magnetic field parameter, g is the acceleration due to gravity, m is the mass of the particles, T is the temperature, T_0 is the reference temperature, λ is the thermal conductivity and γ is the coefficient of heat transfer. Since motion is in the x-direction, using the analysis in Ajadi [1], the continuity equation (2.2) becomes

$$u = u(y), \quad v = v(y) \quad \text{and} \quad p = p(x).$$

Thus the equations (2.1)-(2.5), in one dimensional form bocome

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}u + \frac{KN}{\rho}(v-u) + g\beta(T-T_0), \tag{2.6}$$

$$\frac{\partial v}{\partial t} = g\beta(T-T_0) + \frac{K}{M}(u-v), \tag{2.7}$$

$$\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial y^2} - \gamma(T-T_0). \tag{2.8}$$

Due to the oscillatory nature of the walls, the relevant boundary condition related to this problem are

$$\begin{aligned} &u = a_1 e^{-i\lambda_1 t}, \quad y = 0, \qquad u = a_2 e^{-i\lambda_2 t}, \quad y = d, \\ &T = \sinh(\wedge y), \qquad\qquad\qquad\qquad t = 0, \end{aligned} \tag{2.9}$$

$$T = 0 \quad y = 0, \quad \text{and} \quad T = e^{-i\omega t}\sinh(\wedge d), \quad y = d, \qquad t > 0.$$

We non-dimensionalize the above equations using the variables,

$$x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad p' = \frac{h^2 p}{\rho\nu^2}, \quad t' = \frac{\nu}{h^2}t, \quad u' = \frac{uh}{\nu}, \quad T' = \frac{T-T_0}{T_w-T_0}, \quad v' = \frac{vh}{\nu}.$$

Thus, the equations (2.6)-(2.8) become

$$\frac{\partial u'}{\partial t'} = -\frac{dp'}{dx'} + \frac{\partial^2 u'}{\partial y'}^2 - \frac{\sigma B_0^2 h^2 u'}{\rho\nu} + \frac{KNh^2}{\rho\nu}(v'-u') + \frac{h^3 g\beta}{\nu^2}(T_w-T_0)T', \tag{2.10}$$

$$\frac{\partial v'}{\partial t'} = \frac{g\beta h^3}{\nu^2}(T_w-T_0)T' + \frac{Kh^2\nu}{m\nu}(u'-v'), \tag{2.11}$$

$$\frac{\partial T'}{\partial t'} = \frac{\lambda}{\nu\rho C_p}\frac{\partial^2 T'}{\partial y'}^2 - \frac{\gamma h^2}{\nu}T'. \tag{2.12}$$

After dropping primes in equations (2.10)-(2.12), we have

$$\frac{\partial u}{\partial t} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} - Mu + D_1(v-u) + G_r T, \tag{2.13}$$

$$\frac{\partial v}{\partial t} = G_r T + D_2(u-v), \tag{2.14}$$

$$\frac{\partial T}{\partial t} = \frac{1}{p_r}\frac{\partial^2 T}{\partial y^2} - QT, \tag{2.15}$$

where

$$M = \frac{\sigma B_0 h}{\rho\nu}, \quad D_1 = \frac{KNh^2}{\rho\nu}, \quad D_2 = \frac{Kh^2}{m\nu}, \quad G_r = \frac{g\beta h^3(T_w - T_0)}{\nu^2},$$

$$P_r = \frac{\nu\rho C_p}{\lambda}, \quad P_r = \frac{\nu\rho C_p}{\lambda}, \quad Q = \frac{\gamma h^2}{\nu} \text{ and } m = \frac{\rho}{N}.$$

The solution of (2.15) is

$$T(y,t) = e^{-i\omega t}\sinh(\wedge y), \quad \text{where } \wedge = \sqrt{P_r(Q - i\omega)}. \tag{2.16}$$

In order to solve (2.13) and (2.15) by analytical procedure, there is need to consider some particular cases of pressure gradient ($\frac{dp}{dx}$).

2.1. Constant Pressure ($\frac{dp}{dx} = 0$)

Taking derivative of (2.13), we have

$$u_{tt} = u_{yyt} - Mu_t + D_1(v_t - u_t) + G_r T_t \tag{2.17}$$

and expressing as

$$(u - v) = \frac{[-\frac{\partial p}{\partial x} + u_{yy} - Mu + G_r T - u_t]}{D_1} \qquad (2.17)a$$

(2.14) becomes

$$v_t = G_r T + \frac{D_2}{D_1}[-\frac{dp}{dx} + u_{yy} - Mu + G_r T - u_t]. \qquad (2.17)b$$

Substitute (2.17)a into (2.17), we have

$$u_{tt} = u_{yyt} - (M + D_1 + D_2)u_t + D_2 u_{yy} - D_2 Mu - D_2\frac{\partial p}{\partial x} + G_r[T_t + (D_1 + D_2)T]. \tag{2.18}$$

Assuming constant pressure (i.e $\frac{dp}{dx} = 0$), (2.18) reduces to

$$u_{tt} = u_{yyt} - (M + D_1 + D_2)u_t + D_2 u_{yy} - D_2 Mu + G_r[T_t + (D_1 + D_2)T]. \tag{2.19}$$

Due to the nature of the boundary conditions, we take ansatz in the form

$$u(y,t) = a_1 f(y)e^{-i\lambda_1 t} + a_2 g(y)e^{-i\lambda_2 t}. \tag{2.20}$$

Substituting (2.20) into (2.19), we have

$$-i\lambda_1^2 a_1 f e^{-i\lambda_1 t} - i\lambda_2^2 a_2 g e^{-i\lambda_2 t} = \left(-i\lambda_1 a_1 f'' e^{-i\lambda_1 t} - i\lambda_2 a_2 g'' e^{-i\lambda_2 t}\right) +$$

$$(M + D_1 + D_2)\left(i\lambda_1 a_1 f e^{-i\lambda_1 t} + i\lambda_2 a_2 g e^{-i\lambda_2 t}\right) + D_2\left(a_1 f'' e^{-i\lambda_1 t} + a_2 g'' e^{-i\lambda_2 t}\right)$$

$$-D_2M\left(a_1fe^{-i\lambda_1 t}+a_2ge^{-i\lambda_2 t}\right)+G_r[-i\omega+(D_1+D_2)]e^{-i\omega t}\sinh(\wedge y). \quad (2.21)$$

Assuming that assuming that $\lambda_1=\omega$ and collecting terms, we obtain

$$a_1e^{-i\lambda_1 t}: \quad -i\lambda_1^2a_1f=-i\lambda_1a_1f''+(M+D_1+D_2)i\lambda_1a_1f+a_1D_2Af''$$
$$-D_2Mf+\frac{G_r}{a_1}(D_1+D_2-i\omega)\sinh(\wedge y), \quad (2.22)$$

$$a_2e^{-i\lambda_2 t}: \quad -i\lambda_2^2g=-i\lambda_2g''+i(M+D_1+D_2)B\lambda_2g+D_2g''-D_2Mg. \quad (2.23)$$

After rearrangement, equations (2.22) and (2.23) may be written as

$$f''+\frac{[(i\lambda_1)(M+D_1+D_2)+(i\lambda_1^2)-D_2M]}{D_2-i\lambda_1}f$$
$$=\frac{G_r(i\omega-D_1-D_2)}{a_1(D_2-i\lambda_1)}\sinh(\wedge y) \quad (2.24)$$

and

$$g''+\frac{[(i\lambda_2)(M+D_1+D_2)+(i\lambda_2^2)]}{D_2-i\lambda_2}g=0, \quad (2.25)$$

which can be expressed as

$$f''+P^2f=G_rR\sinh(\wedge y), \quad (2.26)$$

$$g''+S^2g=0, \quad (2.27)$$

where,

$$P^2=\frac{i\lambda_1(M+D_1+D_2)+i\lambda_1^2-D_2M}{D_2-i\lambda_1}, \quad S^2=\frac{i\lambda_2(M+D_1+D_2)+i\lambda_2^2}{D_2-i\lambda_2},$$

$$R=\frac{(i\omega-D_1-D_2)}{a_1(D_2-i\lambda_1)}.$$

Combining (2.20) and (2.16)

$$a_1e^{-i\lambda_1 t}=a_1f(0)e^{-i\lambda_1 t}+a_2g(0)e^{-i\lambda_2 t}\Rightarrow f(0)=1, \quad g(0)=0,$$
$$a_2e^{-i\lambda_2 t}=a_1f(d)e^{-i\lambda_1 t}+a_2g(d)e^{-i\lambda_2 t}\Rightarrow g(d)=1, \quad f(d)=0.$$

Let the particular solution ($f=f_p$) of (2.26) be

$$f_p = A_1\cosh(\theta y)+B_1\sinh(\theta y),$$

then

$$f_p''=A_1\theta^2\cosh(\theta y)+B_1\theta^2\sinh(\theta y).$$

After subsitution equation (2.26) becomes

$$A_1\theta^2\cosh(\theta y)+B_1\theta^2\sinh(\theta y)+P^2(A_1\cosh(\theta y)+B_1\sinh(\theta y))=G_r\sinh(\wedge y).$$

Let $\theta = \wedge$,

$$A_1 = 0, \quad B_1 = \frac{G_r R}{P^2 + \wedge^2}.$$

Hence

$$f_p(y) = \frac{G_r R}{p^2 + \wedge^2} \sinh(\wedge y),$$

$$f_c(y) = A_2 \cos(Py) + B_2 \sin(Py).$$

Combining the homogeneous and particular solutions, we obtain

$$f = f_c + f_p = A_2 \cos(Py) + B_2 \sin(Py) + \frac{G_r R}{P^2 + \wedge^2} \sinh(\wedge y),$$

where A_2, B_2 are constant to be determined. The boundary conditions

$$f(0) = 1 \text{ and } f(d) = 0,$$

we have, $A_2 = 1$ and $B_2 = -\frac{\cos(Pd)}{\sin(Pd)} - \frac{G_r R}{P^2+\wedge^2}\frac{\sinh(\wedge d)}{\sin(Pd)}$.

$$f = \frac{\sin P(d-y)}{\sin Pd} + \frac{G_r R}{(P^2 + \wedge^2)\sin(Pd)}(\sinh(\wedge y)\sin(Pd) - \sinh(\wedge d)\sin(Py)). \quad (2.28)$$

Similarly,

$$g(y) = C \cos(Sy) + D \sin(Sy),$$

where C and D are constant to be determined. Using the boundary conditions,

$$g(0) = 0 \quad \text{and} \quad g(d) = 1,$$

we have $C = 0$ and $D = \frac{1}{\sin Sd}$. Hence,

$$g(y) = \frac{\sin Sy}{\sin Sd}. \quad (2.29)$$

Substituting (2.28) and (2.29) into (2.20), we obtain

$$u(y,t) = a_1 \left[\frac{\sin P(d-y)}{\sin Pd} + G_r R \left(\frac{\sinh(\wedge y)\sin(Pd) - \sinh(\wedge d)\sin(Py)}{(P^2+\wedge^2)\sin(Pd)}\right)\right] \times e^{-i\lambda_1 t} + a_2 \frac{\sin Sy}{\sin Sd} e^{-i\lambda_2 t}. \quad (2.30)$$

Substituting for u in $(2.17)a$ and simplifying, we obtain

$$v = \frac{a_1}{D_1}\Bigg[(M + D_1 - i\lambda_1 + P^2)\frac{\sin(d-y)P}{\sin Pd} + G_r \Bigg(\sinh(\wedge y) + \frac{R}{P^2+\wedge^2}(M + D_1 + \wedge^2 - i\lambda_1)$$

$$(\sinh(\wedge y)\sin(Pd)-\sinh(\wedge d)\sin(Py))]\,e^{-i\lambda_1 t}+a_2\left[\frac{(M+D_1+S^2-i\lambda_2)}{D_1}\right] \times\frac{\sin Sy}{\sin Sd}e^{-i\lambda_2 t}. \quad (2.31)$$

Solutions (2.30) and (2.31) may be simplified as

$$u(y,t)=a_1\left[\frac{\sin P(d-y)}{\sin Pd}+G_rX\left(\sinh(\wedge y)\sin(Pd)-\sinh(\wedge d)\sin(Py)\right)\right] times e^{-i\lambda_1 t}+a_2\frac{\sin Sy}{\sin Sd}e^{-i\lambda_2 t}, \quad (2.32)$$

$$v(y,t)=a_1\left[\frac{W\sin(d-y)P}{D_1\sin Pd}+\frac{G_r}{D_1}\sinh(\wedge y)+XY\left(\sinh(\wedge y)\sin(Pd)\right.\right. \left.\left.-\sinh(\wedge d)\sin(Py)\right)\right]e^{-i\lambda_1 t}+a_2\frac{Z\sin Sy}{D_1\sin Sd}e^{-i\lambda_2 t}, \quad (2.33)$$

where

$$W=M+D_1-i\lambda_1+P^2, \qquad X=\frac{R}{P^2+\wedge^2},$$
$$Y=D_1+M+\wedge^2-i\lambda_1 \ \text{ and } \ Z=M+D_1+S^2-i\lambda 2.$$

We note that these results are generalization of known ones in literature. In particular, the special case of isothermal flow, $T_0=T_w=T$(i.e $G_r=0$), we recovered the result of Ganguly and Lahiri [6] and Ajadi [1]. In particular, for lower fixed plate ($y=0$) and upper oscillating plate ($a_1=\lambda_1=0$), we recovered the results of Ganguly and Lahiri [6]

$$u(y,t)=a_2\frac{\sin sy}{\sin Sd}e^{-i\lambda_2 t} \ \text{ and } \ v(y,t)=\frac{a_2 Z}{D_1}\frac{\sin Sy}{\sin Sd}e^{-i\lambda_2 t}. \quad (2.34)$$

Conversely, case of lower oscillating plate ($y=0$) and upper fixed plate ($a_2=\lambda_1=0$)

$$u(y,t)=a_1\left[\frac{\sin P(d-y)}{\sin Pd}+G_rX(\sinh(\wedge y)\sin(Pd)-\sinh(\wedge d)\sin(Py))\right]e^{-i\lambda_1 t},$$

and

$$v(y,t)=a_1\left[\frac{W}{D_1}\frac{\sin(d-y)P}{\sin Pd}+\frac{G_r}{D_1}\sinh(\wedge y)\right. \left.+XY\left(\sinh(\wedge y)\sin(Pd)-\sinh(\wedge d)\sin(Py)\right)\right]e^{-i\lambda_1 t}. \quad (2.35)$$

The case in which both plates are non-oscillatory, we obtain the trivial solutions

$$u=0 \quad \text{and} \quad v=0. \quad (2.36)$$

3. Constant Pressure Gradient ($\frac{dp}{dx} = P_0$)

At constant pressure gradient, the special case (2.34) does not reflect the heat contribution (radiation). This may not be unconnected with the form of ansatz. Hence, we are constrained to adopt solutions of the form used by Soundalgekar et al [9] and Singh et al [10], where one plate is fixed ($y = 0$) and the other ($y = d$) is oscillating. At the fixed plate the no-slip condition is valid. Thus, non-dimensional initial and boundary conditions relevant to the problems are:

$$u(y,t) = V(y,t) = T(y,t) = 0, \quad y = 0, \quad t > 0,$$
$$u(y,t) = v(y,t) = T(y,t) = 1 + \epsilon e^{-i\omega t}, \quad y = d. \tag{3.1}$$

We also propose the form of solutions

$$u(y,t) = u_0(y) + \epsilon u_1(y) e^{-i\omega t},$$
$$v(y,t) = v_0(y) + \epsilon v_1(y) e^{-i\omega t},$$
$$T(y,t) = T_0(y) + \epsilon T_1(y) e^{-i\omega t},$$
$$\frac{dp}{dx} = P_0. \tag{3.2}$$

Substituting (3.2) into (2.13)-(2.14), we obtain

$$T_0'' - P_r Q T_0 = 0, \tag{3.3}$$
$$T_1'' - P_r(i\omega - Q)T_1 = 0, \tag{3.4}$$
$$G_r T_1 + D_2(u_1 - v_1) + i\omega v_1 = 0, \tag{3.5}$$
$$G_r T_0 + D_2(u_0 - v_0) = 0, \tag{3.6}$$
$$G_r T_0 + u_0'' - M u_0 + D_1(v_0 - u_0) = P_0 = \frac{dp}{dx} \tag{3.7}$$
$$G_r T_1 + u_1'' - M u_1 + D_1(v_1 - u_1) + i\omega u_1 = 0, \tag{3.8}$$

where

$$\begin{aligned} u_0 = u_1 = v_0 = v_1 = T_0 = T_1 = 0 \text{ on } \quad y = 0, \\ u_0 = u_1 = v_0 = v_1 = T_0 = T_1 = 1 \text{ on } \quad y = d. \end{aligned} \tag{3.9}$$

The solutions of (3.3) and (3.4) are

$$T_0 = \frac{\sinh \sqrt{P_r Q} y}{\sinh \sqrt{P_r Q} d}, \qquad T_1 = \frac{\sinh \sqrt{P_r(i\omega - Q)} y}{\sinh \sqrt{P_r(i\omega - Q)} d}.$$

Then

$$T(y,t) = \frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d} + \epsilon\frac{\sinh\sqrt{P_r(i\omega - Q)}y}{\sinh\sqrt{P_r(i\omega - Q)}d}e^{-i\omega t}. \tag{3.10}$$

Combining (3.6) and (3.7), we have

$$(v_0 - u_0) = \frac{G_r}{D_2}T_0,$$

$$G_rT_0 + u_0'' - Mu_0 + \frac{D_1}{D_2}(v_0 - u_0) = P_0,$$

$$u_0'' - Mu_0 = p_0 - G_r\left(1 + \frac{D_1}{D_2}\right)T_0 = P_0 - G_r\left(1 + \frac{D_1}{D_2}\right)\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d}. \tag{3.11}$$

The complimentary and particular solutions of (3.11) are

$$u_{0,c} = A_2e^{\sqrt{M}y} + B_2e^{-\sqrt{M}y}, \tag{3.12}$$

$$u_{0,p} = C_2\sinh\sqrt{P_rQ}y + D_2\cosh\sqrt{P_rQ}y + E_2, \tag{3.13}$$

where

$$C_2 = \frac{G_r(1 + \frac{D_1}{D_2})}{(M-1)\sinh\sqrt{P_rQ}d}, \quad D_2 = 0 \text{ and } E_2 = -\frac{P_0}{M}.$$

Hence,

$$u_0 = A_2e^{\sqrt{M}y} + B_2e^{-\sqrt{M}y} - \frac{P_0}{M} + \frac{G_r(1 + \frac{D_1}{D_2})\sinh\sqrt{P_rQ}y}{(M-1)\sinh\sqrt{P_rQ}d}. \tag{3.14}$$

Using the boundary conditions (3.9), we have

$$A_2 = \frac{1 - \frac{G_r(1+\frac{D_1}{D_2})}{M-1} - \frac{P_0}{M}(e^{-\sqrt{M}d} - 1)}{2\sinh\sqrt{M}d} \quad \text{and} \quad B_2 = \frac{P_0}{M} - A_2.$$

Hence, the solution of (3.11) becomes,

$$u_0 = [1 - \frac{G_r(1 + \frac{D_1}{D_2})}{M-1} - \frac{P_0}{M}(e^{-\sqrt{M}d} - 1)]\frac{\sinh\sqrt{M}y}{\sinh\sqrt{M}d} + \frac{P_0}{M}(e^{-\sqrt{M}y} - 1) + \frac{G_r(1 + \frac{D_1}{D_2})}{M-1}\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d}. \tag{3.15}$$

Inserting (3.15) into (3.6), we obtain

$$v_0 = u_0 + \frac{G_r}{D_2}T_0 = u_0 + \frac{G_r}{D_2}\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d},$$

$$v_0 = [1 - \frac{G_r(1 + \frac{D_1}{D_2})}{M-1} - \frac{P_0}{M}(e^{-\sqrt{M}d} - 1)]\frac{\sinh\sqrt{M}y}{\sinh\sqrt{M}d} + \frac{P_0}{M}(e^{-\sqrt{M}y} - 1)$$

$$+ G_r(\frac{1}{D_2}\frac{(1+\frac{D_1}{D_2})}{M-1})\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d}. \quad (3.16)$$

Similarly, combining (3.5) and (3.8), we have

$$u_1'' - Lu_1 = -G_r(1+\frac{D_1}{D_2-i\omega})\frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}, \quad (3.17)$$

where $L = (M + D_1 - \frac{D_1D_2}{D_2-i\omega} - i\omega)$. The complimentary and particular solutions become

$$u_{1,c} = A_3e^{\sqrt{L}y} + B_3e^{-\sqrt{L}y}, \quad (3.18)$$

$$u_{1,p} = E_3\sinh\sqrt{P_r(i\omega-Q)}y + F_3\cosh\sqrt{P_r(i\omega-Q)}y. \quad (3.19)$$

We combine $u_{1,c}$ and $u_{1,p}$, we have

$$u_1 = A_3e^{\sqrt{L}y} + B_3e^{-\sqrt{L}y} + \frac{G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{L}y}{\sinh\sqrt{L}d}, \quad (3.20)$$

where

$$F_3 = 0, \quad E_3 = -\frac{G_r(1+\frac{D_1}{D_2-i\omega})}{[L-Pr(i\omega-Q)]\sinh\sqrt{L}d}.$$

Applying the boundary conditions in (3.9) to (3.20), we have

$$u_1 = \frac{1-G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{L}y}{\sinh\sqrt{L}d} + \frac{G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}, \quad (3.21)$$

where

$$A_3 = -B_3 = \frac{1-G_r(1+\frac{D_1}{D_2-i\omega})}{[L-Pr(i\omega-Q)]2\sinh\sqrt{L}d}.$$

And from (3.5)

$$v_1 = \frac{D_2}{D_2-i\omega}u_1 + \frac{G_r}{D_2-i\omega}T_1,$$

which is explicitly written as

$$v_1 = \frac{D_2}{D_2-i\omega}\left[\frac{1-G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\right]\frac{\sinh\sqrt{L}y}{\sinh\sqrt{L}d} + \left[\frac{D_2G_r}{D_2-i\omega}(\frac{(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}) + \frac{G_r}{D_2-i\omega}\right]\frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}. \quad (3.22)$$

Therefore

$$u(y,t) = u_0 + \epsilon u_1 e^{-i\omega t},$$

$$u = \left[1 - \frac{G_r(1+\frac{D_1}{D_2})}{M-1} - \frac{P_0}{M}(e^{-\sqrt{M}d}-1)\right]\frac{\sinh\sqrt{M}y}{\sinh\sqrt{M}d} + \frac{P_0}{M}(e^{-\sqrt{M}y}-1) + \frac{G_r(1+\frac{D_1}{D_2})}{M-1}\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d}$$
$$+\epsilon\left[\frac{1-G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{L}y}{\sinh\sqrt{L}d} + \frac{G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}\right] \times e^{-i\omega t}, \quad (3.23)$$

$$v(y,t) = v_0 + \epsilon v_1 e^{-i\omega t},$$

$$v = \left[[1 - \frac{G_r(1+\frac{D_1}{D_2})}{M-1} - \frac{P_0}{M}(e^{-\sqrt{M}d}-1)\right]\frac{\sinh\sqrt{M}y}{\sinh\sqrt{M}d} + \frac{P_0}{M}(e^{-\sqrt{M}y}-1) + G_r(\frac{1}{D_2}\frac{(1+\frac{D_1}{D_2})}{M-1})\frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d}$$
$$+\epsilon\left[\left[\frac{D_2}{D_2-i\omega}\frac{1-G_r(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}\frac{\sinh\sqrt{L}y}{\sinh\sqrt{L}d}\right] + \left[\frac{D_2G_r}{D_2-i\omega}(\frac{(1+\frac{D_1}{D_2-i\omega})}{L-Pr(i\omega-Q)}) + \frac{G_r}{D_2-i\omega}\right] \times \frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}\right] e^{-i\omega t}, \quad (3.24)$$

$$T(y,t) = T_0 + \epsilon T_1 e^{-i\omega t},$$

$$T(y,t) = \frac{\sinh\sqrt{P_rQ}y}{\sinh\sqrt{P_rQ}d} + \epsilon\frac{\sinh\sqrt{P_r(i\omega-Q)}y}{\sinh\sqrt{P_r(i\omega-Q)}d}e^{-i\omega t}. \quad (3.25)$$

4. Skin Friction and Heat Transfer (Nusselt Number)

The shear stress at the boundaries may be evaluated from the skin friction τ_i, while the rate of heat transfer is estimated using the Nusselt number (N). In particular, skin frictions of the fluid (2.32), the particle (2.33) and the Nusselt number (2.16) at the boundaries $y = 0$ and $y = d$ are

$$\tau_1 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = a_1 \left[-P\frac{\cos dP}{\sin Pd} + G_r X(\wedge \sin Pd - P\sinh \wedge d)\right] e^{-i\lambda_1 t} + \frac{a_2 S}{\sin Sd} e^{-i\lambda_2 t}, \quad (4.1)$$

$$\tau_2 = \left(\frac{\partial u}{\partial y}\right)_{y=d} = a_1 \left[\frac{-P}{\sin Pd} + G_r X(\wedge \cosh \wedge d \sin Pd - P \sinh \wedge d \cos P\wedge)\right] \times e^{-i\lambda_1 t} + a_2 \frac{S\cos Sd}{\sin Sd} e^{-i\lambda_2 t}. \quad (4.2)$$

The skin friction of the dust particle,

$$\tau_3 = \left(\frac{\partial v}{\partial y}\right)_{y=0} = -a_1 \frac{PW}{D_1}\frac{\cos Pd}{\sin Pd} + \frac{G_r}{D_1}\left[\wedge + XY(\wedge \sin Pd - P\sinh \wedge d)\right] e^{-i\lambda_1 t} + \frac{a_2 ZS}{D_1 \sin Sd} e^{-i\lambda_2 t}, \quad (4.3)$$

$$\tau_4 = \left(\frac{\partial v}{\partial y}\right)_{y=d} = -a_1 \frac{PW}{D_1 \sin Pd} + \frac{G_r}{D_1}\left[\wedge \cosh \wedge d + XY(\wedge \cosh \wedge d \sin Pd - P \sinh \wedge d \cos Pd)\right] e^{-i\lambda_1 t} + \frac{a_2 ZS \cos Sd}{D_1 \sin Sd} e^{-i\lambda_2 t}. \quad (4.4)$$

The rate of heat transfer is

$$N_1 = \wedge e^{-i\omega t} \quad \text{and} \quad N_2 = \wedge e^{-i\omega t}\cosh(\wedge d). \quad (4.5)$$

As stated earlier, the above results are generalized forms of known ones. The special case of $a_1 = \lambda_1 = 0$, and $G_r = 0$, we recovered all the previous results for the τ_i and N in Ajadi [1] and Ganguly and Lahiri [6].

5. Conclussion and Discussion

We have examined the non-isothermal flow of a dusty incompressible conducting fluid between two oscillating parallel walls. We obtained closed-form solutions, which apparently show the heat contributions on the fluid and particle velocities. Our results show that temperature will definitely affect both the particle and fluid velocity silmultaneously, although at different degrees. In particular, for zero pressure gradient with solutions (2.30) and (2.31), Figures 1 and 2 represent the spatial (y) development of the fluid velocity (u), while Figures 3 and 4 represent the variations in particle velocity (v) with y for some time.

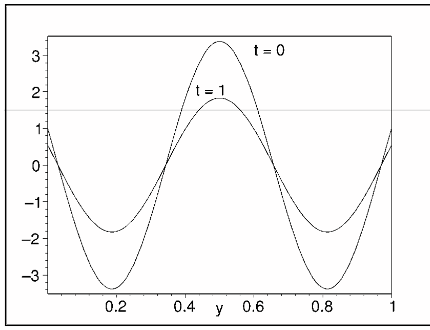

Fig. 1 u vs y $B_0 = 10$,$G_r = 0$

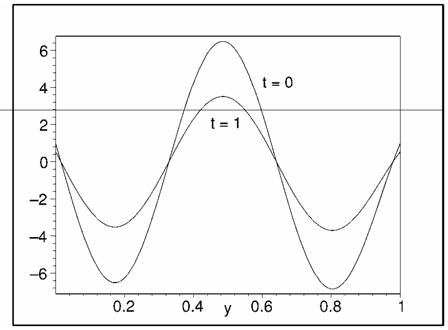

Fig. 2 u vs y $B_0 = 10$, $G_r = 5$

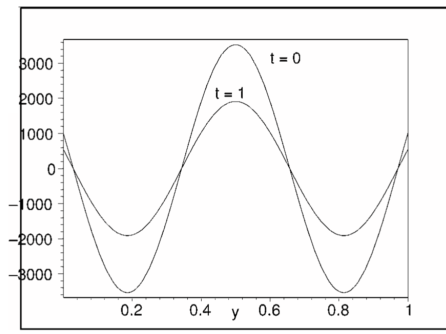

Fig. 3 v vs y $B_0 = 10$, $g = 10$, $G_r = 0$

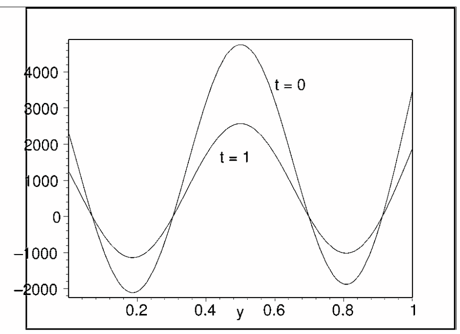

Fig. 4 v vs y $B_0 = 10$, $g = 10$, $G_r = 5$

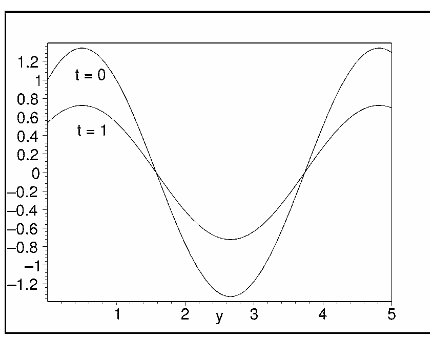

Fig. 5 u vs y $B_0 = 1$, $P_r = 0$, $G_r = 10$

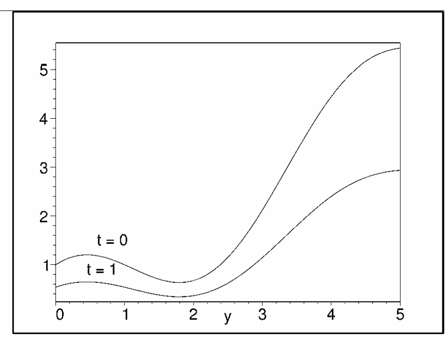

Fig. 6 u vs y $B_0 = 1$, $P_r = 0.005$, $G_r = 10$

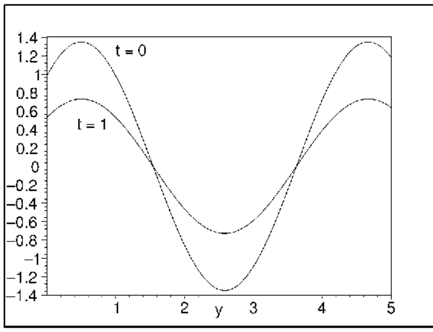

Fig. 7 v vs y $B_0 = 1$, $P_r = 0$, $G_r = 10$

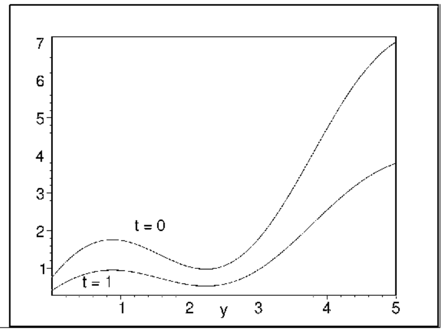

Fig. 8 v vs y $B_0 = 1$, $P_r = 0.005$, $G_r = 10$

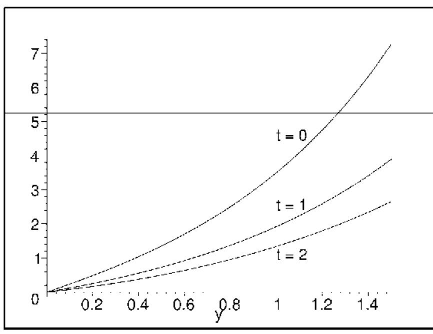

Fig. 9 u vs y $B_0 = 1$, $P_r = 5.63 \times 10\;\,^{4}$, $G_r = 0$

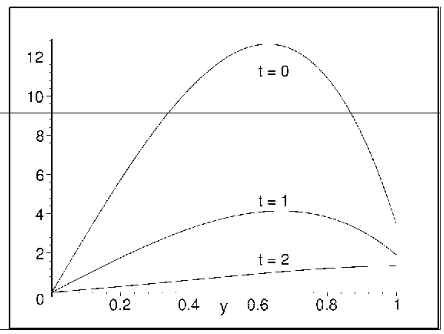

Fig. 10 u vs y $B_0 = 1$, $P_r = 5.63 \times 10\;\,^{4}$, $G_r = 44.4$

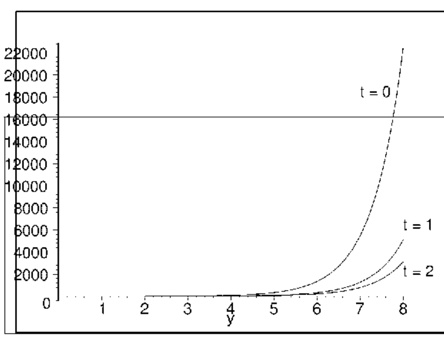

Fig. 11 v vs y $B_0 = 1$, $P_r = 5.63 \times 10\;\,^{4}$, $G_r = 0$

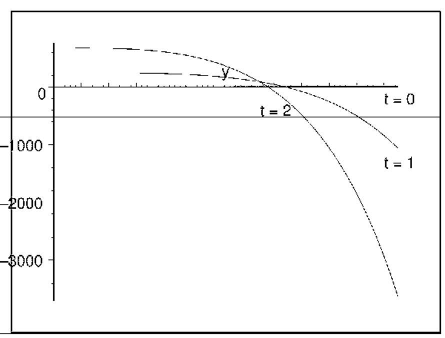

Fig. 12 v vs y $B_0 = 1$, $P_r = 5.63 \times 10^{-4}$, $G_r = 44.4$

It was revealed that the non-isothermal ($Gr \neq 0$) case has a higher velocity profiles than the isothermal case ($Gr = 0$). Similarly, Figures 5 and 6 represent the variation of the fluid velocity (u) with (y), while Figures 7 and 8 represent the variations in particle velocity (v) with y for some time. In the presence of Prandtl number ($Pr \neq 0$), the velocity profiles is higher than the case when $Pr = 0$. For non-zero pressure gradient, Figures 9 and 10, and Figures 11 and 12 are graphical demonstration of solutions (3.23) and (3.24) respectively. The behaviour of these solutions is also similar to the case of zero pressure gradient.

We have shown that when heat transfer is involved in a particulate Newtonian fluid system, the velocity of the fluid and particle will increase. This may be due to decrease in density resulting from temperature increase. In all the cases considered, and at a given space region, the velocities are monotonically decreasing function of time (t). The study would find place in some industrial applications such as such as conveyor belt system, hydraulic system, thermal explosion.

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